Q. 1. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80 . Then which one of the following gives possible values a and b?

i. a = 1, b = 6
ii. a = 3, b = 4
iii. a = 0, b = 7
iv. a = 5, b = 2

sol: 2

Q. 2. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $+\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

i. $\alpha = 2, \beta = 1$
ii. $\alpha = 1, \beta = 1$
iii. $\alpha = 2, \beta = 2$
iv. $\alpha = 1, \beta = 2$

Sol. 2

Q. 3.

The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$.
Then the angle between $\vec{a}$ and $\vec{c}$ is

i. $\frac{\pi}{2}$
ii. π
iii. 0
iv. $\frac{\pi}{4}$

Sol. 2

Q. 4. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

i. a = 6, b = 4
ii. a = 8, b = 2
iii. a = 2, b = 8
iv. a = 4, b = 6

Sol. 1

Q. 5. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to

i. 2
ii. 2
iii. 5
iv. 5

Sol.3

Q. 6. The differential of the family of circles with fixed radius 5 units and centre on the line y = 2 is

i. $(y-2)^2 y'^2 = 25 - (y-2)^2$
ii. $(x-2)^2 y'^2 = 25 - (y-2)^2$
iii. $(x-2) y'^2 = 25 - (y-2)^2$
iv. $(y-2) y'^2 = 25 - (y-2)^2$

Sol. 1

Q. 7. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz, y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

i. 0
ii. 1
iii. 2

iv. - 1

Sol. 2

Q. 8. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

i. If $\det A = \pm 1$, then A^{-1} exists and all its entries are int egers
ii. If $\det A = \pm 1$, then A^{-1} need not exist
iii. If $\det A = \pm 1$, then A^{-1} exist but all its entries are not necessarily int egers
iv. If $\det A = = 1$, then A^{-1} exist and all its entries are non $-$ int egers

Sol. 1

Q. 9. The quadratic equations $x^2 - 6x\ a = 0$ and $x^2 - cx + 6 = 0$ and have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

i. 3
ii. 2
iii. 1
iv. 4

Sol. 2

Q. 10. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

i. $6.8.{}^7C_4$
ii. $7.{}^6C_4.{}^8C_4$
iii. $8.{}^6C_4.{}^7C_4$
iv. $6.7.{}^8C_4$

Sol. 2

Q. 11.

Let $I = \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx$. Then which one of the following is true?

i. $I < \frac{2}{3}$ and $J > 2$

ii. $I < \frac{2}{3}$ and $J < 2$

iii. $I > \frac{2}{3}$ and $J > 2$

iv. $I < \frac{2}{3}$ and $J > 2$

Sol. 4

Q. 12. The area of the plane region bounded by the curve $x + 2y^2 = 0$ and $3y^2 = 1$ is equal to

i. $\frac{2}{3}$

ii. $\frac{4}{3}$

iii. $\frac{5}{3}$

iv. $\frac{1}{3}$

Sol. 2

Q. 13.

The value of $\sqrt{2} \int \frac{\sin x\,dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is

i. $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

ii. $x - \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

iii. $x + \log\ \cos\left(x - \frac{\pi}{4}\right)\Big| + c$

iv. $x - \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

Sol. 1

Q. 14. *The statement $p \to (q \to p)$ is equivalent to*

i. $p \to (p \wedge q)$
ii. $p \to (p \leftrightarrow q)$
iii. $p \to (p \to q)$
iv. $p \to (p \vee q)$

Corredt answer is (4)

Q. 15.

The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ *is*

i. $\frac{4}{17}$
ii. $\frac{5}{17}$
iii. $\frac{6}{17}$
iv. $\frac{3}{17}$

Sol. 3

Q. 16. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of A.

Assume that $A^2 = I$

$$\text{Statement} - 1: \text{If } A \neq I \text{ and } A \neq -I, \text{ then } \det A = -1.$$

$$\text{Statement} - 2: \text{If } A \neq I \text{ and } A \neq -I, \text{ then } tr(A) \neq 0$$

i. Statement-1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1.
ii. Statement-1 is true, Statement-2 is false
iii. Statement-1 is false, Statement-2 is true.
iv. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Sol. 2

Q. 17. Let p be the statement 'x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".

Statement-1: r is equivalent to either q or p.

$$\text{Statement} - 2: r \text{ is equivalent to } \sim(p \leftrightarrow \sim q)$$

i. Statement-1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1.
ii. Statement-1 is true, Statement-2 is false.
iii. Statement-1 is false, Statement-2 is true.
iv. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol. non

Q. 18. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1: The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.

Statement-2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

i. Statement-1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1.
ii. Statement-1 is true, Statement-2 is false.
iii. Statement-1 is false, Statement-2 is true.
iv. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol. 3

Q. 19.

$$\textit{Statement}-1:\sum_{r=0}^{n}(r+1)\,{}^{n}C_{r}=(n+2)\,2^{n-1}$$

$$\textit{Statement}-2:\sum_{r=0}^{n}(r+1)\,{}^{n}C_{r}\,x^{r}=(1+x)^{n}-nx\,(1+x)^{n-1}$$

i. Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.
ii. Statement-1 is true, Statement-2 is false.
iii. Statement-1 is false, Statement-2 is true.
iv. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol. 4

Q. 20.

$$\textit{Statement}-1:\ \textit{For every natural number } n\geq 2,\ \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+.....+\frac{1}{\sqrt{2}}>\sqrt{n}$$

$$\textit{Statement}-1:\ \textit{For every natural number } n\geq 2,\ \sqrt{n\,(n+1)}<n+1$$

i. Statement-1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1.
ii. Statement-1 is true, Statement-2 is false.
iii. Statement-1 is false, Statement-2 is true.
iv. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol. 4

Q. 21.

The conjugate of a complex number is $\frac{1}{i-1}$.

Then that complex number is

i. $\frac{-1}{i+1}$

ii. $\frac{1}{i-1}$

iii. $\frac{-1}{i-1}$

iv. $\frac{1}{i+1}$

Sol. 1

Q. 22. Let R be the real line. Consider the following subsets of the plane

$$S = \{(x, y) \; y = x + 1 \text{ and } 0 < x < 2\}$$
$$T = \{(x, y) \; x - y \text{ is an integer}\}$$

Which one of the following is true?

i. S is an equivalence relation on R but T is not
ii. T is an equivalence relation on R but S is not
iii. Neither S nor T is an equivalence relation on R
iv. Both S and T are equivalence relations on R

Sol: 2

Q. 23.

Let $f : N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.

Show that f is invertible and its inverse is

i. $g(y) = \frac{y+3}{4}$
ii. $g(y) = \frac{y-3}{4}$
iii. $g(y) = \frac{3y+4}{3}$
iv. $g(y) = 4 + \frac{y+3}{4}$

Sol: 2

Q. 24. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is $60^{0.}$ He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45^{0}. Then the height of the pole is.

i. $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)m$
ii. $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}\ m$
iii. $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}\ m$
iv. $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

Sol: 4

Q. 25. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

i. 1
ii. $\frac{2}{5}$
iii. $\frac{3}{5}$
iv. 0

Sol: 1

Q. 26. It is given that the events A and B are such that

$$P(A) = \frac{1}{4}, P(A|B) = \frac{1}{2} \text{ and } P(B|A) = \frac{2}{3}. \text{ Then } P(B) \text{ is}$$

i. $\frac{2}{3}$
ii. $\frac{1}{2}$
iii. $\frac{1}{6}$
iv. $\frac{1}{3}$

Sol: 4

Q. 27. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

i. $\frac{4}{3}$
ii. $\frac{5}{3}$
iii. $\frac{8}{3}$
iv. $\frac{2}{3}$

Sol: 3

Q. 28. A parabola has the origin as its focus and the line x = 2 as the directrix. Then the vertex of the parabola is at

i. (0, 1)
ii. (2, 0)

iii. (0, 2)
iv. (1, 0)

Sol: 1

Q. 29. The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

i. (- 3, - 4)
ii. (3, 4)
iii. (3, - 4)
iv. (- 3, 4)

Sol: 1

Q. 30. The perpendicular bisector of the line segment joining p (1, 4), and Q (k, 3), has y-intercept –4. Then a possible value of k is

i. - 2
ii. - 4
iii. 1
iv. 2

Sol: 1

Q. 31. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

i. 12
ii. 4
iii. - 4
iv. - 12

Sol: 4

Q. 32. Suppose the cubic x^3 - px + qhas three distinct real roots where p > 0 and q > 0. Then which one of the following holds?

i. The cubic min ima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
ii. The cubic max ima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
iii. The cubic min ima at $\sqrt{\frac{p}{3}}$ and max ima at $-\sqrt{\frac{p}{3}}$
iv. The cubic min ima at $-\sqrt{\frac{p}{3}}$ and max ima at $\sqrt{\frac{p}{3}}$

Sol: 3

Q. 33. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 \_ 30x - 560 = 0$ have?

i. 3
ii. 5
iii. 7
iv. 1

Sol: 4

Q. 34.

$$Let\ f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & if \quad x \neq 1 \\ 0 & if \quad x = 1 \end{cases}.\ Then\ which\ one\ of\ the\ following\ is\ true?$$

i. f is differentiable at x = 0 but not at x = 1
ii. f is differentiable at x = 1 but not at x = 0
iii. f is neither differentiable at x = 0 nor at x = 1
iv. f is differentiable at x = 0 and at x = 1

Sol: 1

Q. 35. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition y (1) = 1 is

i. $y = xe^{(x-1)}$
ii. y = x In x +x
iii. y = In x +x
iv. $y = x\ln x + x^2$

Sol: 4