

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable. Any missing data may be assumed suitably with justification.*

**Note :** Answer Question No. 1 and any three from Group-A and any two from Group-B.

**1. Choose the correct answer for any ten of the following :**

$$10 \times 1 = 10$$

- (i) An open system is one in which
- Heat and work cross the boundary of the system, but the mass of the working substance does not
  - Mass of working substance crosses the boundary of the system but the heat and work do not.
  - Both the heat and work as well as mass of the working substances cross the boundary of the system.
  - Neither the heat and work nor the mass of the working substances crosses the boundary of the system.
- (ii) Which of the following is an intensive property of a thermodynamic system?
- Volume
  - Temperature
  - Mass
  - Energy.
- (iii) During throttling process
- internal energy does not change
  - pressure does not change
  - entropy does not change
  - enthalpy does not change
  - volume change is negligible.
- (iv) Kelvin-Planck's law deals with
- conservation of energy
  - conservation of heat
  - conservation of mass
  - conversion of heat into work
  - conversion of work into heat.
- (v) If a heat engine attains 100% thermal efficiency, it violates
- zeroth law of thermodynamics
  - first law of thermodynamics
  - second law of thermodynamics
  - law of conservation of energy.
- (vi) For an irreversible process entropy change is
- greater than  $\delta Q/T$
  - equal to  $\delta Q/T$
  - less than  $\delta Q/T$
  - equal to zero.
- (vii) The continuity equation (at two sections 1 and 2) for an incompressible fluid is given as
- $\rho_1 A_1 V_1^2 = \rho_2 A_2 V_2^2$
  - $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$
  - $A_1 V_1 = A_2 V_2$
  - $\rho_1^2 A_1 V_1 = \rho_2^2 A_2 V_2$
- (viii) Euler's equation is written as
- $\frac{dp}{\rho} + v^2 \cdot dv + g \cdot dz = 0$
  - $\frac{dp}{\rho} + v \cdot dv + g \cdot dz = 0$

$$(c) \frac{dp}{\rho} + v^2 \cdot dv + g^2 \cdot dz = 0 \quad (d) \frac{dp}{\rho} + v^2 \cdot dv + g \cdot dz = 0$$

(ix) Kinematics viscosity is defined as equal to

(a) dynamic viscosity  $\times$  density (b) dynamic viscosity/density (c) dynamic viscosity/pressure (d) pressure  $\times$  density.

(x) For pipes, turbulent flow occurs when Reynolds number is

(a) less than 2000 (b) between 2000 and 4000 (c) more than 4000 (d) less than 4000.

(xi) The change of entropy when heat is absorbed by the gas is

(a) positive (b) negative (c) positive or negative (d) none of these.

(xii) Spherical shape of droplets of mercury is due to

(a) high density (b) high surface tension (c) high adhesion (d) low vapour pressure.

Ans. (i)(c) ; (ii)(b) ; (iii)(d) ; (iv)(d) ; (v)(c) ; (vi)(a) ; (vii)(c) ; (viii)(b) ; (ix)(b) ; (x)(c) ; (xi)(a) ; (xii)(b).

### Group-A

(Answer any three)

2..(a) Explain intensive property. 2

(b) Prove that for polytropic process  $W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-n}$  4

(c) A gas is enclosed in a piston-cylinder assembly as a system. The gas is initially at a pressure of 500 kPa and occupies a volume of 0.2 m<sup>3</sup>. The gas is taken to the final state where  $P_2 = 100$  kPa by following two different processes. Calculate the work done by the gas in each case.

(i) The volume of the gas is inversely proportional to the pressure

(ii) The process follows the path  $PV^\gamma = \text{constant}$  where  $\gamma = 1.4$ . 6

Ans. (a) Thermodynamic properties which are independent of the mass of the system are called intensive properties. Example : pressure, temperature.

Ans. (b) For a polytropic process,  $PV^n = k$

where,  $k = \text{constant}$ .

$$\text{Work done} = W = \int P \, dv.$$

$$= \int_{v_1}^{v_2} k v^{-n} \, dv = k \left[ \frac{V^{1-n}}{1-n} \right]_{v_1}^{v_2} = k \frac{1}{1-n} \left[ v_2^{1-n} - v_1^{1-n} \right]$$

$$= \frac{1}{1-n} \left[ k v_2^{1-n} - k v_1^{1-n} \right] = \frac{1}{1-n} \left[ v_2 \cdot k v_2^{-n} - v_1 \cdot k v_1^{-n} \right] = \frac{1}{1-n} \left[ P_2 V_2 - P_1 V_1 \right]$$

Ans. (c) Initial Pressure =  $P_1 = 500$  kPa

Initial Volume =  $V_1 = 0.2$  m<sup>3</sup>.

Final Pressure =  $P_2 = 100$  kPa.

(i)  $V \propto \frac{1}{P}$  i.e.,  $PV = k$  [  $k = \text{constant}$  ]

$$\text{Work done} = W = \int P \, dv$$

$$\text{Final volume} = V_2$$

$$= \int_{v_1}^{v_2} \frac{k}{v} \, dv$$

$$P_1 V_1 = P_2 V_2$$

$$= k [\ln V]_{v_1}^{v_2}$$

$$v_2 = \frac{P_1 V_1}{P_2}$$

$$= k [\ln v_2 - \ln v_1]$$

$$= \frac{500 \times 0.2}{100}$$

$$= k \ln \frac{v_2}{v_1}$$

$$= 1 \text{ m}^3$$

$$= 100 \ln \left( \frac{1}{0.2} \right)$$

$$k = P_1 V_1 = P_2 V_2 = 500 \times 0.2$$

$$\therefore W = 100 \ln 5 \text{ kJ} = 160.94 \text{ kJ}$$

$$= 100$$

(ii)  $PV^\gamma = k$

$$\text{Work done} = W = \int P \, dv$$

$$\text{Final volume} = V_2$$

$$= k \int_{v_1}^{v_2} V^{-\gamma} \, dv$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$= k \left[ \frac{V^{1-\gamma}}{1-\gamma} \right]_{v_1}^{v_2}$$

$$V_2 = \left( \frac{P_1 V_1^\gamma}{P_2} \right)^{\frac{1}{\gamma}}$$

$$= \frac{k}{1-\gamma} [V_2^{1-\gamma} - V_1^{1-\gamma}]$$

$$= 0.63 \text{ m}^3$$

$$= \frac{1}{1-\gamma} [V_2 \cdot k V_2^{-\gamma} - V_1 \cdot k \cdot V_1^{-\gamma}]$$

$$= \frac{1}{1-\gamma} [V_2 P_2 - V_1 P_1] = \frac{1}{1-1.4} [0.63 \times 100 - 0.2 \times 500] \therefore W = 92.5 \text{ kJ}$$

3. (a) Explain first law of thermodynamics for a closed system undergoing a change of state.
- (b) A turbine operates under steady flow conditions, receiving steam at the following state :  
 Pressure 1.2 MPa, Temperature 188°C. Enthalpy 2785 kJ/kg. Velocity 33.3 m/sec and elevation 3m. The steam leaves the turbine at the following state :  
 Pressure 20 kPa, Enthalpy 2512 kJ/kg, Velocity 100 m/sec and elevation 0m. Heat is lost to the surroundings at the rate of 0.29 kJ/kg. If the rate of steam flow through the turbine is 0.42 kg/sec. What is the power output of the turbine in kW?

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**Ans. (a)** If a system undergoes a change of state during which both heat transfer and work transfer are involved, the net energy transfer is accumulated within the system.

If  $Q$  is the amount of heat transferred to the system and  $W$  is the amount of work transferred from the system during the process, the net energy transfer ( $Q - W$ ) is stored in the system.

Energy in storage is neither heat nor work, known as internal energy.

$$Q - W = \Delta E$$

$\Delta E$  = internal energy = increase in the energy of the system.

**Ans. (b)** Given,

$$P_i = 1.2 \text{ MPa}$$

$$P_e = 20 \text{ kPa}$$

$$= 1200 \text{ kPa}$$

$$h_e = 2512 \text{ kJ/kg}$$

$$t_i = 188^\circ\text{C}$$

$$C_e = 100 \text{ m/s}$$

$$h_i = 2785 \text{ kJ/kg}$$

$$Z_e = 0 \text{ m}$$

$$C_i = 33.3 \text{ m/s}$$

$$Z_i = 3 \text{ m}$$

where,  $P$  = Pressure ;  $t$  = temp.

$$\dot{Q} = -0.29 \text{ kJ/s}$$

$h$  = enthalpy ;  $C$  = speed

$$\dot{m}_i = \dot{m}_e = \dot{m} = 0.42 \text{ kg/s}$$

$z$  = elevation

The turbine necessarily undergoes one steady flow process. The energy balance equation for a steady flow process is :

$$0 = \dot{Q} - \dot{W} + \dot{m} \left[ (h_i - h_e) + g(z_i - z_e) + \frac{C_i^2 - C_e^2}{2} \right]$$

$$\Rightarrow 0 = -0.29 - \dot{W} + 0.42 \left[ (2785 - 2512) + 9.8(3 - 0) \times 10^{-3} + \left( \frac{33.3^2 - 100^2}{2} \times 10^{-3} \right) \right]$$

$$\therefore \dot{W} = 112.51 \text{ kW}$$

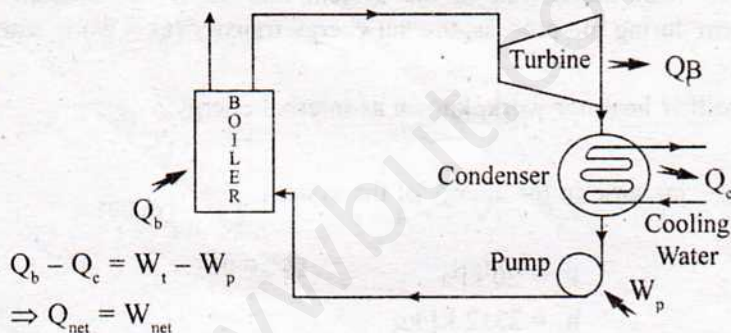
4. (a) What is cyclic heat engine? 4

(b) A Carnot cycle has an efficiency of 32%. Assuming that the lower temperature is kept constant, determine the percentage increase of the upper temperature of the cycle if the cycle efficiency is raised to 48%. 8

**Ans. (a)** A heat engine cycle is a thermodynamic cycle in which there is a net heat transfer to the system and a net work transfer from the system. The system which executes a heat engine cycle is called a cyclic heat engine. A simple steam power plant is an example of a

cyclic heat engine. For a cyclic heat engine, the first law of thermodynamics is  $\sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$

A schematic block diagram of a simple steam power plant is shown.



**Ans. (b)**  $1 - \frac{T_2}{T_1} = 0.32$  and  $1 - \frac{T_2}{T_1'} = 0.48$

$$\therefore T_1 = \frac{T_2}{0.68} \text{ and } T_1' = \frac{T_2}{0.52}$$

[  $T_2$  = lower temp,  $T_1$  = upper temp.,  $T_1'$  = revised upper temp. ]

Percentage increase in upper temp.

$$= \frac{T_1' - T_1}{T_1} \times 100 = \left( \frac{T_1'}{T_1} - 1 \right) \times 100 = \left( \frac{0.68}{0.52} - 1 \right) \times 100 = \frac{0.16}{0.52} \times 100 = 30.77\%$$

5. (a) Write the principle of entropy. 3

(b) Calculate the entropy change of 10 gm of water at  $20^\circ\text{C}$ , when it is converted to ice at  $-10^\circ\text{C}$ . Assume the specific heat of water to remain constant at  $4.2 \text{ J/gmK}$  and that of ice to be half of the value and taking the latent heat of fusion of ice at  $0^\circ\text{C}$  to be  $335 \text{ J/gm}$ . 9

**Ans. (a)** For any infinitesimal process undergone by a system, for the total mass  $ds \geq \frac{dQ}{T}$ .

For an isolated system,  $ds_{\text{iso}} \geq 0$ , [ as  $dQ = 0$  ]

∴ For a reversible process,  $ds_{iso} = 0 \Rightarrow s = \text{constant}$ .

For an irreversible process,  $ds_{iso} > 0$ .

So the entropy of an isolated system can never decrease. This is known as principle of increase of entropy, or simply, the entropy principle.

**Ans. (b)** Entropy decrease of water as it is cooled from  $20^{\circ}\text{C}$  to  $0^{\circ}\text{C}$ .

$$\begin{aligned} \Delta s_1 &= s_2 - s_1 = mc_p \ln \frac{T_2}{T_1} \\ &= 10 \times 4.2 \ln \frac{273}{293} \\ &= -2.97 \text{ J/k.} \end{aligned}$$

Entropy decrease of water as it freezes into ice at  $0^{\circ}\text{C}$ ,

$$\Delta s_2 = s_3 - s_2 = -\frac{335 \times 10}{273} = -12.27 \text{ J/k}$$

Entropy decrease of ice as it is cooled from  $0^{\circ}\text{C}$  to  $10^{\circ}\text{C}$ ,

$$\begin{aligned} \Delta s_3 &= s_4 - s_3 = \int \frac{dQ}{T} = \int_{273}^{263} \frac{mC_p dT}{T} \\ &= mC_p \ln \frac{263}{273} \\ &= 10 \times 2.1 \ln \frac{263}{273} = -0.78 \text{ J/k} \end{aligned}$$

∴ Total entropy change of the system

$$\begin{aligned} &= -(2.97 + 12.27 + 0.78) \\ &= -16.02 \text{ J/k} \end{aligned}$$

**6. (a)** Prove that entropy change for an ideal gas

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

**(b)** A vessel of volume  $0.04 \text{ m}^3$  contains a mixture of saturated water and saturated steam at a temperature of  $250^{\circ}\text{C}$ . The mass of the liquid present is  $9 \text{ kg}$ . Find the pressure, the mass, the specific volume, the enthalpy, the entropy and the internal energy.

**Ans. (a)** By second law of thermodynamics,  $\delta Q_{rev} = T ds$

And by first law of thermodynamics,  $\delta Q_{rev} = \delta W_{rev} + du$

$T ds = du + P dv$  .....(1) (For reversible process)

Now, we know,  $H = u + p v$  [  $H = \text{Enthalpy}$  ]

$dH = du + P dv + V dP = T ds + V dP$  [ From 1 ]

$$T ds = dH - V dP \dots\dots\dots (2)$$

Now, from the definition of the specific heats,

$$C_p = \frac{1}{m} \left( \frac{\partial H}{\partial T} \right)_p \quad [ \text{The final relation involves } C_p \text{ only} ]$$

$$C_p = \left( \frac{\partial H}{\partial T} \right)_p \dots\dots\dots (3)$$

From (2) & (3)

$$T ds = C_p dT - V dP$$

$$ds = C_p \frac{dT}{T} - \frac{V}{T} dP \quad \left[ \text{We know, } PV = RT, \frac{V}{T} = \frac{R}{P} \right]$$

$$ds = C_p \frac{dT}{T} - \frac{V}{T} dP$$

Integrating,

$$\int_1^2 ds = C_p \int_1^2 \frac{dT}{T} - R \int_1^2 \frac{dP}{P}$$

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

**Ans. (b)** Given :  $V = 0.04 \text{ m}^3$ ,  $m_f = 9 \text{ kg}$ ,  $t = 250^\circ\text{C}$ .

Contains a mixture of standard steam and saturated water.

On consulting the temperature index steam we get,

At  $t = 250^\circ\text{C}$

$$P_{\text{sat}} = 3973.0 \text{ kPa}$$

$$V_f = 0.001251 \text{ m}^3/\text{kg}$$

$$V_f = 1080.37 \text{ kJ/kg}$$

$$V_{fg} = 0.04887 \text{ m}^3/\text{kg}$$

$$V_{fg} = 1522.00 \text{ kJ/kg}$$

$$V_g = 0.05013 \text{ m}^3/\text{kg}$$

$$V_g = 2602.37 \text{ kJ/kg}$$

$$h_f = 1085.34 \text{ kJ/Kg}$$

$$S_f = 2.7927 \text{ kJ/Kgk}$$

$$h_{fg} = 1716.18 \text{ kJ/Kg}$$

$$s_{fg} = 3.2802 \text{ kJ/Kgk}$$

$$h_g = 2801.52 \text{ kJ/Kg}$$

$$s_g = 6.0729 \text{ kJ/Kgk}$$

Therefore,  $V_f = m_f v_f$  [  $V_f = \text{vol. of liquid phase}$  ]

$$= (9 \times 0.001251) \text{ m}^3$$

$$= 0.01126 \text{ m}^3$$

$$\therefore v_g = v - v_f = (0.04 - 0.01126) \text{ m}^3 = 0.02874 \text{ m}^3$$

$$m_g = \frac{V_g}{v_g} = \frac{0.02874}{0.05013} = 0.5733 \text{ kg}$$

∴ The total mass,  $m = m_f + m_g = (9 + 0.5733) \text{ kg} = 9.5733 \text{ kg}$ .

Therefore, the dryness fraction of the mixture  $x = \frac{m_g}{m_f + m_g} = \frac{0.5733}{9.5733} = 0.0598$

Now, Enthalpy :  $h = h_f + x h_{fg}$   
 $= 1085.34 + 0.0598 \times 1716.18 = 1187.967 \text{ kJ/kg}$ .

Volume :  $v = v_f + x v_{fg}$   
 $= 0.001251 + 0.0598 \times 0.04887 = 0.004173 \text{ kJ/kg}$

Internal energy :  $u = u_f + x u_{fg}$   
 $= 1080.37 + 0.0598 \times 1522.00 = 1171.3856 \text{ kJ/kg}$ .

Entropy :  $s = s_f + x s_{fg}$   
 $= 2.7927 + 0.0598 \times 3.2802 = 2.988 \text{ kJ/kgk}$ .

### Group-B

(Answer any two questions)

7. (a) State Newton's law of viscosity. 2  
 (b) What is Bulk Modulus of Elasticity? 2  
 (c) The space between two large flat and parallel walls 25mm apart is filled with a liquid of absolute viscosity  $0.7 \text{ Ns/m}^2$ . Within this space a thin flat plate, 250 mm  $\times$  250 mm is towed at a velocity of 150 mm/s at a distance of 6 mm from one wall, the plate and its movement being parallel to the walls. Assuming linear variations of velocity between the plate and the walls, determine the force exerted by the liquid on the plate. 8

Soln. : (a) Newton's law of viscosity :

The viscous force  $F$  acting tangentially on a layer of the liquid is proportional to the area of the layer ( $A$ ) and the velocity gradient  $\frac{dv}{dx}$ . ∴  $F \propto A \frac{dv}{dx}$ .

∴  $F = -\eta A \frac{dv}{dx}$ ,  $\eta$  = co-efficient of viscosity of the liquid.

Soln. : (b) Bulk modulus of elasticity : within elastic limit, it is the ratio of normal stress to volumetric strain.

If by applying a pressure  $dp$ , the decrease in the fluid volume is  $dv$ , then the bulk modulus of elasticity is,  $E_v = -\frac{dp}{dv/v}$ .

$v$  = original fluid volume.

Soln. : (c)  $F_1$  = shear force on the upper side of the plate.

$F_2$  = shear force on the lower side of the plate.

$F$  = Total force exerted by the liquid on the plate



$A =$  Area of the flat plate  $= 250 \text{ mm} \times 250 \text{ mm}$ .

Shear stress on the upper side of the flat plate

$$= \tau_1 = \mu \left( \frac{du}{dy} \right)_{y=6 \text{ mm}} = 0.7 \times \frac{150}{6} \text{ N/m}^2$$

$$F_1 = \tau_1 \times A$$

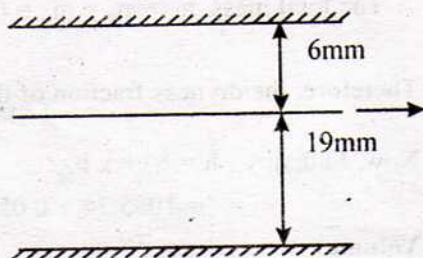
Shear stress on the lower side of the flat plate

$$= \tau_2 = \mu \left( \frac{du}{dy} \right)_{y=19 \text{ mm}} = 0.7 \times \frac{150}{19} \text{ N/m}^2$$

$$F_2 = \tau_2 \times A$$

$$F = F_1 + F_2 = A(\tau_1 + \tau_2)$$

$$= 250 \times 250 \times 10^{-6} \times 0.7 \left( \frac{150}{6} + \frac{150}{19} \right) \text{ N} = 1.43 \text{ N}.$$



8. (a) The velocity distribution for a two-dimensional incompressible flow is given by

$$u = -\frac{x}{x^2 + y^2}, \quad v = -\frac{y}{x^2 + y^2}. \quad \text{Show that it satisfies continuity.} \quad 4$$

(b) Water is flowing through two different pipes A and B to which an inverted differential manometer having an oil of sp. gr. 0.9 is connected. The pressure in the pipe A is 2.5m of water. Find the pressure in the pipe B for the manometer readings as shown in Fig. 1. 8

Soln. : (a)  $u = -\frac{x}{x^2 + y^2}$   $v = -\frac{y}{x^2 + y^2}$

$$\frac{\partial u}{\partial x} = -\frac{(x^2 + y^2) - 2x^2}{(x^2 + y^2)^2} = -\frac{y^2 - x^2}{(x^2 + y^2)^2} = -\frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$\frac{\partial v}{\partial y} = -\frac{(x^2 + y^2) - 2y^2}{(x^2 + y^2)^2} = -\frac{x^2 - y^2}{(x^2 + y^2)^2} = \frac{y^2 - x^2}{(x^2 + y^2)^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{(x^2 - y^2)}{(x^2 + y^2)^2} + \frac{(y^2 - x^2)}{(x^2 + y^2)^2} = 0.$$

Soln. : (b)

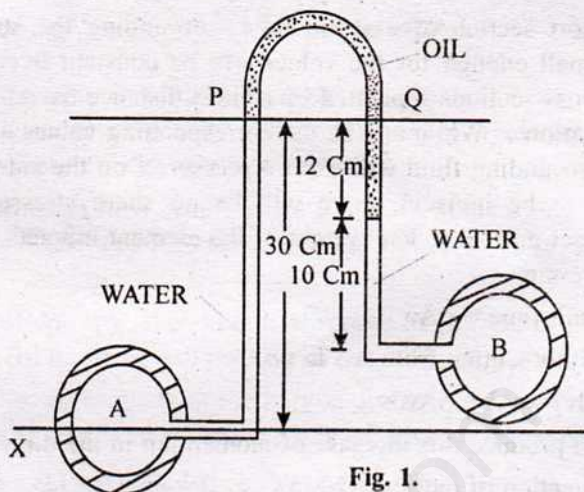


Fig. 1.

Pressure in the pipe A =  $P_A = 2.5\text{m}$  of  $\text{H}_2\text{O}$  sp. gr. of oil = 0.9

$P_B =$  Pressure in pipe B. Take the line XY as reference.

Now, equating the pressures at P & Q we get,

$P_A - 30\text{ cm of H}_2\text{O} = P_B - 10\text{ cm of H}_2\text{O} - 12\text{ cm of oil column.}$

$\Rightarrow P_B = P_A - 30\text{ cm of H}_2\text{O} + 10\text{ cm of H}_2\text{O} + 12\text{ cm of oil column}$

$\Rightarrow P_B = (250 - 30 + 10)\text{ cm of H}_2\text{O} + 12 \times 0.9\text{ cm of H}_2\text{O} = 240.8\text{ cm of H}_2\text{O}$   
 $= 2.408\text{ m of H}_2\text{O}$

Ans. The pressure in the pipe B is 2.408m of  $\text{H}_2\text{O}$ .

9. (a) Derive Euler's equation of motion along a streamline. 5

(b) A vertical venturimeter shown in Fig. 2 has an area ratio of 5. It has a throat diameter of 1 cm. When oil of sp. gr. 0.8 flows through it the mercury in the differential gauge indicates a difference in height of 12 cm. Find the discharge through the venturimeter. 7

Soln. : (a)

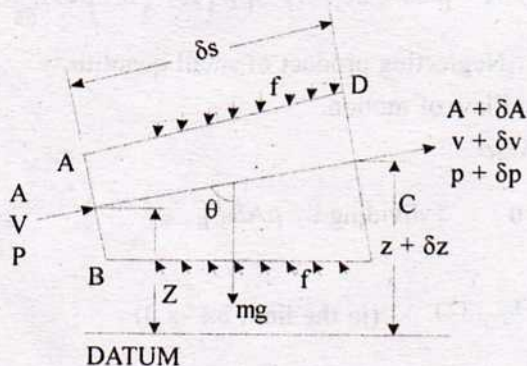


Figure shows a short section of a steam tube surrounding the streamline and having a cross-sectional area small enough for the velocity to be constant over the cross section. AB and CD are the two cross-sections separated by a short distance  $\delta s$ . At AB, area is  $A$ , velocity  $V$ , pressure  $p$  and elevation  $z$ . While at CD, the corresponding values are  $A + \delta A$ ,  $V + \delta v$ ,  $p + \delta p$  and  $z = \delta z$ . The surrounding fluid will exert a pressure  $f$  on the sides of the element and if the fluid is assumed to be inviscid, there will be no shear stresses on the sides of the streamtube and  $f$  will act normally. The weight of the element  $mg$  acts vertically downward at an angle  $\theta$  to the centreline.

$$\text{Mass flowing per unit time} = \rho Av$$

$$\text{Rate of increase of momentum from AB to CD}$$

$$= \rho Av [(v + \delta v) - v] = \rho Av \delta v$$

The forces acting to produce this increase of momentum in the direction of motion are

$$\text{force due to } p \text{ in direction of motion} = pA$$

$$\text{force due to } p + \delta p \text{ opposing motion} = (p + \delta p)(A + \delta A)$$

$$\text{Force due to } f \text{ producing a component in the direction of motion} = f\delta A.$$

$$\text{Force due to } mg \text{ producing a component opposing motion} = mg \cos\theta$$

$\therefore$  Resultant force in the direction of motion

$$= pA - (p + \delta p)(A + \delta A) + f\delta A - mg \cos\theta \dots\dots (1)$$

The value of  $f$  varies from  $p$  at AB to  $p + \delta p$  at CD and can be taken as  $(p + k\delta p)$ , where  $k$  is a fraction.

$$\text{Weight of element, } mg = \rho g \times \text{volume} = \rho g \left( A + \frac{1}{2} \delta A \right) \delta s$$

$$\cos\theta = \frac{\delta z}{\delta s}$$

$\therefore$  from (1), Resultant force in the direction of motion

$$= -p\delta A - A\delta p - \delta p \cdot \delta A + p\delta A + k\delta p \cdot \delta A - \rho g \left( A + \frac{1}{2} \delta A \right) \cdot \delta s \cdot \frac{\delta z}{\delta s}$$

$$= -A\delta p - \rho g A \cdot \delta z \quad (\text{Neglecting product of small quantities})$$

Applying Newton's 2nd law of motion,

$$\rho Av \cdot \delta v = -A\delta p - \rho g A \cdot \delta z$$

$$\Rightarrow \frac{1}{\rho} \frac{\delta p}{\delta s} + v \frac{\delta v}{\delta s} + g \frac{\delta z}{\delta s} = 0 \quad [\text{dividing by } \rho A \delta s]$$

$$\Rightarrow \frac{1}{\rho} \frac{dp}{ds} + v \frac{dv}{ds} + g \frac{dz}{ds} = 0 \dots\dots (2) \quad (\text{in the limit } \delta s \rightarrow 0)$$

This is known as EULER'S EQUATION, which is the relationship between pressure  $p$ , velocity  $v$ , density  $\rho$ , elevation  $z$  along a streamline for steady flow.

It cannot be integrated until the relationship between density  $\rho$  and pressure  $p$  is known.

For an incompressible fluid, for which  $\rho$  is constant, integration of equation (2), along a streamline, with respect to  $s$ , gives the Bernonlli's equation.

**Soln. : (b)** Given, sp. gr. of oil = 0.8 =  $\rho$

$$P_1 - P_2 = 12 \text{ cm of Hg column} \\ = 12 \times 13.6 \times 980 \text{ dyne/cm}^2$$

$$\frac{A_1}{A_2} = 5 \text{ where, } A_1 = \text{area of cross-section at (1)}$$

$A_2 =$  area of cross-section at (2)

Applying Bernonllis theorem at section (1) & (2),

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} = z_2 - z_1$$

$$\Rightarrow \frac{13.6 \times 12}{0.8} + \frac{V_1^2 - V_2^2}{2g} = 10$$

$$\therefore V_2^2 - V_1^2 = 194g \times 2 \dots\dots(1)$$

From continuity eqn.  $V_1 A_1 = V_2 A_2$

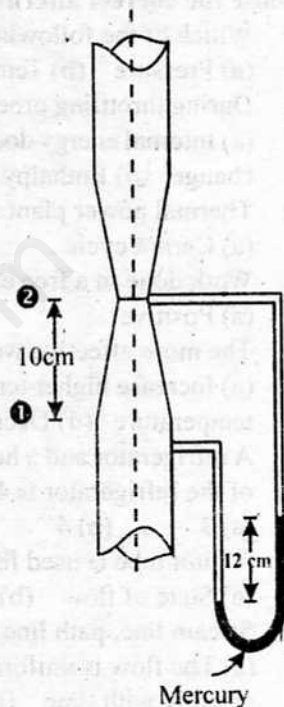
$$\therefore V_2 = V_1 \frac{A_1}{A_2} = 5V_1$$

From (1)  $24V_1^2 = 194g \times 2$

$$\therefore V_1^2 = \frac{97}{6}g \quad \therefore V_2 = 5\sqrt{\frac{97}{6}g}$$

$\therefore$  discharge through the venturimeter

$$= V_2 A_2 = 5\sqrt{\frac{97g}{6}} \times \frac{\pi}{4} \times (0.01)^2 \text{ m}^3/\text{s} \\ = 4.94 \times 10^{-4} \text{ m}^3/\text{sec.}$$



Mercury

Fig. 2

where  $P_1 =$  pressure at position (1)

$P_2 =$  Pressure at position (2)

$V_1 =$  Velocity at (1)

$V_2 =$  Velocity at (2)

$Z_1 =$  Elevation of (1)

$Z_2 =$  Elevation of (2)