SOLUTION & ANSWER FOR ISAT-2010 – PAPER - I
VERSION – A

[PHYSICS, CHEMISTRY & MATHEMATICS]

1. A block A with initial velocity $\vec{v}_0$ strikes a spring tied to a block ------

Ans : $\vec{v}_1 + \vec{v}_2 = \vec{v}_0$

Sol: At maximum compression, no relative velocity.
$v_1 = v_2 = v'$ for conservation of momentum,
$2\,mv' = mv \Rightarrow v' = \dfrac{v}{2}$
$\vec{v}_1 + \vec{v}_2 = \vec{v}_0$

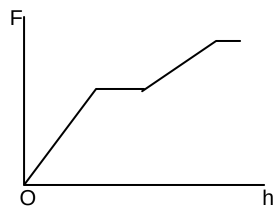
2. A particle moves in the first quadrant of the x-y plane under the action of a -----------

Ans : $v_x = u(1 + t / t_0)$

Sol: $v_x = \dfrac{dx}{dt}$ = (option given)
$x = \int (\text{option given})\,dt$
This gives x = 0 at t = 0 and at no other value of t only for option (c).

3. The figure, on the right, shows a jar filled with two liquids of densities ρ and --------------------

Ans :



Sol: Higher density liquid is lower.
As the cylinder is getting immersed, the buoyancy force increases from zero linear till it is fully immersed. Once fully immersed, until it reaches the interface buoyancy force is constant.

When it crosses the interface and moves into the higher density liquid, again, the buoyancy force increases linearly.

Once fully immersed in the second liquid the buoyancy force is constant.

4. An infinitely large surface of uniform charge density σ has a disc of radius R cut out -------------

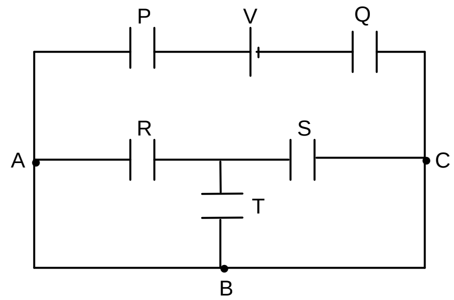
Ans : $\dfrac{\sigma}{2\varepsilon_0}\dfrac{a}{\sqrt{R^2 + a^2}}$

Sol: For a disc of charge density σ and radius R the electric field at a distance a
$= \dfrac{\sigma}{2\varepsilon_0}\left[1 - \dfrac{a}{\left(R^2 + a^2\right)^{1/2}}\right]$
Hence in the given problem, subtract from the field $\dfrac{\sigma}{2\varepsilon_0}$ of the infinite sheet the above value.
Hence $E = \dfrac{\sigma}{2\varepsilon_0}\dfrac{a}{\sqrt{R^2 + a^2}}$

5. All the five capacitors shown in the figure have the same capacitance C. The battery -------------

Ans : Zero

Sol:

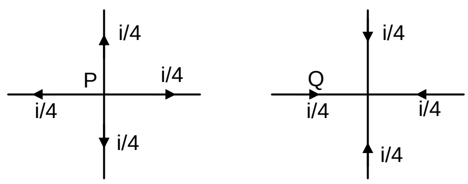


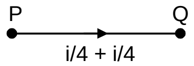
Points A, B and C are at the same potential. Hence no charge on R, S and T.

6. The figure shows a wire mesh of infinite extent, such that the resistance ---------------------

Ans : $\dfrac{1}{2}$ A

Sol:





$= \frac{i}{2} = \frac{1}{2}$ A

7. In a Young's double slit experiment, the separation between the slits --------------------

Ans : 0.25 mm

Sol: Position of the first dark fringe is given by

$$\frac{\beta}{2} = \frac{\lambda D}{2d}$$

The least distance of the dark fringe from the centre is for the smaller wavelength.

$$\therefore \frac{500 \times 10^{-6} \times 1}{2 \times 10^{-3}} = 0.25 \text{ mm}$$

8. The electric field $\vec{E}\left(\vec{r}, t\right)$ and the magnetic field -----------------

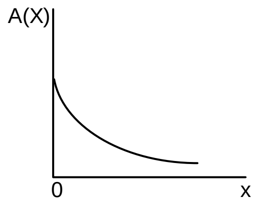
Ans : $\frac{c}{2}.\varepsilon_0 E_0^2 \hat{k}$

Sol: The rate of energy flow across a unit area is the time average over a cycle of the pointing vector.

$$\therefore S = \frac{1}{2}\varepsilon_0 c E_0^2$$

9. A transverse wave travels on a taut string stretched along the x-axis. The linear mass ---------------

Ans :



Sol: Intensity of the wave down the string of linear density is proportional to

$$A^2 \times \mu \times v \propto \frac{A^2 \mu}{\sqrt{\mu}} \propto A^2\sqrt{\mu}$$

Since energy independent of x,

$$A^2 \propto \frac{1}{\sqrt{\mu}}$$

10. A spherical soap bubble of radius R is blown from a tiny drop of a soap ---------------

Ans : $8\pi T r^2$

Sol: Total surface energy = T. area

$= T \times 2 \times 4\pi r^2$

$= 8\pi r^2$ T

11. In a column of air at a given temperature T, the density ρ is found to vary with the ------------------

Ans : $\left[\frac{k_B T}{mg}\right]^2$

Sol: Probability of finding a molecule at height z is

$$P(z) \propto \int_0^\infty z\rho_0 e^{\left(\frac{-mgz}{k_B T}\right)} dz$$

$$\propto \left[\frac{1}{\frac{mg}{k_B} T}\right]^2 = \left[\frac{k_B T}{mg}\right]^2$$

12. The mass density in a nucleus near its center, in units of kg/m^3, is in the range ----------------

Ans : 10^{15} to 10^{20}

Sol: Density at the centre

$$\propto \frac{\rho_0}{1 + e^{-\frac{R}{a}}} = \frac{\rho_0}{1 + e^{-2}}$$

$$= \frac{0.17}{1.135} \text{ nucleons / fm}^3$$

$$\cong \frac{0.15 \times 10^{-27}}{10^{-45}} \sim 1.5 \times 10^{17} \text{ kg m}^{-3}$$

13. For the nucleus ^{214}Te, the value of r for which the nucleon density falls to half its ----------------

Ans : 6 to 7 fm

Sol: $\frac{\rho_0}{1 + e^{\left(\frac{r}{a} - 12\right)}} = \frac{\rho_0}{2\left(1 + e^{-12}\right)}$

$\therefore 1 + e^{\left(\frac{r}{a} - 12\right)} = 2(1 + e^{-12})$

$\Rightarrow e^{-12} [e^{r/a} - 2] = 1$

$(e^{r/a} - 2) = e^{12}$

$\therefore \frac{r}{a} \cong 12$

(neglecting of no comparison to e^{12})

$\therefore r = 12 a = 0.55 \times 12$ fm = 6.6 fm

14. The plots of p(r) versus r for ^{28}Si and another nucleus X are shown ------------------

Ans : ^{142}Ba

Sol: $\frac{\rho_0}{1+e^{(r/a-10)}} = \frac{\rho_0}{2(1+e^{-10})}$

$\therefore 1 + e^{(r/a-10)} = 2(1 + e^{-10})$

$e^{-10}(e^{r/a} - 2) = 1$

$\therefore (e^{r/a} - 2) = e^{10} \Rightarrow r \cong 10\ a = 5.5$ fm

nearest value of A is ^{142}Ba

15. A vessel, fitted with a weightless, frictionless piston of 0.025 m^2 …

Ans : 75%

Sol: Work done = $-P\Delta V$

$= -1\ (atm) \times 0.025 \times 10^3\ (dm^3)$

$Fe + 2HCl \rightarrow FeCl_2 + H_2$

Number of moles of Fe = Number of moles of H_2

Work done = $-nRT$

$= -n \times 0.0821\ (L\ atm\ mol^{-1}\ K^{-1}) \times 300\ K$

$\therefore n = \frac{25}{0.0821 \times 300} = 1$

mass of Fe = 55.85

% purity = $\frac{55.85 \times 100}{75} \cong 75\%$

16. A solution at 298 K is separated from the pure solvent by a semi-permeable membrane. Difference in the height of the solution ……..

Ans : 250.20 K

Sol: Osmotic pressure, $\pi = hdg$

$= 0.9 \times 1 \times 10^3 \times 9.8\ N\ m^{-2}$

$= \frac{0.9 \times 9.8 \times 10^3}{1.013 \times 10^5}$ atm

$\pi = MST$

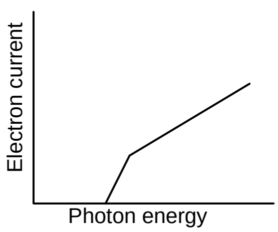
$M = \frac{0.9 \times 9.8 \times 10^3}{1.013 \times 10^5 \times 0.0821 \times 298} = 3.5 \times 10^{-3}$

$\Delta Tf = K_f \times m$

$= 30 \times 3.5 \times 10^{-3} = 0.1$

$\therefore$ F. P = 250.2 K

17. Ans :



Sol: The electron current occurs above the threshold voltage. Initially the current rises steeply and then slowly attains saturation.

18. In the following statements,

A Ideal gases are liquids only

Ans : B, C, D

Sol: Ideal gases cannot be liquefied as there is no attraction between the molecules. Real gases show ideal behavior at low pressure and high temperature. None of the gas is ideal.

19. The correct order for the rule of S_N…….

Ans : B> C > A> D

Sol: The relative stability of carbocation formed by hydrolysis is B> C > A> D

20. The aromatic species among the following are…

Ans : D & E

Sol: (D) is tropyllium cation which is aromatic according to Huckel's rule. (E) is aromatic because is forms cyclobutenyl dication with 2 delocalised π electrons.

21. The reactions that give products with dipole moment are………

Ans : A, C & D

Sol: The product in (A) is cis but-2-ene. Reaction (C) gives pent-2-ene. All these products have dipole moments.

22. The total number of stereoisomers for the following compound is…..

Ans : 4

Sol: The compound contains two double bonds and the group linked to the two double bonds are different. Hence four stereoisomers are possible. They are E-E, Z-Z, E-Z and Z-E.

23. The correct order of ligand field strength is

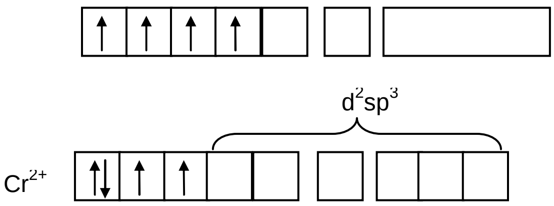
Ans : $Cl^- < H_2O < NH_3 < CO$

Sol: CO is a strong field ligand. The correct order of field strength is $Cl^- < H_2O < NH_3 < CO$

24. The complex exhibiting a spin-only magnetic moment…….

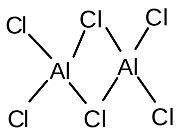
Ans : $Na_2[Cr[NCS]_4\ [NH_3]_2]$

Sol: In $Na_2[Cr[NCS]_4\ [NH_3]_2]$ the central atom is in +2 oxidation state and the hybridization is d^2sp^3. There are two unpaired electrons.



25. The most stable species among the following is

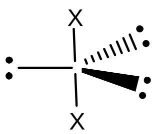
Ans :



Sol: Al_2Cl_6 is a stable species existing in solid **stat**e.

26. According to the VSEPR model, ………

Ans :

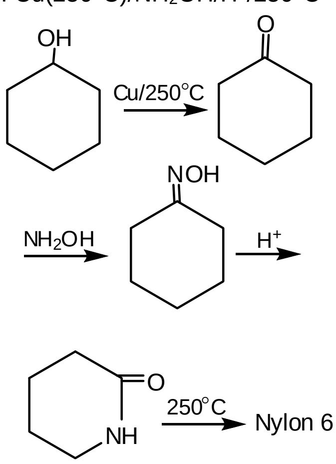


Sol: lone pair arrange at 120° to each other to form linear molecule.

27. Cyclohexanol is converted to Nylon-6 by……

Ans : Cu(250°C)/NH_2OH/H^+/250°C

Sol:



Caprolactum

28. Some gases in column X may be associated with options in Y.

Ans : $SO_2 \rightarrow$ M, O, & R

Sol: SO_2 is present in Troposphere. It is a component of classical smog. It is also responsible for acid rain.

29. Let x be the 7^{th} term from the beginning and y be the 7^{th} term ….

Ans : 9

Sol: $x = T_7$; $y = T_{n-6}$; $\left(3^{1/3} + \frac{1}{4^{1/3}}\right)^n$

$$T_{r+1} = {}^nC_r \,.\, \left(3^{1/3}\right)^{n-r}\left(\frac{4}{4^{1/3}}\right)^r$$

$$\frac{x}{y} = \frac{1}{12} \Rightarrow \frac{{}^nC_6 \cdot \left(3^{1/3}\right)^{n-6} \cdot \left(\frac{1}{4^{1/3}}\right)^6}{{}^nC_{n-6} \cdot \left(3^{1/3}\right) \cdot \left(\frac{1}{4^{1/3}}\right)^{n-6}} = \frac{1}{12}$$

$$\Rightarrow 3^{n/3-4} \,.\, 4^{+n/3-4} = \frac{1}{12} = 3^{-1}.4^{-1}$$

$\Rightarrow n = 9$

30. The number of functions f: {1, 2, …n} $\rightarrow$ {1, 2, …m}, where m, n, are positive….

Ans : m^{n-1}

Sol: {1, 2, …n} $\rightarrow$ {1, 2, …m}
f(1) = 1
$\therefore$ for the remaining (n–1) elements there are m choices each $\Rightarrow m^{n-1}$

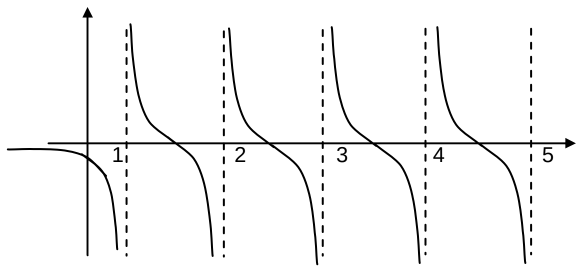
31. The number of real roots of the equation in the interval
(1/(x – 1)) + (1/(x – 2)) + …+1/(x – 5) = 1…

Ans : 4

Sol: In the question 'in the interval' is repeated twice. Please read after deleting the phrase the first time it appears

$$\frac{1}{(x-1)} + \frac{1}{(x-2)} + ... + \frac{1}{(x-5)} = 1$$

It is obvious that f(x) $\rightarrow -\infty$ when x $\rightarrow 1^-$, 2^-, 3^-, 4^-, 5^-. Also that f(x) $\rightarrow +\infty$ when x $\rightarrow 1^+$, 2^+, 3^+, 4^+, 5^+ and in (1, 2), (2, 3), (3, 4), (4, 5), f(x) is continuous.



The approximate graph y = f(x) is given above. Therefore number of roots = 4

Aliter

The given function can be modified to

$(x-2)(x-3)(x-4)(x-5) + (x-1)(x-3)(x-4)(x-5) + (x-1)(x-2)(x-4)(x-5) + (x-1)(x-2)(x-3)(x-5) + (x-1)(x-2)(x-3)(x-4) - (x-1)(x-2)(x-3)(x-4)(x-5) = f(x)$

When x = 1, f(x) is +ve
x = 2, f(x) is –ve
x = 3, f(x) is +ve
x = 4, f(x) is –ve
x = 5, f(x) is +ve

Since there are 5 sign changes, there are 4 roots.

32. Let y be the solution of the problem
$y' - y = 1 + 5e^{-x}$, $y(0) = y_0$ …

Ans : $-\frac{7}{2}$

Sol: The general solution is

$y = Ce^{x} - 1 - \frac{5}{2}e^{-x}$

$y(0) = y_0 \Rightarrow C - \frac{7}{2} = y_0$

$C = y_0 + \frac{7}{2}$

But given $\lim_{x\to\infty} |y(x)|$ is finite

$\Rightarrow C \to 0 \quad \Rightarrow y_0 = -\frac{7}{2}$

33. The distance from a point (k, 0) on the positive x-axis to the tangent ….

Ans : $\frac{19}{4}$

Sol: $y^2 = 4x \qquad y^1 = \frac{4}{2y}$

$2yy' = 4$ slope $= (y')_{\left(\frac{1}{4},1\right)} = \frac{4}{2} = 2$

Equation : tangent

$\Rightarrow y - 1 = 2\left(x - \frac{1}{4}\right)$

$= \frac{2(4x-1)}{2}$

$2y - 2 = 4x - 1$

$4x - 2y + 1 = 0$

$\frac{|4k+1|}{\sqrt{16+4}} = 2\sqrt{5}$

$\frac{4k+1}{\sqrt{2}\ 2\sqrt{5}} = 2\sqrt{5}$

$4k + 1 = 4 \times 5$

$4k = 20 - 1 = 19$

$k = \frac{19}{4}$

34. The point (–3, 2) undergoes the following …

Ans : $\left(\frac{1}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

Sol: (–3, 2)

Reflection about $y = -x \Rightarrow (-2, 3)$

Reflection about $x = 0 \Rightarrow (2, 3)$

Translation by 2 units along $y \Rightarrow (2, 1)$

Rotation by $\frac{\pi}{4}$ anticlock wise about origin

$\Rightarrow (2+i)\times\frac{1}{\sqrt{2}}(1+i)$

$= \frac{1}{\sqrt{2}}(1+3i)$

$\therefore$ The point is $\left(\frac{1}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

35. The equation of the plane passing through the intersection of the planes x + 2y + z – 1 = 0..

Ans : –4

Sol: Equation : plane through intersection is

$(1 + 2\lambda)x + (2 + \lambda)y + (1 + 3\lambda)z - 1 - 2\lambda = 0$

$1 + 2\lambda + 2 + \lambda + 1 + 3\lambda = 0$

$4 + 6\lambda = 0$

$\lambda = \frac{-4}{6} = \frac{-2}{3}$

$x + 2y + z - 1 - \frac{2}{3}(2x + y + 3z - 2) = 0$

$3x + 6y + 3z - 3 - 4x - 2y - 6z + 4 = 0$

$-x + 4y - 3z + 1 = 0$

$x - 4y + 3z - 1 = 0$

$\therefore k = -4$

36. The principal value of

$\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$ is …

Ans : $\frac{\pi}{3}$

Sol: $\sin^{-1}\sin\left(\frac{5\pi}{9}+\frac{\pi}{9}\right)=\sin^{-1}\sin\left(\frac{6\pi}{9}\right)$

$=\sin^{-1}\sin\left(\frac{2\pi}{3}\right)$,

i.e., $\sin^{-1}\sin\left(\pi-\frac{2\pi}{3}\right)=\sin^{-1}\sin\frac{\pi}{3}=\frac{\pi}{3}$

37. Four numbers are chosen at random without replacement from the first 15 …

Ans : $1-\frac{{}^{8}C_4}{{}^{15}C_4}$

Sol: Total number of ways of choosing 4 out of 15 = ${}^{15}C_4$

For product to be even atleast are of 4 chosen numbers should be even.

P(product even)

= 1 – P(No even numbers)

$= 1-\frac{{}^{8}C_4}{{}^{15}C_4}$

38. Let M be a non-singular matrix of order 5×5….

Ans : $|M|^{-4}$

Sol: $|\text{adj } M| = |M|^4$

$|M^{-1}|=\frac{1}{|M|}$

$|\text{adj } M^{-1}| = |M^{-1}|^4=\frac{1}{|M|^4}=|M|^{-4}$

39. Let P = $\begin{bmatrix}1 & 2 & 0\\ -1 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$ and Q be such that …

Ans : real numbers

Sol: $Qx = \lambda X \Rightarrow (Q - \lambda I)X = 0$

$\Rightarrow |Q - \lambda I| = 0$ (Θ X is non zero)

$|P| = 1(1 - 4) - 2(-1 -2) = -3 + 6 = 3$

$Q = \frac{1}{3}\begin{pmatrix}-3 & -2 & 4\\ -3 & 1 & -2\\ -3 & 0 & 3\end{pmatrix}$

$Q - \lambda I = \frac{1}{3}\begin{pmatrix}-3-\lambda & -2 & 4\\ -3 & 1-\lambda & -2\\ -3 & 0 & 3-\lambda\end{pmatrix}$

$|Q - \lambda I| = -3(4 - 4(1 - \lambda)$

$+ (3 - \lambda)[-(-3-\lambda)(1 - \lambda) -6]$

$= (1 - \lambda)(9 - \lambda^2 + 6\lambda) = 0$

$\Rightarrow$ All real roots $\Rightarrow \lambda \in R$

40. The value of determinant $\begin{vmatrix}x & a & a & a & a\\ a & x & a & a & a\\ a & a & x & a & a\\ a & a & a & x & a\\ a & a & a & a & x\end{vmatrix}$ is

Ans : $(x + 4a)(x - a)^4$

Sol: $(x+4a)\begin{vmatrix}1 & 1 & 1 & 1 & 1\\ a & x & a & a & a\\ a & a & x & a & a\\ a & a & a & x & a\\ a & a & a & x & a\end{vmatrix}$

$(x+4a)\begin{vmatrix}1 & 0 & 0 & 0 & 0\\ a & x-a & 0 & 0 & 0\\ a & 0 & x-a & 0 & 0\\ a & 0 & 0 & x-a & 0\\ a & 0 & 0 & 0 & x-a\end{vmatrix}$

$(x + 4a)(x - a)^4$