AIEEE–CBSE–ENG–03

1. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases} \text{ is}$$

(A) one–one but not onto (B) onto but not one–one
(C) one–one and onto both (D) neither one–one nor onto

2. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral triangle, then
(A) $a^2 = b$ (B) $a^2 = 2b$
(C) $a^2 = 3b$ (D) $a^2 = 4b$

3. If z and ω are two non–zero complex numbers such that $|z\omega| = 1$, and $\text{Arg}(z) - \text{Arg}(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to
(A) 1 (B) – 1
(C) i (D) – i

4. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then
(A) x = 4n, where n is any positive integer
(B) x = 2n, where n is any positive integer
(C) x = 4n + 1, where n is any positive integer
(D) x = 2n + 1, where n is any positive integer

5. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$ $(1, b, b^2)$ and $(1, c, c^2)$ are non–coplanar, then the product abc equals
(A) 2 (B) – 1
(C) 1 (D) 0

6. If the system of linear equations
$x + 2ay + az = 0$
$x + 3by + bz = 0$
$x + 4cy + cz = 0$
has a non–zero solution, then a, b, c
(A) are in A. P. (B) are in G.P.
(C) are in H.P. (D) satisfy a + 2b + 3c = 0

7. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{b}$ are in
(A) arithmetic progression (B) geometric progression
(C) harmonic progression (D) arithmetic–geometric–progression

8. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(A) 2 (B) 4
(C) 1 (D) 3

9. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)\, x^2 + (3a - 1)\, x + 2 = 0$ is twice as large as the other, is

(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

I10. If A = $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(A) $\alpha = a^2 + b^2, \beta = ab$ (B) $\alpha = a^2 + b^2, \beta = 2ab$

(C) $\alpha = a^2 + b^2, \beta = a^2 - b^2$ (D) $\alpha = 2ab, \beta = a^2 + b^2$

11. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(A) 140 (B) 196

(C) 280 (D) 346

12. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

(A) $6! \times 5!$ (B) 30

(C) $5! \times 4!$ (D) $7! \times 5!$

13. If 1, ω, ω^2 are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$ is equal to

(A) 0 (B) 1

(C) ω (D) ω^2

14. If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals

(A) ${}^{n+2}C_r$ (B) ${}^{n+2}C_{r+1}$

(C) ${}^{n+1}C_r$ (D) ${}^{n+1}C_{r+1}$

15. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is

(A) 32 (B) 33

(C) 34 (D) 35

16. If x is positive, the first negative term in the expansion of $(1 + x)^{27/5}$ is

(A) 7^{th} term (B) 5^{th} term

(C) 8^{th} term (D) 6^{th} term

17. The sum of the series $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots\ldots\ldots$ upto ∞ is equal to

(A) $2 \log_e 2$ (B) $\log_2 2 - 1$

(C) $\log_e 2$ (D) $\log_e\left(\frac{4}{e}\right)$

18. Let f (x) be a polynomial function of second degree. If f (1) = f (– 1) and a, b, c are in A. P., then f′ (a), f′ (b) and f′ (c) are in

(A) A.P. (B) G.P.

(C) H. P. (D) arithmetic–geometric progression

19. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1) (x_2, y_2) and (x_3, y_3)
(A) lie on a straight line
(B) lie on an ellipse
(C) lie on a circle
(D) are vertices of a triangle

20. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is
(A) $a \cot\left(\frac{\pi}{n}\right)$
(B) $\frac{a}{2} \cot\left(\frac{\pi}{2n}\right)$
(C) $a \cot\left(\frac{\pi}{2n}\right)$
(D) $\frac{a}{4} \cot\left(\frac{\pi}{2n}\right)$

21. If in a triangle ABC $a \cos^2\left(\frac{C}{2}\right) + c \cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c
(A) are in A.P.
(B) are in G.P.
(C) are in H.P.
(D) satisfy $a + b = c$

22. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the Δ ABC is
(A) $\frac{8}{3}$
(B) $\frac{16}{3}$
(C) $\frac{32}{3}$
(D) $\frac{64}{3}$

23. The trigonometric equation $\sin^{-1} x = 2 \sin^{-1} a$, has a solution for
(A) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$
(B) all real values of a
(C) $|a| < \frac{1}{2}$
(D) $|a| \geq \frac{1}{\sqrt{2}}$

24. The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1} \frac{3}{5}$ at point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is
(A) 20 m
(B) 40 m
(C) 60 m
(D) 80 m

25. The real number x when added to its inverse gives the minimum value of the sum at x equal to
(A) 2
(B) 1
(C) – 1
(D) – 2

26. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is
(A) $\frac{7n}{2}$
(B) $\frac{7(n+1)}{2}$
(C) $7n(n + 1)$
(D) $\frac{7n(n+1)}{2}$

27. If f (x) = x^n, then the value of f (1) $- \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + ... + \frac{(-1)^n f^n(1)}{n!}$ is

(A) 2^n (B) 2^{n-1}
(C) 0 (D) 1

28. Domain of definition of the function f (x) = $\frac{3}{4-x^2}$ + $\log_{10}(x^3 - x)$, is

(A) (1, 2) (B) (– 1, 0) ∪ (1, 2)
(C) (1, 2) ∪ (2, ∞) (D) (– 1, 0) ∪ (1, 2) ∪ (2, ∞)

29. $\lim_{x \to \pi/2} \frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is

(A) $\frac{1}{8}$ (B) 0
(C) $\frac{1}{32}$ (D) ∞

30. If $\lim_{x \to 0} \frac{\log(3+x)-\log(3-x)}{x}$ = k, the value of k is

(A) 0 (B) $-\frac{1}{3}$
(C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

31. Let f (a) = g (a) = k and their n^{th} derivatives f^n (a), g^n (a) exist and are not equal for some n. Further if $\lim_{x \to a} \frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)}$ = 4, then the value of k is

(A) 4 (B) 2
(C) 1 (D) 0

32. The function f (x) = log (x + $\sqrt{x^2+1}$), is
(A) an even function (B) an odd function
(C) a periodic function (D) neither an even nor an odd function

33. If f (x) = $\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then f (x) is

(A) continuous as well as differentiable for all x
(B) continuous for all x but not differentiable at x = 0
(C) neither differentiable nor continuous at x = 0
(D) discontinuous everywhere

34. If the function f (x) = $2x^3 - 9ax^2 + 12a^2x + 1$, where a > 0, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals
(A) 3 (B) 1
(C) 2 (D) $\frac{1}{2}$

35. If f (y) = e^y, g (y) = y; y > 0 and F (t) = $\int_0^t f(t-y)\, g(y)\, dy$, then

(A) F (t) = $1 - e^{-t}(1 + t)$ (B) F (t) = $e^t - (1 + t)$
(C) F (t) = $t e^t$ (D) F (t) = $t e^{-t}$

36. If f (a + b – x) = f (x), then $\int_a^b x\, f(x)\, dx$ is equal to

(A) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (B) $\frac{a+b}{2}\int_a^b f(x)dx$
(C) $\frac{b-a}{2}\int_a^b f(x)dx$ (D) $\frac{a+b}{2}\int_a^b f(a+b-x)dx$

37. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\, dt}{x \sin x}$ is

(A) 3 (B) 2
(C) 1 (D) 0

38. The value of the integral I = $\int_0^1 x(1-x)^n dx$ is

(A) $\frac{1}{n+1}$ (B) $\frac{1}{n+2}$
(C) $\frac{1}{n+1} - \frac{1}{n+2}$ (D) $\frac{1}{n+1} + \frac{1}{n+2}$

39. $\lim_{n\to\infty} \frac{1+2^4+3^4+......+n^4}{n^5} - \lim_{n\to\infty} \frac{1+2^3+3^3+......+n^3}{n^5}$ is

(A) $\frac{1}{30}$ (B) zero
(C) $\frac{1}{4}$ (D) $\frac{1}{5}$

40. Let $\frac{d}{dx}$F (x) = $\left(\frac{e^{\sin x}}{x}\right)$, x > 0. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx$ = F (k) – F (1), then one of the possible values of k, is

(A) 15 (B) 16
(C) 63 (D) 64

41. The area of the region bounded by the curves y = |x – 1| and y = 3 – |x| is

(A) 2 sq units (B) 3 sq units
(C) 4 sq units (D) 6 sq units

42. Let f (x) be a function satisfying f′ (x) = f (x) with f (0) = 1 and g (x) be a function that satisfies f (x) + g (x) = x^2. Then the value of the integral $\int_0^1 f(x)\, g(x)\, dx$, is

(A) $e - \frac{e^2}{2} - \frac{5}{2}$ (B) $e + \frac{e^2}{2} - \frac{3}{2}$

(C) $e - \frac{e^2}{2} - \frac{3}{2}$ (D) $e + \frac{e^2}{2} + \frac{5}{2}$

43. The degree and order of the differential equation of the family of all parabolas whose axis is x–axis, are respectively

(A) 2, 1 (B) 1, 2

(C) 3, 2 (D) 2, 3

44. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$, is

(A) $(x - 2) = k\ e^{-\tan^{-1} y}$ (B) $2x e^{2\tan^{-1} y} + k$

(C) $x\ e^{\tan^{-1} y} = \tan^{-1} y + k$ (D) $x e^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

45. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of 'c' is

(A) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (B) $a_1^2 + a_2^2 + b_1^2 - b_2^2$

(C) $\frac{1}{2}(a_1^2 + a_2^2 - b_1^2 - b_2^2)$ (D) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

46. Locus of centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, – b cos t) and (1, 0), where t is a parameter, is

(A) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ (B) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(C) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$ (D) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

47. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then

(A) p = q (B) p = – q

(C) pq = 1 (D) pq = – 1

48. a square of side a lies above the x–axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \frac{\pi}{4})$ with the positive direction of x–axis. The equation of its diagonal not passing through the origin is

(A) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

(B) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(C) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

(D) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

49. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(A) 2 < r < 8 (B) r < 2

(C) r = 2 (D) r > 2

50. The lines 2x – 3y = 5 and 3x – 4y = 7 are diameters of a circle having area as 154 sq units. Then the equation of the circle is

(A) $x^2 + y^2 + 2x - 2y = 62$ (B) $x^2 + y^2 + 2x - 2y = 47$

(C) $x^2 + y^2 - 2x + 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 62$

51. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then

(A) $t_2 = -t_1 - \dfrac{2}{t_1}$ (B) $t_2 = -t_1 + \dfrac{2}{t_1}$

(D) $t_2 = t_1 - \dfrac{2}{t_1}$ (D) $t_2 = t_1 + \dfrac{2}{t_1}$

52. The foci of the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{144}-\dfrac{y^2}{81}=\dfrac{1}{25}$ coincide. Then the value of b^2 is

(A) 1 (B) 5
(C) 7 (D) 9

53. A tetrahedron has vertices at O (0, 0, 0), A (1, 2, 1), B (2, 1, 3) and C (– 1, 1, 2). Then the angle between the faces OAB and ABC will be

(A) $\cos^{-1}\left(\dfrac{19}{35}\right)$ (B) $\cos^{-1}\left(\dfrac{17}{31}\right)$
(C) 30^0 (D) 90^0

54. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

(A) 1 (B) 2
(C) 3 (D) 4

55. The lines $\dfrac{x-2}{1}=\dfrac{y-3}{1}=\dfrac{z-4}{-k}$ and $\dfrac{x-1}{k}=\dfrac{y-4}{2}=\dfrac{z-5}{1}$ are coplanar if

(A) k = 0 or – 1 (B) k = 1 or – 1
(C) k = 0 or – 3 (D) k = 3 or – 3

56. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ will be perpendicular, if and only if

(A) $aa' + bb' + cc' + 1 = 0$ (B) $aa' + bb' + cc' = 0$
(C) $(a + a')(b + b') + (c + c') = 0$ (D) $aa' + cc' + 1 = 0$

57. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(A) 26 (B) $11\dfrac{4}{13}$
(C) 13 (D) 39

58. Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a′, b′, c′ from the origin, then

(A) $\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}+\dfrac{1}{a'^2}+\dfrac{1}{b'^2}+\dfrac{1}{c'^2} = 0$ (B) $\dfrac{1}{a^2}+\dfrac{1}{b^2}-\dfrac{1}{c^2}+\dfrac{1}{a'^2}+\dfrac{1}{b'^2}-\dfrac{1}{c'^2} = 0$

(C) $\dfrac{1}{a^2}-\dfrac{1}{b^2}-\dfrac{1}{c^2}+\dfrac{1}{a'^2}-\dfrac{1}{b'^2}-\dfrac{1}{c'^2} = 0$ (D) $\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}-\dfrac{1}{a'^2}-\dfrac{1}{b'^2}-\dfrac{1}{c'^2} = 0$

59. $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to

(A) 0 (B) – 7
(C) 7 (D) 1

60. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non–coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w})\cdot(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals

(A) 0 (B) $\vec{u}\cdot\vec{v}\times\vec{w}$

(C) $\vec{u}\cdot\vec{w}\times\vec{v}$ (D) $3\vec{u}\cdot\vec{v}\times\vec{w}$

61. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a

(A) square (B) rhombus

(C) rectangle (D) parallelogram but not a rhombus

62. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$, and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(A) $\sqrt{18}$ (B) $\sqrt{72}$

(C) $\sqrt{33}$ (D) $\sqrt{288}$

63. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is

(A) 20 units (B) 30 units

(C) 40 units (D) 50 units

64. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is unit vector such that $\vec{u}\cdot\hat{n}=0$ and $\vec{v}\cdot\hat{n}=0$, then $|\vec{w}\cdot\hat{n}|$ is equal to

(A) 0 (B) 1

(C) 2 (D) 3

65. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set

(A) is increased by 2 (B) is decreased by 2

(C) is two times the original median (D) remains the same as that of the original set

66. In an experiment with 15 observations on x, then following results were available:

$\sum x^2 = 2830$, $\sum x = 170$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is

(A) 78.00 (B) 188.66

(C) 177.33 (D) 8.33

67. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

68. Events A, B, C are mutually exclusive events such that P (A) = $\frac{3x+1}{3}$, P (B) = $\frac{1-x}{4}$ and P (C) = $\frac{1-2x}{2}$. The set of possible values of x are in the interval

(A) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (B) $\left[\frac{1}{3}, \frac{2}{3}\right]$

(C) $\left[\frac{1}{3}, \frac{13}{3}\right]$ (D) [0, 1]

69. The mean and variance of a random variable having a binomial distribution are 4 and 2 respectively, then P (X = 1) is

(A) $\frac{1}{32}$ (B) $\frac{1}{16}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

70. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2$ is

(A) 3 : 1 : 1 (B) 2 : 3 : 2

(C) 1 : 2 : 3 (D) 2 : 3 : 1

71. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in

(A) arithmetic–geometric progression (B) A.P.

(C) G.P. (D) H.P.

72. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle, the moment of the couple thus formed is $\vec{H}$. If instead, the forces $\vec{P}$ are turned through an angle α, then the moment of couple becomes

(A) $\vec{G}\sin\alpha - \vec{H}\cos\alpha$ (B) $\vec{H}\cos\alpha + \vec{G}\sin\alpha$

(C) $\vec{G}\cos\alpha - \vec{H}\sin\alpha$ (D) $\vec{H}\sin\alpha - \vec{G}\cos\alpha$

73. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle with respect to the first is least after a time

(A) $\frac{u\sin\alpha}{f}$ (B) $\frac{f\cos\alpha}{u}$

(C) $u\sin\alpha$ (D) $\frac{u\cos\alpha}{f}$

74. Two stones are projected from the top of a cliff h meters high, with the same speed u so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan\theta$ equals

(A) $\sqrt{\frac{2u}{gh}}$ (B) $2g\sqrt{\frac{u}{h}}$

(C) $2h\sqrt{\frac{u}{g}}$ (D) $u\sqrt{\frac{2}{gh}}$

75. A body travels a distances s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by

(A) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$ (B) $\frac{2s}{\frac{1}{f}+\frac{1}{r}}$

(C) $\sqrt{2s(f+r)}$ (D) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$

Solutions

1. Clearly both one – one and onto
Because if n is odd, values are set of all non–negative integers and if n is an even, values are set of all negative integers.
Hence, (C) is the correct answer.

2. $z_1^2 + z_2^2 - z_1z_2 = 0$
$(z_1 + z_2)^2 - 3z_1z_2 = 0$
$a^2 = 3b$.
Hence, (C) is the correct answer.

5. $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$
$(1 + abc)\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$
$\Rightarrow abc = -1$.
Hence, (B) is the correct answer

4. $\frac{1+i}{1-i} = \frac{(1+i)^2}{2} = i$
$\left(\frac{1+i}{1-i}\right)^x = i^x$
$\Rightarrow x = 4n$.
Hence, (A) is the correct answer.

6. Coefficient determinant = $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$
$\Rightarrow b = \frac{2ac}{a+c}$.
Hence, (C) is the correct answer

8. $x^2 - 3|x| + 2 = 0$
$(|x| - 1)(|x| - 2) = 0$
$\Rightarrow x = \pm 1, \pm 2$.
Hence, (B) is the correct answer

7. Let α, β be the roots
$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$
$\alpha + \beta = \frac{\alpha^2 + \beta^2 - 2\alpha\beta}{(\alpha+\beta)}$
$\left(-\frac{b}{a}\right) = \frac{b^2 - 2ac}{c^2}$
$\Rightarrow 2a^2c = b(a^2 + bc)$

$\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.

Hence, (C) is the correct answer

10. $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$A^2 = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$= \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$

$\Rightarrow \alpha = a^2 + b^2, \beta = 2ab.$

Hence, (B) is the correct answer.

9. $\beta = 2\alpha$

$3\alpha = \frac{3a-1}{a^2 - 5a + 3}$

$2\alpha^2 = \frac{2}{a^2 - 5a + 6}$

$\frac{(3a-1)^2}{a(a^2 - 5a + 3)^2} = \frac{1}{a^2 + 5a + 6}$

$\Rightarrow a = \frac{2}{3}$.

Hence, (A) is the correct answer

12. Clearly $5! \times 6!$

(A) is the correct answer

11. Number of choices = ${}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$

= 140 + 56.

Hence, (B) is the correct answer

13. $\Delta = \begin{vmatrix} 1 + \omega^n + \omega^{2n} & \omega^n & \omega^{2n} \\ 1 + \omega^n + \omega^{2n} & \omega^{2n} & 1 \\ 1 + \omega^n + \omega^{2n} & 1 & \omega^n \end{vmatrix}$

= 0

Since, $1 + \omega^n + \omega^{2n} = 0$, if n is not a multiple of 3

Therefore, the roots are identical.

Hence, (A) is the correct answer

14. ${}^nC_{r+1} + {}^nC_{r-1} + {}^nC_r + {}^nC_r$

$= {}^{n+1}C_{r+1} + {}^{n+1}C_r$

$= {}^{n+2}C_{r+1}$.

Hence, (B) is the correct answer

17. $\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} - \dots\dots\dots$

$= 1 - \frac{1}{2} - \frac{1}{2} + \frac{1}{3} + \frac{1}{3} - \frac{1}{4} - \dots\dots\dots$

$= 1 - 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - ....\right)$

$= 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\right) - 1$

= 2 log 2 – log e

$= \log\left(\frac{4}{e}\right)$.

Hence, (D) is the correct answer.

15. General term = $^{256}C_r\,(\sqrt{3})^{256-r}\,[(5)^{1/8}]^r$
From integral terms, or should be 8k
⇒ k = 0 to 32.
Hence, (B) is the correct answer.

18. $f(x) = ax^2 + bx + c$
f (1) = a + b + c
f (– 1) = a – b + c
⇒ a + b + c = a – b + c also 2b = a + c
f′ (x) = 2ax + b = 2ax
$f'(a) = 2a^2$
f′ (b) = 2ab
f′ (c) = 2ac
⇒ AP.
Hence, (A) is the correct answer.

19. Result (A) is correct answer.

20. (B)

21. $a\left(\frac{1+\cos C}{2}\right) + c\left(\frac{1+\cos A}{2}\right) = \frac{3b}{2}$
⇒ a + c + b = 3b
a + c = 2b.
Hence, (A) is the correct answer

26. f (1) = 7
f (1 + 1) = f (1) + f (1)
f (2) = 2 × 7
only f (3) = 3 × 7

$\sum_{r=1}^{n} f(r) = 7\,(1 + 2 + \ldots\ldots\ldots + n)$

$= 7\,\frac{n(n+1)}{2}$.

25. (B)

23. $-\frac{\pi}{4} \le \frac{\sin^2 x}{2} \le \frac{\pi}{4}$

$-\frac{\pi}{4} \le \sin^{-1}(a) \le \frac{\pi}{4}$

$\frac{1}{2} \le |a| \le \frac{1}{\sqrt{2}}$.

Hence, (D) is the correct answer

27. LHS = $1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \ldots\ldots\ldots$

$= 1 - {}^nC_1 + {}^nC_2 - \ldots\ldots\ldots$

$= 0$.

Hence, (C) is the correct answer

30. $\lim_{x \to 0} \frac{\frac{1}{3+x} + \frac{1}{3-x}}{1} = \frac{2}{3}$.

Hence, (C) is the correct answer.

28. $4 - x^2 \ne 0$

$\Rightarrow x \ne \pm 2$

$x^3 - x > 0$

$\Rightarrow x (x + 1) (x - 1) > 0$.

Hence (D) is the correct answer.

29. $\lim_{x \to \pi/2} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)(1 - \sin x)}{4\left(\frac{\pi}{4} - \frac{x}{2}\right)(\pi - 2x)^2}$

$= \frac{1}{32}$.

Hence, (C) is the correct answer.

32. $f(-x) = -f(x)$

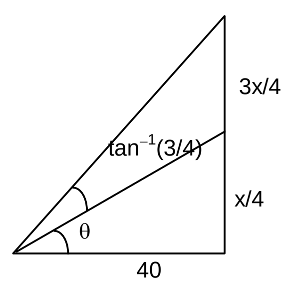
Hence, (B) is the correct answer.

1. $\sin(\theta + \alpha) = \frac{x}{40}$

$\sin a = \frac{x}{140}$

$\Rightarrow x = 40$.

Hence, (B) is the correct answer



34. $f(x) = 0$ at $x = p, q$

$6p^2 + 18ap + 12a^2 = 0$

$6q^2 + 18aq + 12a^2 = 0$

$f''(x) < 0$ at $x = p$

and $f''(x) > 0$ at $x = q$.

30. Applying L. Hospital's Rule

$\lim_{x \to 2a} \frac{f(a)g'(a) - g(a)f'(a)}{g'(a) - f'(a)} = 4$

$$\frac{k(g'(a) - ff'(a))}{(g'(a) - f'(a))} = 4$$

k = 4.

Hence, (A) is the correct answer.

36. $\int_a^b x\, f(x)\, dx$

$= \int_a^b (a + b - x)\ f(a + b - x)\, dx.$

Hence, (B) is the correct answer.

33. f′ (0)

f′ (0 – h) = 1

f′ (0 + h) = 0

LHD ≠ RHD.

Hence, (B) is the correct answer.

37. $\lim_{x \to 0} \frac{\tan(x^2)}{x \sin x}$

$= \lim_{x \to 0} \frac{\tan(x^2)}{x^2\left(\frac{\sin x}{x}\right)}$

= 1.

Hence (C) is the correct answer.

38. $\int_0^1 x\,(1 - x)^n\, dx = \int_0^1 x^n\,(1 - x)$

$= \int_0^1 (x^n - x^{n+1}) = \frac{1}{n+1} - \frac{1}{n+2}.$

Hence, (C) is the correct answer.

35. $F(t) = \int_0^t f(t - y)\, f(y)\, dy$

$= \int_0^t f(y)\, f(t - y)\, dy$

$= \int_0^t e^y\,(t - y)\, dy$

$= x^t - (1 + t).$

Hence, (B) is the correct answer.

34. Clearly f″ (x) > 0 for x = 2a ⇒ q = 2a < 0 for x = a ⇒ p = a

or $p^2 = q \Rightarrow a = 2$.

Hence, (C) is the correct answer.

40. $F'(x) = \frac{e^{\sin x}}{3^x}$

$= \int \frac{3}{x} e^{\sin x}\, dx = F(k) - F(1)$

$= \int_1^{64} \frac{e^{\sin x}}{x}\, dx = F(k) - F(1)$

$= \int_1^{64} F'(x)\, dx = F(k) - F(1)$

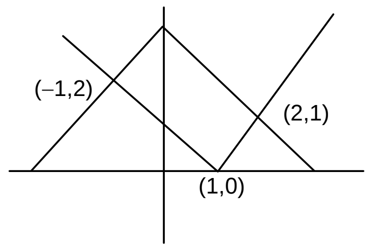
$F(64) - F(1) = F(k) - F(1)$

$\Rightarrow k = 64.$

Hence, (D) is the correct answer.

41. Clearly area = $2\sqrt{2} \times \sqrt{2}$

= sq units



45. Let p (x, y)

$(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2$

$(a_1 - a_2)\, x + (b_1 - b_2)\, y + \frac{1}{2}\,(b_2^2 - b_1^2 + a_2^2 - a_1^2) = 0.$

Hence, (A) is the correct answer.

46. $x = \frac{a\cos t + b\sin t + 1}{3}$, $y = \frac{a\sin t - b\cos t + 1}{3}$

$\left(x - \frac{1}{3}\right)^2 + y^2 = \frac{a^2 + b^2}{9}.$

Hence, (B) is the correct answer.

43. Equation $y^2 = 4a\ 9x - h)$

$2yy_1 = 4a \Rightarrow yy_1 = 2a$

$yy_2 = {y_1}^2 = 0.$

Hence (B) is the correct answer.

42. $\int_0^1 f(x)\,[x^2 - f(x)]\, dx$

solving this by putting $f'(x) = f(x)$.

Hence, (B) is the correct answer.

50. Intersection of diameter is the point (1, – 1)

$\pi s^2 = 154$

$\Rightarrow s^2 = 49$

$(x - 1)^2 + (y + 1)^2 = 49$

Hence, (C) is the correct answer.

47. (D)

49. $\frac{dx}{dy}\ (1 + y^2) = (e^{\sin^{-1} y} - x)$

$$\frac{dx}{dy} + \frac{x}{1+y^{\alpha}} = \frac{e^{sub^{-1}-y}}{1+y^2}$$

52. $$\frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

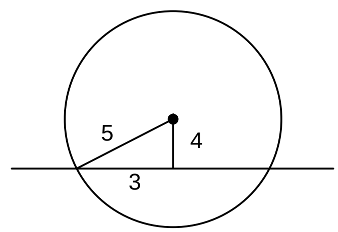
$\Rightarrow e_1 = \frac{5}{4}$

$ae_2 = \sqrt{1-\frac{b^2}{16}} \times 4 = 3$

$\Rightarrow b^2 = 7.$

Hence, (C) is the correct answer.

54. (C)



69. np = 4

npq = 2

$q = \frac{1}{2}, p = \frac{1}{2}$

n = 8

$p\,(x = 1) = {}^8C_1 \left(\frac{1}{2}\right)^8$

$= \frac{1}{32}.$

Hence, (A) is the correct answer.

49. $(x-1)^2 + (y-3)^2 = r^2$

$(x-4)^2 + (y+2)^2 - 16 - 4 + 8 = 0$

$(x-4)^2 + (y+2)^2 = 12.$

67. Select 2 out of 5

$= \frac{2}{5}.$

Hence, (D) is the correct answer.

65. $0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$12x + 4 + 3 - 3x + 6 - 12x \le 1$

$0 \le 13 - 3x \le 12$

$3x \le 13$

$\Rightarrow x \ge \frac{1}{3}$

$x \le \frac{13}{3}.$

Hence, (C) is the correct answer.

3. $\text{Arg}\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$

$|z\omega| = 1$

$\bar{z}\,\omega = -i$ or $+i$.