where Q_1 is the heat transferred to the heat engine from a high temperature reservoir at T_1 and Q_2 is the heat rejected to a low temperature reservoir at T_2.

From the thermodynamic temperature scale

$$\frac{Q_1}{Q_2}=\frac{T_1}{T_2}$$

Then the efficiency of a Carnot heat engine becomes

$$\eta_{\text{ther, Carnot}}=1-\frac{T_2}{T_1}$$

The efficiency of engine is given by $\eta=1-\dfrac{T_2}{T_1}$

Let T_1 be increased by ΔT, keeping T_2 constant,

$$\eta_2=1-\frac{T_2-\Delta T}{T_1}$$

$$\eta_2-\eta_1=1-\frac{T_2-\Delta T}{T_1}-1+\frac{T_2}{T_1+\Delta T}=\frac{T_1T_2-(T_1+\Delta T)(T_2-\Delta T)}{T_1(T_1+\Delta T)}=\frac{(T_1+T_2)\Delta T(\Delta T)^2}{T_1(T_1+\Delta T)}$$

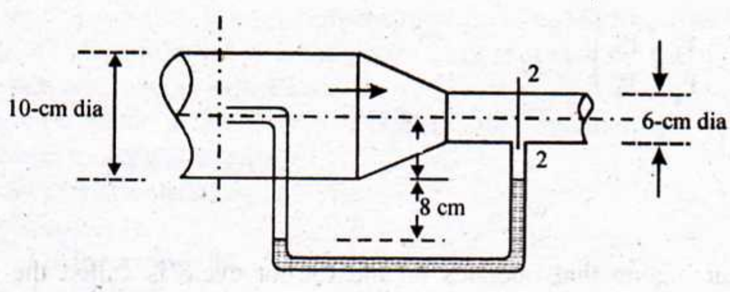
Since $T_1>T_2$, $\eta_2-\eta_1>0$

The more effective way to increase the efficiency engine is to decrease the lower temperature T_2.

12. (a) Derive an expression of actual discharge through a venturimeter tube. 5

(b) Write Bernoulli's equation and describe the various terms in it.
What are the assumptions involved in derivation of Bernoulli's equations? 3 + 2

(c) In the figure, the fluid flowing in (W_{air}= 12N/m^3) and the manometric fluid is merian red oil (specific gravity = 0.827). Assuming no loss, compute the flow rate. 5



Ans. (a) There is no loss of energy and applying Bernoulli's equation.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$V_2^2 - V_1^2 = 2g\left[\frac{(P_1 - P_2)}{\rho g}(z_1 - z_2)\right]$$

From the continuity equation, $A_1V_1 = A_2V_2$

$$V_2 = \frac{A_1}{A_2}V_1$$

Substituting in the equation (1), we have

$$V_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] = 2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]$$

$$V_1 = \frac{A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

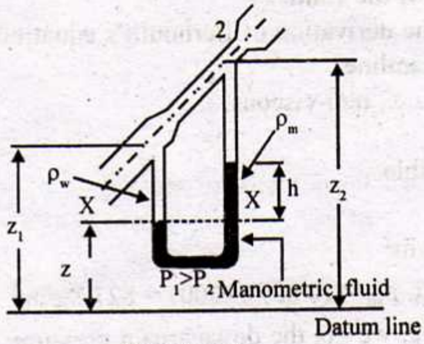
Volume flow rate,

$$Q = A_1V_1 = \frac{A_1A_2}{\sqrt{A_1{}^2 - A_2{}^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

$$Q = \frac{A_1A_2}{\sqrt{A_1{}^2 - A_2{}^2}}\sqrt{2gH}$$

where $H = \left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]$

The value of H in the equation (4) can be found from the reading of the U-tube differential manometer. Assuming that the connections to the gauge are filled with the fluid flowing in the pipeline, which has a density ρ_w and that the density of manometric fluid is ρ_m. Then, since pressures at level XX must be same in both limbs,



Venturimeter

$$P_X = P_1 + \rho_w g (z_1 - z) = P_2 + \rho_w g (z_2 - z - h) + \rho_m \Delta h g$$

$$H = \left[\frac{(p_1 - p_2)}{\rho g} + (z_1 - z_2)\right] = h\left(\frac{\rho_m}{\rho_w} - 1\right)$$

Equation (5) can be written as, $Q = \dfrac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh\left(\dfrac{\rho_m}{\rho_w} - 1\right)}$

A coefficient of discharge C_d is, therefore, introduced, which is defined as the ratio of actual dischange to that of theoretical discharge.

$$C_d = \frac{Q_{act}}{Q_{th}}$$

The usual value of C_d varies from 0.95 to 0.99.

Actual discharge, $Q_{act} = C_d \times Q_{theo}$

$$Q_{act} = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh\left(\frac{\rho_m}{\rho_w} - 1\right)}$$

Ans. (b) Bernoulli's equation can be written as $\dfrac{P}{\rho g} + \dfrac{V^2}{2g} + z = C$

The constant C is known as Bernoulli's constant which is constant for a streamline and varies from one streamline to another.

$\dfrac{P}{\rho g}$ is the pressure head (pressure energy per unit weight). It represents the height of a fluid column that produces that produces the static pressure P. $\dfrac{V^2}{2g}$ is the velocity head (kinetic energy per unit weight). It represents the elevation needed for a fluid to reach the velocity V during frictionless free fall, *z* is the potential head (potential energy per unit weight). It represents the potential energy of the fluid.

The assumptions made in the derivation of Bernoulli's equations are as follows :

(i) the flow is along a streamline.

(ii) the flow is frictionless i.e., non-viscous.

(iii) the flow is steady.

(iv) the fluid is incompressible.

Ans. (c) $p_1 = 12$ N/m^2

Density of air, $\rho_{air} = 1.2$ kg/m^2

Density of manometric fluid, $\rho_m = (0.827)(1000) = 827$ kg/m^2

From the manometer reading, we get the downstream pressure.

$p_1 - p_2 = (\rho_m - \rho_{air})gh = (827 - 1.2)(9.81)(0.08) = 648.09$ Pa

Applying the Bernoulli equation between section 1-1 and 2-2, we have

$$p_1 + \frac{1}{2}\rho_{air}V_1^2 = p_2 + \frac{1}{2}\rho_{air}V_2^2$$

or. $$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho_{air}}} = \sqrt{\frac{2(648\cdot09)}{1\cdot2}} = 32\cdot87 \text{ m/s}$$

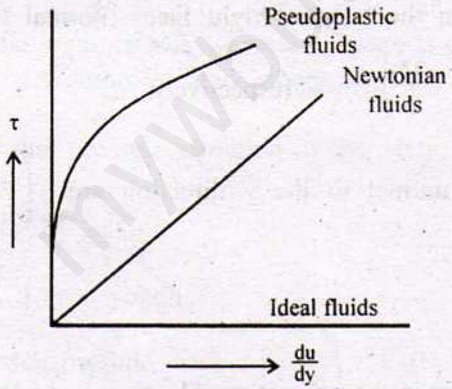
Flow rate, $Q = A_2V_2 = \frac{\pi}{4}(0\cdot06)^2(32\cdot87) = 0\cdot0929$ m³/s.

13. (a) Draw the rheological curve for a class of Newtonian and non-Newtonian fluid. 3

(b) State and prove hydrostatic law of a fluid. what is the stagnation pressure at a point in a fluid flow? 3+2

(c) Determine the pressure difference between points A and B. Specific gravities of benzene, kerosene and air are 0.88, 0.82 and 1.2×10^{-3} respectively.

Ans. (a) The Newtonian fluids behave according to Newton's law of viscosity that shear stress is linearly proportional to the velocity gradient for parallel flow. Thus for Newtonian fluids, the plot of shear stress against velocity gradient is a straight line passing through the origin. Common fluids, such as water and air are Newtonian fluids.



Rheological diagram

There are certain fluids where the linear relationship between the shear stress and the deformation rate (velocity gradient in parallel flow) is not valid. Because of the deviation from Newton's law of viscosity they are commonly termed as non-Newonian fluids. Examples of pseudoplastic fluids are blood, milk, gelatine, etc.

Ans. (b) The hydrostatic law states that the rate of decrease of pressure in the vertically upward direction is equal to the specific weight of the fluid. Mathematically,

$$\frac{dp}{dz} = -\rho g$$

Consider a differential fluid element at rest in rectangular Cartesian coordinates with the z-axis vertically upwards.

The forces acting on the fluid element at rest, are of surface forces and body forces. The only body force in thermodynamics is the gravity force.

The body force acting on the fluid element = $\rho g\ \Delta x\ \Delta y\ \Delta z$ and is acting vertically downward. The body force along the z direction is

$-\rho g\ \Delta x\ \Delta y\ \Delta z$

Since the fluid element is at rest, the shear stress acting on the element will be zero. The only surface force is the pressure force.

Let the pressure at the centre of the element be P.

The pressure at the bottom face (normal to the z axis) of the element is $P - \frac{\partial P}{\partial z} \cdot \frac{\Delta z}{2}$

The pressure at the top face (normal to the z-axis) of the element is $P + \frac{\partial P}{\partial z} \cdot \frac{\Delta z}{2}$

The force exerted on the bottom face is $\left(P - \frac{\partial P}{\partial z} \cdot \frac{\Delta z}{2}\right)\Delta x \cdot \Delta y$

The force exerted on the top face is $\left(P + \frac{\partial P}{\partial z} \cdot \frac{\Delta z}{2}\right)\Delta x \cdot \Delta y$

Similarly the forces acting on the left and right faces (normal to the x direction) are $\left(P - \frac{\partial P}{\partial x} \cdot \frac{\Delta x}{2}\right)\Delta y \cdot \Delta z$ and $\left(P + \frac{\partial P}{\partial x} \cdot \frac{\Delta x}{2}\right)\Delta y \cdot \Delta z$ respectively.

The forces acting on faces normal to the y direction are $\left(P - \frac{\partial P}{\partial y} \cdot \frac{\Delta y}{2}\right)\Delta x \cdot \Delta z$ and $\left(P + \frac{\partial P}{\partial y} \cdot \frac{\Delta y}{2}\right)\Delta x \cdot \Delta z$ respectively.

Net forces acting on the element in the z direction, $\delta F_z = -\frac{\partial P}{\partial z} \cdot \Delta x \Delta y \Delta z - \rho g \Delta x \Delta y \Delta z$

Similarly, net forces acting on the element in the x direction,

$$\delta F_x = -\frac{\partial P}{\partial x} \cdot \Delta x \Delta y \Delta z$$

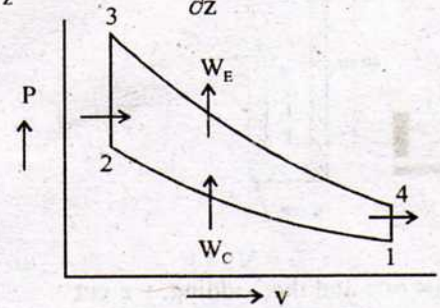
Net forces acting on the element in the y direction, $\delta F_y = -\frac{\partial P}{\partial y} \cdot \Delta x \Delta y \Delta z$

From Newton's second law, a force balance in the x, y and z direction gives :

$$\sum F_x = ma_x = 0 \qquad \frac{\partial P}{\partial x} = 0$$

$$\sum F_y = ma_y = 0 \qquad \frac{\partial P}{\partial y} = 0$$

$$\sum F_z = ma_z = 0 \qquad \frac{\partial P}{\partial z} = -\rho g$$



Variation of fluid pressure

$$\frac{\partial P}{\partial x} = 0 \qquad \frac{\partial P}{\partial y} = 0 \qquad \frac{\partial P}{\partial z} = -\rho g$$

Two points at the same elevation in the same continuous mass of fluid at rest have the same pressure.

P is a function of z only $\frac{dP}{dz} = -\rho g$

The sum of the static pressure and dynamic pressure is called the stagnation pressure. The pressure at a point in a frictionless flow where the fluid velocity is zero is known as the stagnation pressure.

Ans. (c) We know that pressure variation in any static fluid is described by the relation

$$\frac{dp}{dz} = -\frac{dp}{dh} = -\rho g$$

or, $dp = \rho g dh$ or, $\int_{p_1}^{p_2} dp = \int_{h_1}^{h_2} \rho g dh$

Assuming constant density fluid, we get $P_2 - P_1 = Pg(h_2 - h_1)$

Beginning at point A and applying the equation between successive points along the manometer gives

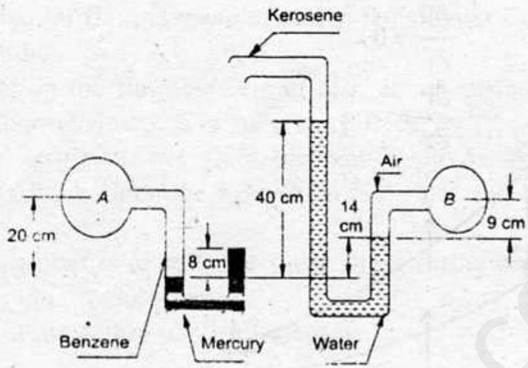
$$p_C - p_A = \rho_{Benzene}\, gh_1$$

$$p_D - p_C = \rho_{Mercury}\, gh_2$$

$$p_E - p_D = \rho_{Kerosene}\, gh_3$$

$$p_F - p_E = \rho_{Water}\, gh_4$$

$$p_B - p_F = \rho_{air}\, gh_5$$



Multiplying each equation by minus one and then adding, we get

$p_A - p_B = \rho_{Benzene}\, gh_1 + \rho_{Mercury}\, gh_2 + \rho_{Kerosene}\, gh_3 - \rho_{Water}\, gh_4 + \rho_{air}\, gh_5$

$= -0.88(1000)(9.81)(0.2) + 13.6(1000)(9.81)(0.08)$

$+ 0.82(1000)(9.81)(0.32) - (1000)(9.81)(0.26) + 1.2 \times 10^{-3}(1000)(9.81)(0.09)$

$= 8971.32 \text{ Pa} \approx 8.97 \text{ kPa}.$