DSA – 230 (MAT – UG) (R)

First Year B.Sc. Degree Examination, August/September 2008
Directorate of Correspondence Course
MATHEMATICS (Paper – I) (Repeaters)

Time : 3 Hours

Max. Marks : 90

Note : *Answer any **SIX full** questions of the following choosing atleast **one** from **each** Part.*

PART – A

1. a) i) Find $\phi(72)$. **2**

   ii) Find the g.c.d. of 506 and 1155. **2**

   b) Show that $712! + 1 \equiv 0 \pmod{719}$. **5**

   c) Solve the simultaneous congruences $x \equiv 2 \pmod 5$ and $3x \equiv 1 \pmod 8$ using chinese remainder theorem. **6**

2. a) i) Define a partition of nonempty set. **2**

   ii) Define $f : R \to R$ by $f(x) = x^2$ and $g : R \to R$ by $g(x) = \sin x \ \forall x \in R$ show that $g \circ f \neq f \circ g$. **2**

   b) Let $f : X \to Y$ be a subjective function and c be any subset of Y. Show that $f[f^{-1}(c)] = c$. **5**

   c) Show that $f : R - \{1\} \to R - \{3\}$ defined by $f(x) = \frac{3x-2}{x-1}$ is a bijective map find a formula for f^{-1} **6**

PART – B

3. a) i) Show that the function $f(x) = |x|$ is continuous but not differentiable at $x = 0$. **2**

   ii) Find $\frac{dy}{dx}$ if $x^y = y^x$. **2**

   b) Discuss the continuity of the function f(x) defined by $f(x) = \begin{cases} x^2+2 & \text{if } x>1 \\ 2x+1 & \text{if } x=1 \\ 3 & \text{if } x<1 \end{cases}$ at $x = 1$. **5**

   c) If $x = \tan(\log y)$ prove that $(1 + x^2)\, y_{n+2} + (2nx + 2x - 1)\, y_{n+1} + n(n + 1)y_n = 0$. **6**

P.T.O.

4. a) i) Find $\frac{ds}{dt}$ for the parametric curve $x = a\cos^3 t$; $y = a\sin^3 t$. **2**

ii) Show that the curvature at all points on the circle $x^2 + y^2 = a^2$ is constant. **2**

b) Obtain the pedal equation for the curve $r = a(1 + \cos\theta)$. **5**

c) Show that the evolute of the parabole $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. **6**

PART – C

5. a) i) Find the equation of the plane passing through the point (2, 3, 4) and parallel to the plane $5x - 6y + 7z = 3$. **2**

ii) Find the point of intersection of the line $x = t - 1, y = 3t - 3, z = -2t + 2$ and the plane $3x + 4y + 5z = 5$. **2**

b) Does the three points (2, –1, 4), (3, 2, 6) and (1, –4, 2) are collinear. **5**

c) Show that the two lines l_1 and l_2 intersect hence find the point of intersection where

$l_1 : x = 2t + 1, \quad y = -3t - 1, \quad z = 8t - 10$

$l_2 : x = s + 4, \quad y = -4s - 3, \quad z = 7s - 1$ **6**

6. a) i) Find the equation of the sphere whose centre is at (1, 2, 3) and which passes through the point (0, –1, –2). **2**

ii) Find the singular points on the curve $y^2 = 2ax^2 - x^3$. **2**

b) Find all the asymptotes to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$. **5**

c) Find the surface area of the solid obtained by revolving the cardiode $r = a(1 + \cos\theta)$ about the initial line. **6**

PART – D

7. a) i) Express the matrix $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 4 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices. **2**

ii) Find the real value of λ for which the system of equations $2x - y + 2z = 0$, $3x + y - z = 0$, and $\lambda x - 2y + z = 0$ has a non-trival solution. **2**

b) Find the rank of the matrix $\begin{pmatrix} 2 & 4 & 3 & 4 \\ 1 & 2 & -1 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{pmatrix}$. **5**

c) Test the following non-homogeneous equation for consistency and solve :

$2x + 3y - z + 5t = 1,\qquad 3x - y + 2z - 7t = 2$

$4x - y - 3z + 6t = -1,\qquad x - 2y + 4z - 7t = 0$ **6**

8. a) i) Evaluate $\int \frac{1}{\sqrt{a+x} + \sqrt{b+x}}\,dx$. **2**

ii) Evaluate $\int_0^{\pi/2} \sin^6 x \cos^8 x\,dx$. **2**

b) Evaluate $\int \frac{dx}{9+16\cos^2 x}$. **5**

c) Evaluate $\int_0^{\pi} \frac{\sin^4\theta}{(1+\cos\theta)^2}\,d\theta$. **6**
