AIEEE EXAMINATION PAPER 2010
Code-A

PHYSICS, CHEMISTRY , MATHEMATICS

Time : - 3 Hours **Max. Marks:- 432** **Date : 25/04/10**

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.

2 . The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions. The maximum marks are **432.**

5. There are **three** parts in the question paper.
The distribution of marks subjectwise in each part is as under for each correct response.
Part A – Physics (144 Marks) –Questions No. 1 to 20 and 23 to 26 consist of **FOUR (4)** marks each and Questions No. 21 to 22 and 27 to 30 consist of **EIGHT (8)** marks each for each correct response.
Part B – Chemistry (144 Marks) – Questions No. 31 to 39 and 43 to 57 consist of **FOUR (4)** marks each and Questions No.40 to 42 and 58 to 60 consist of **EIGHT (8)** marks each for each correct response.
Part C – Mathematics (144 Marks) – Questions No.61 to 66, 70 to 83 and 87 to 90 consist of **FOUR (4)** marks each and Questions No. 67 to 69 and 84 to 86 consist of **EIGHT (8)** marks each for each correct response

6. Candidates will be awarded marks as stated above in instructions No. 5 for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. Use **Blue/Black Ball Point Pen only** for writing particulars/marking responses on **Side-1** and **Side-2** of the Answer Sheet. **Use of pencil is strictly prohibited**.

8. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.

9. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 2 pages (Pages 38 – 39) at the end of the booklet.

10. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.

11. The **CODE** for this Booklet is **A.** Make sure that the **CODE** printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet

12. Do not fold or make any stray marks on the Answer Sheet.

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PHYSICS

Directions: Questions number 1 – 3 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is -

(1) planar

(2) convex

(3) concave

(4) convex near the axis and concave near the periphery

Ans.[1]

2. The speed of light in the medium is –

(1) maximum on the axis of the beam

(2) minimum on the axis of the beam

(3) the same everywhere in the beam

(4) directly proportional to the intensity I

Ans. [2]

3. As the beam enters the medium, it will -

(1) travel as a cylindrical beam

(2) diverge

(3) converge

(4) diverge near the axis and converge near the periphery

Ans. [1]

Directions: Questions number 4 – 5 are based on the following paragraph.

A nucleus of mass M + Δm is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

4 The speed of daughter nuclei is –

(1) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$

(2) $c\frac{\Delta m}{M+\Delta m}$

(3) $c\sqrt{\frac{2\Delta m}{M}}$

(4) $c\sqrt{\frac{\Delta m}{M}}$

Sol. $\Delta mc^2 = 2 \times \frac{1}{2} \times \left(\frac{M}{2}\right) v^2$

$v^2 = \frac{2\Delta mc^2}{M}$, $v = c\sqrt{\frac{2\Delta m}{M}}$

Ans. (3)

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then –

(1) $E_1 = 2E_2$ (2) $E_2 = 2E_1$

(3) $E_1 > E_2$ (4) $E_2 > E_1$

Ans. [4]

Directions: Questions number 6 – 7 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

6. **Statement- 1:** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase –

Statement – 2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because the range of frequencies present in the incident light.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** the correct explanation of Statement-1.

(4) Statement-1 is false, Statement-2 is true.

Ans. [1]

7. **Statement- 1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement- 2: Principle of conservation of momentum holds true for all kinds of collisions.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1

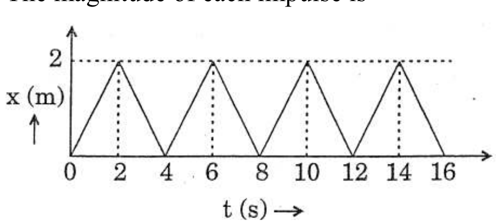
(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** the correct explanation of Statement-1.

(4) Statement-1 is false, Statement-2 is true.

Sol. Both are true but not explain the Ist.

Ans. (3)

8. The figure shows the position – time (x – t) graph of one-dimensional motion of the body of mass 0.4 kg. The magnitude of each impulse is **–**



(1) 0.2 Ns (2) 0.4 Ns

(3) 0.8 Ns (4) 1.6 Ns

Sol. From graph,

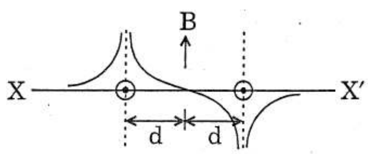
$v_1 = 1\ ms^{-1}$, $v_2 = -1\ ms^{-1}$

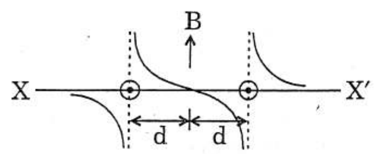
$\therefore \quad J = \int F\,dt = \int dP = m\,\Delta V$

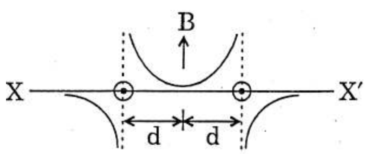
$= 0.4 \times 2 = 0.8$ N.s.

Ans. (3)

9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by -

(1)


(2)


(3)


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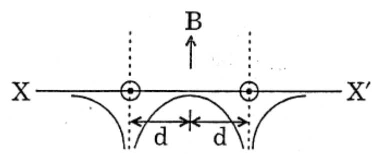
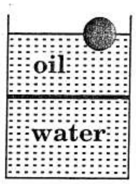
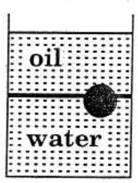


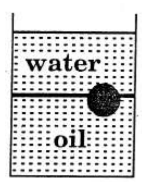
Ans. [2]

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

(1)


(2)


(3)


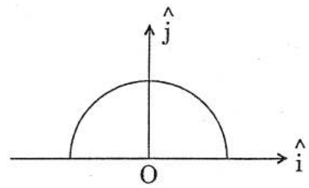
(4)


Sol. $\because \quad \rho_{oil} < \rho < \rho_{water}$,

so ball will not sink in water but sink in oil.

Ans. (3)

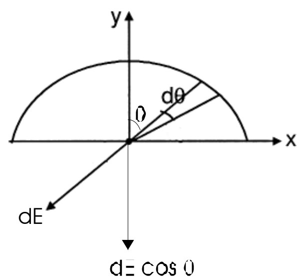
11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is –



(1) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(3) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Sol.



$$E = \int_{-\pi/2}^{\pi/2} dE\cos\theta = 2\int_0^{\pi/2}\frac{k\lambda R d\theta}{R^2}\cos\theta$$

$$\vec{E} = \frac{2}{4\pi\varepsilon_0}\frac{qR}{\pi R\, R^2}[\sin\theta]_0^{\pi/2} = \frac{q}{2\pi^2\varepsilon_0 R^2}[Sin 90 - \sin 0](-\hat{j})$$

$$\vec{E} = \frac{q}{2\pi^2\varepsilon_0 R^2}(-\hat{j})$$

Ans. (4)

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is-

(1) 0.25 (2) 0.5

(3) 0.75 (4) 0.99

Sol. $\eta = \left(1 - \frac{T_2}{T_1}\right)$

$\because \quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\therefore \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{1}{32}\right)^{\gamma-1}$

Putting $\gamma = 7/5$

$$\eta = \left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75$$

Ans. (3)

13. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are -

(1) 4, 4, 2 (2) 5, 1, 2

(3) 5, 1, 5 (4) 5, 5, 2

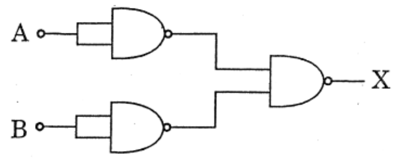
Sol. 23.023 significant fig. 5

0.0003 significant fig. 1

2.1×10^{-3} significant fig. 2

Ans. (2)

14. The combination of gates shown below yields –



(1) NAND gate (2) OR gate

(3) NOT gate (4) XOR gate

Sol. $X = \overline{\overline{A} \cdot \overline{B}} = A + B$

i.e. OR gate

Ans. (2)

15. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called –

(1) γ-rays (2) X-rays

(3) ultraviolet rays (4) microwaves

Sol. $P = n\frac{hc}{\lambda}$

Ans. (2)

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be –

(1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-4}$

(3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$

Sol. ${}^{A}_{Z}X \xrightarrow{(3\alpha+2\,\text{positron})} {}^{A-3\times4}_{Z-3\times2-2\times1}X = {}^{A-12}_{Z-8}X$

$\therefore \frac{\text{No. of Neutrons}}{\text{No. of Protons}} = \frac{(A-12)-(Z-8)}{Z-8}$

$= \frac{A-Z-4}{Z-8}$

Ans. (3)

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto r = R, and ρ(r) = 0 for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by –

(1) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

(2) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(3) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(4) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

Sol. r < R

$$\oint E \cdot dS = \frac{\int \rho_v dv}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \int_0^r \frac{\rho_0}{\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)4\pi r^2 dr$$

$$E \cdot 4\pi r^2 = \frac{\rho_0 4\pi}{\varepsilon_0}\left[\int_0^r \frac{5}{4} r^2 dr - \int_0^r \frac{r^3}{R} dr\right]$$

$$E \cdot 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\left[\frac{5}{4}\frac{r^3}{3} - \frac{r^4}{4R}\right]$$

$$E = \frac{\rho_0}{\varepsilon_0}\left[\frac{5}{4}\frac{r}{3} - \frac{r^2}{4R}\right]$$

$$\boxed{E = \frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)}$$

Ans. (3)

18. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is –

(1) 242 W

(2) 305 W

(3) 210 W

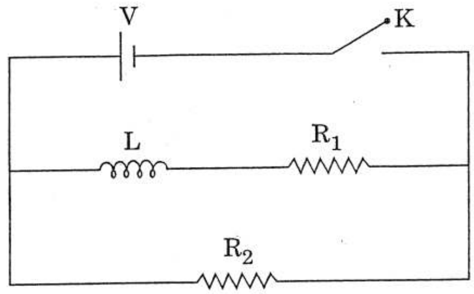
(4) Zero W

Sol. $X_L = X_C$

$$P = \frac{V^2}{R} = 242 \text{ W}$$

Ans. (1)

19. In the circuit shown below, the key K is closed at t = 0. the current through the battery is –



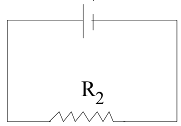
(1) $\frac{V(R_1 + R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(2) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(3) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1 + R_2)}{R_1R_2}$ at t = ∞

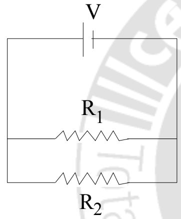
(4) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at t = ∞

Sol. at t = 0



$I = \frac{V}{R_2}$

at t = ∞



$I = \frac{V(R_1 + R_2)}{R_1R_2}$

Ans. (3)

20. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is –

(1) $y^2 = x^2$ + constant

(2) $y = x^2$ + constant

(3) $y^2 = x$ + constant

(4) xy = constant

Sol. $\vec{v} = ky\hat{i} + kx\hat{j}$

$\Rightarrow \frac{dx}{dt} = ky, \ \frac{dy}{dt} = kx$

$\therefore \ \frac{dy}{dx} = \frac{x}{y} \ \Rightarrow \ \int y dy = \int x \, dx$

$\boxed{y^2 = x^2 + \text{constant}}$

Ans. (1)

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1 / t_2 will be –

(1) 2 (2) 1

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Sol. $U = \frac{Q^2}{2C} = \frac{Q_0^2 e^{-\frac{2t}{RC}}}{2C}$ $Q = Q_0 e^{-t/RC}$

$U = \frac{U_0}{2}$

$\frac{Q_0^2}{2\times 2C} = \frac{Q_0^2\, e^{-\frac{2t_1}{RC}}}{2C}$ $\frac{Q_0}{4} = Q_0\, e^{-\frac{t_2}{RC}}$

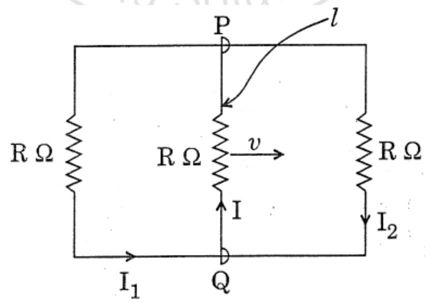
$\frac{1}{2} = e^{-\frac{2t_1}{RC}}$ $\log_e 4 = \frac{t_2}{RC}$

$t_1 = \frac{RC\log_e 2}{2}$ $t_2 = RC\log_e 4$

$\frac{t_1}{t_2} = \frac{1}{4}$

Ans. (4)

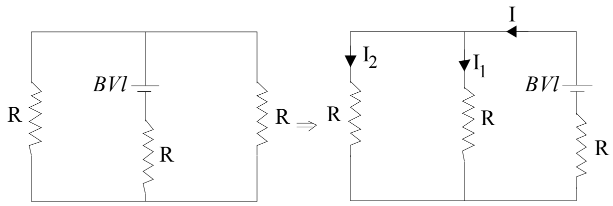
22. A rectangular loop has a sliding connector PQ of length *l* and resistance R Ω and it is moving with a speed *v* as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are –



(1) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ (2) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(3) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$ (4) $I_1 = I_2 = I = \frac{Blv}{R}$

Sol.



$I = \frac{2BVl}{3R}$ $\qquad$ $I_1 = I_2 = \frac{BVl}{3R}$

Ans. (3)

23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by y = 0.02 (m) $\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is –

(1) 6.25 N
(2) 4.0 N
(3) 12.5 N
(4) 0.5 N

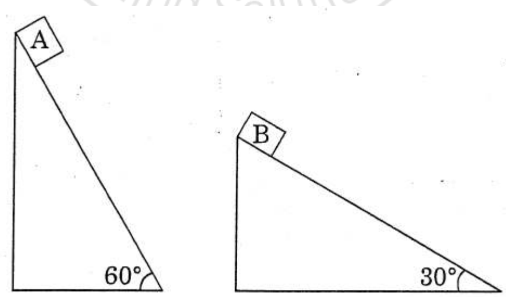
Sol. Putting $\omega = \frac{2\pi}{.04}, k = \frac{2\pi}{0.5}$

in equation $T = \mu v^2 = \mu\left(\frac{\omega}{k}\right)^2$

$= 6.25$ N

Ans. (1)

24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?



(1) 4.9 ms^{-2} in vertical direction
(2) 4.9 ms^{-2} in horizontal direction
(3) 9.8 ms^{-2} in vertical direction
(4) Zero

Sol. $a_{A_{vertical}} = g\sin^2 60° = \frac{3g}{4}$

$a_{B_{vertical}} = g\sin^2 30° = \frac{g}{4}$

So, $a_{AB} = \frac{g}{2} = 4.9\,ms^{-2}$ vertical

Ans. (1)

25. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R, θ) on the circle of radius R is (Here θ is measured from the x-axis)

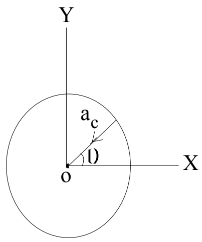
(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(2) $-\frac{v^2}{R}\cos\theta\ \hat{i} + \frac{v^2}{R}\sin\theta\ \hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\ \hat{i} + \frac{v^2}{R}\cos\theta\ \hat{j}$

(4) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\ \hat{j}$

Sol.

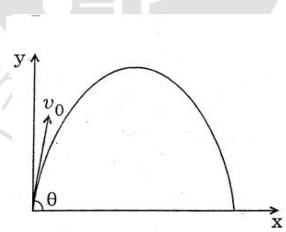


$\vec{a} = a_c \cos\theta(-\hat{i}) + a_c \sin\theta(-\hat{j})$

$\vec{a} = -\frac{V^2}{R}\cos\theta\ \hat{i} - \frac{V^2}{R}\sin\theta\ \hat{j}$

Ans. (4)

26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is –



(1) $\frac{1}{2}mg\,v_0 t^2 \cos\theta\ \hat{i}$

(2) $-mg\,v_0 t^2 \cos\theta\ \hat{j}$

(3) $mg\,v_0 t\cos\theta\ \hat{k}$

(4) $-\frac{1}{2}mg\,v_0 t^2 \cos\theta\ \hat{k}$

where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

Sol. at any time t

$\vec{r} = (v_0 \cos\theta)\,t\,\hat{i} + \left((v_0 \sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$

$\vec{v} = v_0 \cos\theta\,\hat{i} + (v_0 \sin\theta - gt)\,\hat{j}$

so, $\vec{L} = m(\vec{r} \times \vec{v}) = -\frac{1}{2}mgv_0 t^2 \cos\theta\,\hat{k}$

Ans. (4)

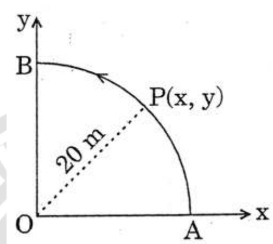
27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is –

(1) 1
(2) 4
(3) 3
(4) 2

Sol. $\frac{1}{1-\frac{\rho}{\sigma}} = \frac{1}{1-\frac{.8}{1.6}} = 2$

Ans. (4)

28. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2s is nearly.



(1) 14 m/s^2
(2) 13 m/s^2
(3) 12 m/s^2
(4) 7.2 m/s^2

Sol. $s = t^3 + 5$

$v = \frac{ds}{dt} = 3t^2$ $\qquad \frac{dv}{dt} = 6t$

$a = \sqrt{(a_r^2 + a_t^2)}$ $\qquad = \sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2} = 14\,m/s^2$

at t = 2s

Ans. (1)

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is –

(1) $\frac{b^2}{6a}$
(2) $\frac{b^2}{2a}$
(3) $\frac{b^2}{12a}$
(4) $\frac{b^2}{4a}$

Sol. $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

so $U_{x=\infty} = 0$

at equilibrium $F = 0 = -\frac{dU}{dx} = -12ax^{-13} + 6bx^{-7} = 0$

$\Rightarrow \quad \frac{1}{x^6} = \frac{b}{2a}$ $\qquad$ So $x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$

$U_{eq} = -\frac{b^2}{4a}$ $\qquad$ $D = \left[0 - \left(-\frac{b^2}{4a}\right)\right] = \frac{b^2}{4a}$

Ans. (4)

30. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. the respective temperature coefficients of their series and parallel combinations are nearly –

(1) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$ (2) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

(3) $\alpha_1 + \alpha_2 \quad \frac{\alpha_1 + \alpha_2}{2}$, (4) $\alpha_1 + \alpha_2 \quad \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$,

In series, $R_1 + R_2 = R_s$

$R(1 + \alpha_1 T) + R(1 + \alpha_2 T) = 2R(1 + \alpha_s T)$

$2R + RT(\alpha_1 + \alpha_2) = 2R + 2R\alpha_s T$

$\alpha_s = \frac{\alpha_1 + \alpha_2}{2}$

In parallel $\frac{1}{Rp} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{1}{\frac{R}{2}(1 + \alpha_p T)} = \frac{1}{R(1 + \alpha_1 T)} + \frac{1}{R(1 + \alpha_2 T)}$$

$2(1 - \alpha_p T) = 1 - \alpha_1 T + 1 - \alpha_2 T$

$\alpha_p = \frac{\alpha_1 + \alpha_2}{2}$

CHEMISTRY

31. In aqueous solution the ionization constants for carbonic acid are $K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$

Selection the correct statement for a saturated 0.034 M solution of the carbonic acid.

(1) The concentration of H^+ is double that of CO_3^{2-}

(2) The concentration of CO_3^{2-} is 0.034 M.

(3) The concentration of CO_3^{2-} is greater than that of HCO_3^-

(4) The concentration of H^+ and HCO_3^- are approximately equal.

Sol. $H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$ $k_1 = 4.2 \times 10^{-7}$

0.034 M x x

$HCO_3^- \rightleftharpoons H^+ + CO_3^{-2}$ $k_1 = 4.8 \times 10^{-11}$

x − y x + 5

Sol. As $k_2 << k_1$, disso in second is negligible

$\therefore$ $x + y \cong x$

and hence $(H^+) \cong [HCO_3^-]$

Ans. (4)

32. Solution product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

(1) 5.0×10^{-8} g (2) 1.2×10^{-10} g

(3) 1.2×10^{-9} g (4) 6.2×10^{-5} g

Sol. $k_{sp} = [Ag^+][Br^-] = 5 \times 10^{-13}$

$[Ag^+] = 0.05M$

$[Br^-] = \frac{5 \times 10^{-13}}{0.05} = 10^{-11} M$

$[Br^-] = [kBr]$

$\therefore$ Mass added in gms = $10^{-11} \times 120$ g

$= 1.2 \times 10^{-9}$ g

Ans. (3)

33. The correct sequence which shows decreasing order of the ionic radii of the elements is

(1) $O^{2-} > F^- > Na^+ > Al^{3+}$ (2) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

(3) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$ (4) $Na^+ > F^- > Mg^{2+} O^{2-} Al^{3+}$

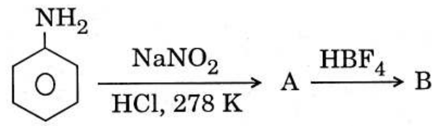
33. **Sol.** Bioelectric series

$r \downarrow \longrightarrow \frac{z}{e} \uparrow$

$\therefore\ O^{-2} > F^{-} > Na^{+} > Mg^{+2} > Al^{+3}$

Ans. (1)

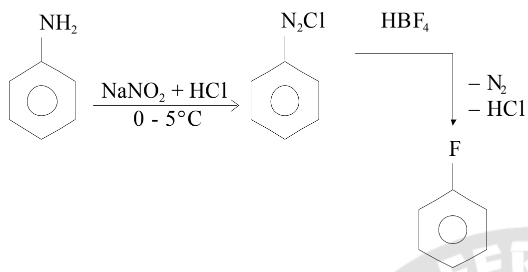
34. In the chemical reactions.



The compounds 'A' and 'B' respectively are

(1) nitrobenzene and chlorobenzene
(2) nitrobenzene and flurobenzene
(3) phenol and benzene
(4) benzene diazonium chloride and flurobenzene

Sol.



Ans. (4)

35. If $10^{-4}\ dm^{-3}$ of water is introduced into a 1.0 dm^{-3} flask at 300 K, how many moles of water are in in the vapour phase when equilibrium is established ?

(Given : Vapour pressure of H_2O at 300 is 3170 pa; R = 8.314 J K^{-1} mol^{-1})

(1) 1.27×10^{-3} mol
(2) 5.56×10^{-3} mol
(3) 1.53×10^{-2} mol
(4) 4346×10^{-2} mol

Sol. PV = nRT

$$3170 \frac{N}{m^2} \times 10^{-3} m^3 = n \times 8.314 \times 300$$

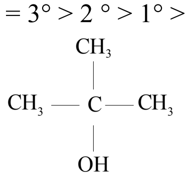
$$n = \frac{3170 \times 10^{-3}}{24.93 \times 10^{-2}} = \frac{31.7 \times 10^{-1}}{24.93 \times 10^{-2}} = 1.287 \times 10^{-1} m$$

Ans. (1)

36. From amongst the following alcohols the one that would react fastest with conc, HCI and anhydrous $ZNCI_2$, is

(1) 1- Butanol
(2) 2- Butanol
(3) 2- Methylpropan -2-ol
(4) 2- Methylpropanol

Sol. Reactivity of LuCaS reagent for alcohol
= 3° > 2 ° > 1° > CH_3



2-methyl. 2-propanol (3° -alcohol) highest reactivity

Ans. (3)

37. If sodium sulphate is considered to be completely dissociated into cations and anions in equeous solution, the change in freezing point of water (ΔT_f), When 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is ($K_f = 1.86 K\ Kg\ mol^{-1}$)

(1) 0.0186 K (2) 0.0372 K

(3) 0.0558 K (4) 0.0744 K

Sol. $Na_2SO_4 \rightleftharpoons 2Na^2 + SO_4^{-2}$

i = 3

$\Delta T_f = iK_f.m.$

= 3 × 1.86 × 0.0

= 0.0558 K

Ans. (3)

38. Three reactions involving $H_2Po_4^-$ are given below:

(1) $H_3Po_4 + H_2O \rightarrow H_3O^+ \rightarrow H_3O^+ H_2Po_4^-$

(2) $H_2Po_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

(3) $H_2Po_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2Po_4^-$ act as an acid?

(1) (i) Only

(2) (ii) Only

(3) (iii) and (ii)

(4) (iii) only

38. **Sol.** (ii) only

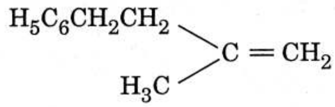
$H_2PO_4^- + H_2O \longrightarrow HPO_4^{-2} + H_3O^+$

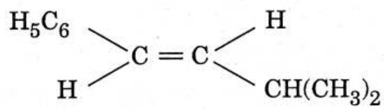
acid base

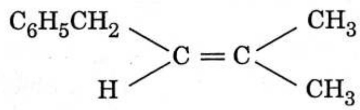
Ans. (2)

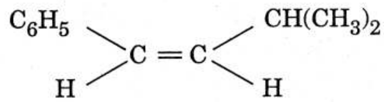
39. The main product of the following reaction is

$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{conc.H_2SO_4} ?$

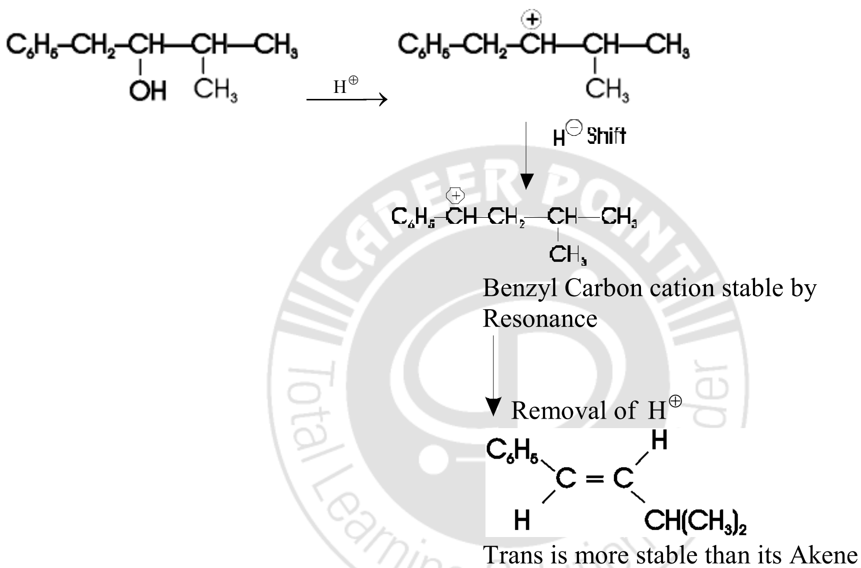
(1) 

(2) 

(3) 

(4) 

Sol.


$H^{\oplus}$
$H^{\ominus}$ Shift
Benzyl Carbon cation stable by Resonance
Removal of $H^{\oplus}$
Trans is more stable than its Akene

Ans. (2)

40. The energy required to break one mole of Cl–Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl–Cl bond is

($C = 3 \times 10^8\ ms^{-1}$ and $N_A = 6.02 \times 10_{23}\ mol^{-1}$)

(1) 494 nm (2) 594

(3) 640 nm (4) 700 nm

Sol. $E = \frac{hc}{\lambda}$

$= \frac{242 \times 10^{+3}}{6.02 \times 10^{23}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$

$\lambda = \frac{6.62 \times 10^{-26} \times 3 \times 6.02 \times 10^{20}}{242}$

$= \frac{6.62 \times 18.06 \times 10^{-6}}{242}$

$= 0.494 \times 10^{-6} = 4.94 \times 10^{-7}$ m

= 494 nm

Ans. (1)

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is

(1) 29.5 (2) 59.0

(3) 47.4 (4) 23.7

41. **Sol.** Equation of NH_3

$= (0.1 \times 20) - (0.1 \times 15) = 0.5$

wt. of NH_3

$= 0.5 \times 17 = 8.5$mg

wt. of 'N'

$= \frac{14}{17} \times 8.5\text{mg} = 7\text{mg}$

% of 'N' = $\frac{7}{29.5} \times 100 = 23.7$

Ans. (4)

42. Ionisation energy of He^+ is 19.6 x 10^{-18} J $atom^{-1}$. The energy of the first stationary state (n = 1) of Li^{2+} is

(1) 8.82 x 10^{-17} J $atom^{-1}$

(2) 4.41 x 10^{-16} J $atom^{-1}$

(3) -4.41×10^{-17} J $atom^{-1}$

(4) -2.2×10^{-15} J $atom^{-1}$

Sol. $\frac{I.E_1}{I.E_2} = \frac{Z_1^2}{Z_2^2}$

$= \frac{19.6\times10^{-18}}{x} = \frac{4}{9}$

$x = \frac{9}{4}\times19.6\times10^{-18} = 44.1\times10^{-18}$ J/atm.

$= 4.41\times10^{-17}$ J/atm

Ans. (3)

43. On mixing, heptane and octane form an ideal solution At 373 K, the vapour pressures of the two liquid components (Heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane =114 g mol^{-1})

(1) 144.5 kPa
(2) 72.0 kPa
(3) 36.1 kPa
(4) 96.2 kPa

Sol. $P_T = P_0x_0 + P_{hep}x_{hep}$

$= 45\times\frac{0.3}{0.55} + 105\times25\frac{0.25}{0.55}$

= 45 × 0.545 + 105 × 0.454

= 72.25 kPa.

Ans. (2)

44. Which one of the following has an optical isomer ?

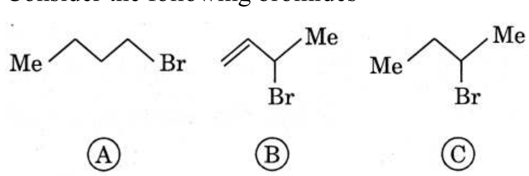
(1) $[Zn(en)_2]^{2+}$
(2) $[Zn(en)(NH_3)_2]^{2+}$
(3) $[CO(en)_3]^{3+}$
(4) $[Co(H_2O)_4(en)]^{3+}$

Sol. $[M(AA)_3]$ type of compound

∴ optically active

Ans. (3)

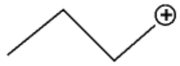
45. Consider the following bromides



The correct order of S_N1 reactivity is

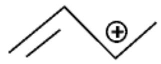
(1) A > B > C
(2) B > C > A
(3) B > A > C
(4) C > B > A

Sol. Reactivity for $SN' \propto$ Stability of corbocation



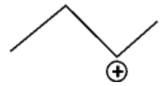
1° carbocation

(A)



stable by resonance

(B)



2° Carbocation

(C)

B > C > A

Ans. (2)

46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is -

(1) ethene (2) propene

(3) 1-butene (4) 2-butene

Sol. Molecular weight = 44 $\therefore [CH_3 - CHO]$

$$CH_3 - CH = O + O = CH - CH_3 \longrightarrow \underset{\text{2-butene}}{CH_3 - CH = CH - CH_3}$$

Ans. (4)

47. Consider the reaction :

$$Cl_2(aq) + H_2S(aq) \rightarrow S(s) + 2H^+(aq) + 2\,Cl^-(aq)$$

The rate equation for this reaction is

Rate = k $[Cl_2][H_2S]$

Which of these mechanisms is/are consistent with this rate equation ?

(A) $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)
$Cl + + HS^- \rightarrow H^+ + Cl^- + S$ (fast)

(B) $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)
$Cl_2 + HS^- \rightarrow 2Cl^- + H^+ + S$ (slow)

(1) A only (2) B only

(3) Both A and B (4) Neither A nor B

Sol. $r = k[Cl_2][H_2S]$

$\therefore$ According to A $\rightarrow r = k[H_2S][Cl_2]$

$\therefore$ According to B $\rightarrow r = k[Cl_2][HS]$

or $K_{eq} = \dfrac{[H^+][HS]}{[H_2S]}$

$[HS] = K_{eq}\dfrac{[H_2S]}{H^+}$

$r = k[Cl_2]\ K_{eq}\dfrac{[H_2S]}{[H^+]}$

$= K'\dfrac{[Cl_2][H_2S]}{[H^+]}$

$\therefore$ (A) Only

Ans. (1)

48. The Gibbs energy for the decomposition of Al_2O_3 at 500 °C is as follows:

$$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2,\ \Delta_rG = +966 \text{kJ mol}^{-1}$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500 °C is at least

(1) 5.0 V (2) 4.5 V

(3) 3.0 V (4) 2.5 V

Sol. $\Delta G = -nFE$ $n = 4$

$966 \times 10^3 = -4 \times 96500 \times E = 2.5$ V

Ans. (4)

49. The correct order of increasing basicity of the given conjugate bases $(R = CH_3)$ is

(1) $RCO\overline{O} < HC \equiv \overline{C} < \overline{N}H_2 < \overline{R}$ (2) $RCO\overline{O} < HC \equiv \overline{C} < \overline{R} < \overline{N}H_2$

(3) $\overline{R} < HC \equiv \overline{C} < RCO\overline{O} < \overline{N}H_2$ (4) $RCO\overline{O} < \overline{N}H_2 < HC \equiv \overline{C} < \overline{R}$

Sol. Confugated acid

RCOOH $CH \equiv CH$ NH_3 $R - H$

Order to A.S. $\Rightarrow RCOOH > CH = CH > NH_3 > R - H$

Order to B.S. $\Rightarrow RCOO^- < CH \equiv C^- < \overline{NH_2} < R^-$

Ans. (1)

50. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

(1) 144 pm (1) 288 pm

(3) 398 pm (4) 618 pm

Sol. $r_{\oplus}^{\bullet} + r_{(-)}^{\bullet} = \frac{q}{2}$

$110 + r_{(-)}^{\bullet} = \frac{508}{2}$

$r_{(-)} = 254 - 110$

$= 144$ nm

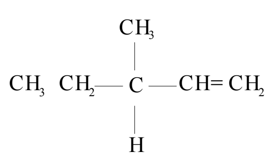
Ans. (1)

51 Out of the following the alkene that exhibits optical isomerism is

(1) 2-methyl-2-pentene (2) 3-methyl-2-pentene

(3) 4-methyl-pentene (4) 3-methyl-1-pentene

Sol.



Due to presence of chiral carbon atom is 4 is show optical isomerism.

Ans. (4)

52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(1) $T = T_e$ (2) $T_e > T$

(3) $T > T_e$ (4) T_e is 5 times T

Sol. $\Delta G = \Delta H - T\Delta S$

+ve +ve

$T > T_e$ for $\Delta G = -ve$

Ans. (3)

53. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively

(1) 48% and 26% (2) 30% and 26%

(3) 26% and 32% (4) 32% and 48%

Sol. ccp : p – f = 74%; 100 – 74 = 26%

bcc : p – f = 68%; 100 – 68 = 32%

Ans. (3)

54. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is

(1) natural rubber (2) teflon

(3) nylon 6,6 (4) polystyrene

Sol. Fact

Ans. (3)

55. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions ?

(1) 8 (2) 9

(3) 10 (4) 11

Sol. $K_{sp} = [Mg^{+2}][OH^-]^2$

$1 \times 10^{-11} = [0.001][OH^-]^2$

$[oh^-] = 10^{-4}$

P on = 4; pn = 10

Ans. (3)

56. The correct order of $E^\circ_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(1) Cr > Mn > Fe > Co (2) Mn > Cr > Fe > Co

(3) Cr > Fe > Mn > Co (4) Fe > Mn > Cr > Co

Sol.

$E^0_{M+2/M}$	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
	–1.67	–1.18	– 0.91	–1.18	– 0.44	– 0.28	– 0.24	+ o.34	– 0.76

Ans. (2)

57. Biuret test is *not* given by

(1) proteins (2) carbohydrates

(3) polypeptides (4) urea

Sol. Carbohydrate does not give biuret test. Due to absence of amide group.
Ans. (2)

58. The time for half life period of a certain reaction A $\rightarrow$ Products is 1 hour. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction ?

(1) 1h (2) 4 h

(3) 0.5 h (4) 0.25 h

Ans.[4]

59. A solution containing 2.675 g of $CoCl_3 \cdot 6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger . The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass =143.5 g mol^{-1}). The formula of the complex is

(At. mass of Ag = 108 u)

(1) $[CoCl(NH_3)_5]Cl_2$ (2) $[Co(NH_3)_6]Cl_3$

(3) $[CoCl(NH_3)_4]Cl$ (4) $[CoCl_3(NH_3)_3]$

Sol. $COCl_3.6NH_3 \longrightarrow AgCl$

$$4.75 \text{ g or } \frac{4.78}{143.5} = 0.03 \text{ m/s}$$

Ans. (2)

60. The standard enthalpy of formation of NH_3 is -46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is -436 kJ mol^{-1} and that of N_2 is -712 kJ mol^{-1}, the average bond enthalpy of $N-H$ bond in NH_3]

(1) -1102 kJ mol^{-1} (2) -964 kJ mol^{-1}

(3) $+352$ kJ mol^{-1} (4) $+1056$ kJ mol^{-1}

Sol. $\frac{1}{2}N_2(g) + \frac{3}{2}H(g) \longrightarrow NH_3$

$$\Delta H_f = \frac{1}{2}B-E_{N-N} + \frac{3}{2}BE_{H-H} - 3.B.E_{N-H}$$

$$-46 = \frac{1}{2}\times(-712) + \frac{3}{2}\times(-436) - 3\times x$$

$$x = \frac{1056}{3} = 352 \text{ kJ/ml.}$$

Ans. (3)

MATHEMATICS

61. Consider the following relations

R = {(x, y) | x, y are real numbers and x = wy for some rational number w};

$S = \{(\frac{m}{n}, \frac{p}{q})$ | m, n, p and q are integers such that n, q ≠ 0 and qm = pn}. Then -

(1) R is an equivalence relation but *S* is not an equivalence relation

(2) Neither R nor *S* is an equivalence relation

(3) *S* is an equivalence relation but R is not an equivalence relation

(4) R and *S* both are equivalence relations

Sol. Probable part of R is

{(0, 1), (0, 2)}

But (1, 0) ∉ R

as 1 = (w) 0

So not symmetric

ie. not equivalence Relation

$\frac{m}{n} S \frac{p}{q} \rightarrow qm = pn$

Reflexive $\frac{m}{n} S \frac{m}{n} \rightarrow mm = mn$

hence function reflexive .

Let $\frac{m}{n} S \frac{p}{q} \rightarrow qm = pn$

Then $\frac{p}{q} S \frac{m}{n} \rightarrow pn = mq$

hence function symmetric

$\frac{m}{n} S \frac{p}{q} \rightarrow mq = pn$ (1)

$\frac{p}{q} S \frac{r}{s} \rightarrow ps = qr$ (2)

eqn. (1)/(2)

$\frac{m}{n} = \frac{r}{s} \rightarrow \frac{m}{n} S \frac{r}{s}$

hence transitive

So S is equivalence relation

Ans. (3)

62. The number of complex numbers z such that |z –1 | = | z + 1| = | z – i| equals -

(1) 0 (2) 1

Sol. $|z - 1| = |z + 1| = |z - i|$

The point z is equidistance from (–1, 0), (1, 0) and (0, 1) is only (0, 0)

hence z is only point (0, 0)

Ans. (2)

63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) – 2 (2) – 1

(3) 1 (4) 2

Sol. Here roots of $x^2 - x + 1 = 0$ are $-\omega$ and $-\omega^2$.

$(-\omega)^{2009} + (-\omega^2)^{2009} = (-\omega)^{2007} \times (-\omega)^2 + (-\omega^2)^{2007} \times (-\omega^2)^2$

$-(\omega^2 + \omega) = 1.$

Ans. (3)

64. Consider the system of linear equations :

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has

(1) Infinite number of solutions (2) Exactly 3 solutions

(3) a unique solution (4) No solution

Sol. Here $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 1\ (1) - 2(1) + 1\ (1) = 0$

$$\Delta_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} = 3\ (1)\ - 2\ (5) + 1\ (12) = 5$$

$\Delta_1 \neq 0$

When $\Delta = 0$ and if $\Delta_1, \Delta_2, \Delta_3$, are not zero then no solution

Ans. (4)

65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is -

(1) 3 (2) 36

(3) 66 (4) 108

Sol. By ${}^3C_2$ way we can select 2 balls from A and By ${}^9C_2$ ways we can select 2 balls from B

Total no. of ways ${}^3C_2 \times {}^9C_2 = 108$

Ans. (4)

66. Let f : (–1, 1) → R be a differentiable function with f(0) = – 1 and f '(0) = 1. Let g(x) = $[f(2f(x) + 2)]^2$,. Then g ' (0) =

(1) 4 (2) –4

(3) 0 (4) –2

Sol. g'(x) = 2 [f(2f(x) + 2)]. f '(2 f(x) + 2) .2f ' (x)

g'(0) = 2 [f(2.f(0)+2)] . f ' (2. f(0) + 2). 2f '(0)

= 2[f(0)]. f '(0) .2

= 2(–1) .(1).2

= – 4

Ans. (2)

67. Let f : R → R be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$

(1) 1 (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) 3

Sol. Function (↑)

f(x) ≤ f(2x) ≤ f(3x)

$1 \le \frac{f(2x)}{f(x)} \le \frac{f(3x)}{f(x)}$

given that $\frac{f(3x)}{f(x)} = 1$

hence $1 \le \frac{f(2x)}{f(x)} \le 1$

hence $\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1$ (by sandwich theorem)

Ans. (1)

68. Let p(x) be a function defined on R such that p'(x) = p'(1 – x), for all x ∈ [0, 1], p(0) = 1 and p(1) = 41. Then $\int_0^1 p(x)dx$ equals -

(1) $\sqrt{41}$ (2) 21

(3) 41 (4) 42

Sol. P'(x) = P' (1 – x)

integrate

P(x) = – P (1 – x) + k ------- (1)

put x = 1

P(1) = – P(0) + k

41 = – 1 + k

K = 42

Put in (1)

P(x) = –P(1 – x) + 42 ------(2)

Now $I = \int_0^1 P(x)dx$

also $I = \int_0^1 P(1-x)dx$

$$2I = \int_0^1 (P(x) + P(1-x))dx$$

using (2) $2I = \int_0^1 42 \ \ dx \qquad = 42(x)_0^1$

I = 21

Ans. (2)

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11},...$ are in an AP with common difference – 2, then the time taken by him to count all notes is -

(1) 24 minutes (2) 34 minutes

(3) 125 minutes (4) 135 minutes

Sol. $a_1 = a_2 = a_3 ..... a_9 = 150$

$a_1 + a_2 + a_3 + ... + a_9 = 1350$

$a_{10} + a_{11} + ... + a_n = 4500 – 1350 = 3150$

$$\frac{n}{2}[2\times 150 + (n-1)(-2)] = 3150$$

$150n – n^2 + n = 3150$

$n^2 – 151n + 3150 = 0$

n = 25 min

hence total time = 25 + 9 = 34 min

Ans. (2)

70. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x – axis, is -

(1) y = 0 (2) y = 1

(3) y = 2 (4) y = 3

Sol. $y = x + \frac{4}{x^2}$

$$\frac{dy}{dx} = 1 - \frac{8}{x^3} = 0$$

$$1 = \frac{8}{x^3}$$

$x^3 = 8$

$x = 2$

at x = 2, $y = x + \frac{4}{x^2}$

$= 2 + \frac{4}{4} = 3$

tangent y – 3 = 0 (x – 2)

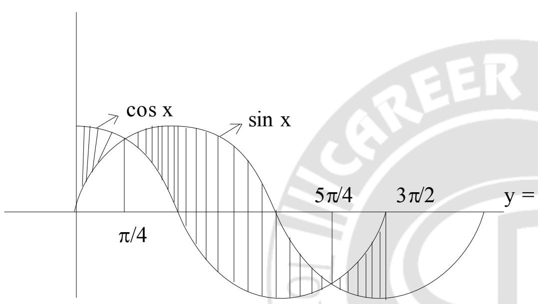
y = 3

Ans. (4)

71. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is -

(1) $4\sqrt{2} - 2$ (2) $4\sqrt{2} + 2$

(3) $4\sqrt{2} - 1$ (4) $4\sqrt{2} + 1$



Sol.

$$\text{Area} = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)dx + \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} (\cos x - \sin x)dx$$

$$= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} + [\sin x + \cos x]_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}$$

$$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (0+1)\right] - \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] + \left[-1 + 0 - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)\right]$$

$$= \sqrt{2} - 1 + \frac{4}{\sqrt{2}} - 1 + \sqrt{2} \quad = 4\sqrt{2} - 2$$

Ans. (1)

72. Solution of the differential equation cos x dy = y (sin x – y) dx, $0 < x < \frac{\pi}{2}$ is -

(1) sec x = (tan x + c) y (2) y sec x = tan x + c

(3) y tan x = sec x + c (4) tan x = (sec x + c) y

Sol. $\cos x \frac{dy}{dx} = y \sin x - y^2$

$\cos x \frac{dy}{dx} - \sin x . y = -y^2$

$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = \sec x$ $\quad -\frac{1}{y} = z$

$\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dz}{dx} + \tan x.z = \sec x$

I.F. = $e^{\int \tan x\, dx}$

Solution of above differential equation is

z. sec x = $\int \sec^2 x\, dx$

$\frac{\sec x}{y} = \tan x + c$

sec x = y (tan x + c)

Ans. (1)

73. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is -

(1) $-\hat{i} + \hat{j} - 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$

(3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

Sol. Let $b = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

given $\vec{a}\,.\,\vec{b} = 3$

$b_2 - b_3 = 3$ (1)

and $\vec{a}\times\vec{b} + \vec{c} = 0$

$\vec{a}\times\vec{b} = -\vec{c}$

$$\begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ b_1 & b_2 & b_3 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

$b_3 + b_2 = -1$ (2)

$-b_1 = 1$ ----- (3)

$-b_1 = 1$ ------(4)

$b_1 = -1$

from (1) and (2)

$b_2 = 1$

$b_3 = -2$

$\vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

Ans. (1)

74. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ) =

(1) (–3, 2) (2) (2, –3)

(3) (–2, 3) (4) (3, –2)

Sol. $\vec{a} \perp \vec{b} \;\therefore \vec{a} \,.\, \vec{b} = 0$

$\vec{b} \perp \vec{c} \therefore \vec{b}.\vec{c} = 0$

$2\lambda + 4 + \mu = 0$ .....(1)

$\vec{a} \perp \vec{c} \therefore \vec{a}.\vec{c} = 0$

$\lambda - 1 + 2\mu = 0$ .....(2)

solving (1) and (2), we get

$\lambda = -3$

$\mu = 2$

Ans. (1)

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) x = 1 (2) 2x + 1 = 0

(3) x = –1 (4) 2x – 1 = 0

Sol. $y^2 = 4x$ comparing with $y^2 = 4ax$

a = 1

Locus of point P will be directrix of given parabola as tangents drawn from P are at right angles, therefore required locus is x = – a

x = – 1

Ans. (3)

76. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is -

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$

(3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

Sol. $\frac{x}{5} + \frac{y}{b} = 1$ (1) Passess through (13, 32)

$\frac{13}{5} + \frac{32}{b} = 1 \Rightarrow 13b + 160 = 5b \Rightarrow b = -20$

so line is $-20x + 5y = -100$ (1)

second line

$\frac{x}{c} + \frac{y}{3} = 1$

3x + cy = 3c (2)

(1) and (2) are parallel

$$\frac{3}{-20} = \frac{c}{5}$$

$$c = \frac{-3}{4}$$

Line $3x - \frac{3}{4}y = -\frac{9}{4}$

12x –3y = – 9

$$-20x + 5y = -9 \times \left(-\frac{5}{3}\right)$$

–20x + 5y = 15 ......(2)

Distance between (1) and (2)

$$= \frac{|-100-15|}{\sqrt{400+25}} \quad = \frac{115}{\sqrt{425}} = \frac{115}{5\sqrt{17}} = \frac{23}{\sqrt{17}}$$

Ans. (4)

77. A line AB in three dimensional space makes angles 45° and 120° with the positive x – axis and the positive y – axis respectively. If AB makes an acute angle θ with the positive z – axis, then θ equals -

(1) 30°
(2) 45°
(3) 60°
(4) 75°

Sol. $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\cos^2 45° + \cos^2 120° + \cos^2 \gamma = 1$

$$\frac{1}{2} + \frac{1}{4} + \cos^2 \gamma = 1$$

$$\cos^2 \gamma = \frac{1}{4}$$

$$\cos \gamma = \pm \frac{1}{2}$$

$$\gamma = 60°$$

Ans. (3)

78. Let S be a non- empty subset of R. Consider the following statement :

P : There is a rational number x ∈ S such that x > 0

Which of the following statements is the negation of the statement P ?

(1) There is a rational number x ∈ S such that x ≤ 0.

(2) There is no rational number x ∈ S such that x ≤ 0.

(3) Every rational number x ∈ S satisfies x ≤ 0.

(4) x ∈ S and x ≤ 0 ⇒ x is not rational

Ans. [3]

79. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and let $\sin(\alpha-\beta)=\frac{5}{13}$, where $0\le\alpha,\beta\le\frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$

(3) $\frac{19}{12}$ (4) $\frac{20}{7}$

Sol. $\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)] = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$ as cos(α + β) = 4/5, sin (α – β) = 5/13

$$\tan 2\alpha = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}} = \frac{\frac{9+5}{12}}{\frac{16-5}{16}} = \frac{56}{33}$$

Ans. (2)

80 The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $-85 < m < -35$ (2) $-35 < m < 15$

(3) $15 < m < 65$ (4) $35 < m < 85$

Sol. $x^2 + y^2 - 4x - 8y - 5 = 0$

centre = (2, 4) and radius = 5

$P < r$ for if line is intersecting the circle at two points

$$P = \left|\frac{3(2)-4(4)-m}{\sqrt{3^2+4^2}}\right| < 5$$

$$|-10-m| < 25$$

$$|10+m| < 25$$

$-35 < m < 15$

Ans. (2)

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is -

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$

(3) 6 (4) $\frac{13}{2}$

Sol. $n_1 = 5$ $n_2 = 5$

$\sigma_1^2 = 4$ $\sigma_2^2 = 5$

$\bar{x}_1 = 2$ $\bar{x}_2 = 4$

sum of data = 10 sum of data = 20

$4 = \frac{1}{5}(sum\ of\ squares) - 4$ $\quad$ $5 = \frac{1}{5}(sum\ of\ squares) - 16$ (as variance $= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$)

sum of squares = 40 $\quad$ sum of squares = 105

$\bar{x} = \frac{10 + 20}{10} = 3$

new variance = $\frac{1}{10}(145) - 9 = \frac{11}{2}$

Ans. (2)

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is -

(1) $\frac{1}{3}$ $\quad$ (2) $\frac{2}{7}$

(3) $\frac{1}{21}$ $\quad$ (4) $\frac{2}{23}$

Sol. Total balls = 3 red balls + 4 blue balls + 2 green balls = 9 balls

required probability = $\frac{{}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^9C_3} = \frac{2}{7}$

Ans. (2)

83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is -

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

Sol. $\tan\left(\frac{\pi}{n}\right) = \frac{\frac{x}{2}}{r} = \frac{x}{2r}$

$r = \frac{x}{2}\cot\left(\frac{\pi}{n}\right)$

and $\sin\frac{\pi}{n} = \frac{x}{2R}$

$R = \frac{x}{2}\operatorname{cosec}\frac{\pi}{n}$

$$\frac{r}{R} = \frac{\cot\left(\frac{\pi}{n}\right)}{\operatorname{cosec}\left(\frac{\pi}{n}\right)} = \cos\left(\frac{\pi}{n}\right)$$

(1) $n = 3, \quad \frac{r}{R} = \frac{1}{2} = .5$

(2) $n = 4, \quad \frac{r}{R} = \frac{1}{\sqrt{2}} = .707$

(3) $n = 5, \quad \frac{r}{R} = \frac{2}{3} = .6$

(4) $n = 6, \quad \frac{r}{R} = \frac{\sqrt{3}}{2}$

(3) is not possible because .6 comes between n = 3 and n = 4 but no integer between n = 3 and n = 4

Ans. (3)

84. The number of 3 × 3 non - singular matrices, with four entries as 1 and all other entries as 0, is -

(1) Less than 4 (2) 5

(3) 6 (4) at least 7

Sol. $A = \begin{vmatrix}1&0&0\\0&1&0\\1&0&1\end{vmatrix}; \begin{vmatrix}1&0&0\\1&1&0\\0&0&1\end{vmatrix}; \begin{vmatrix}1&0&0\\0&1&0\\0&1&1\end{vmatrix}; \begin{vmatrix}1&1&0\\0&1&0\\0&0&1\end{vmatrix}; \begin{vmatrix}1&0&1\\0&1&0\\0&0&1\end{vmatrix}; \begin{vmatrix}1&0&0\\0&1&1\\0&0&1\end{vmatrix}; \begin{vmatrix}1&0&1\\1&0&0\\0&1&0\end{vmatrix}$

So at least 7 non singular matrices are there

Ans. (4)

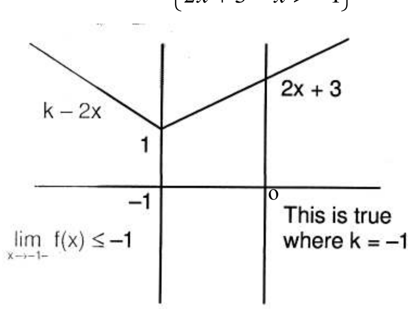
85. Let f : R → R be defined by $f(x) = \begin{cases} k - 2x, & if \ x \le -1 \\ 2x + 3, & if \ x > -1 \end{cases}$

If f has a local minimum at x = –1, then a possible value of k is

(1) 1 (2) 0

(3) $-\frac{1}{2}$ (4) –1

Sol. f:R → R f(x) = $\begin{Bmatrix} k - 2x & x \le -1 \\ 2x + 3 & x > -1 \end{Bmatrix}$ f ' (x) = $\begin{Bmatrix} -2 & x < -1 \\ 2 & x > 1 \end{Bmatrix}$



$k - 2x = +1$

$k = -1$

Ans. (4)

Directions : Questions number 86 to 90 are Assertion – Reason type questions. Each of these questions contains two statements:

Statement – 1 (Assertion) and

Statement – 2 (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer.

You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement – 1 :

The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement – 2 :

If the four chosen numbers form an AP, then the set of all possible values of common difference is {$\pm$ 1, $\pm$ 2, $\pm$ 3, $\pm$ 4 , $\pm$ 5}

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is true.

Sol. S : 1 required no of groups

(1,2,3,4) ……………………………….. (17,18,19,20) = 17 ways
(1,3,5,7) ……………………………….. (14,16,18,20) = 14 ways
(1,4,7,10) ……………………………….. (11,14,17,20) = 11 ways
(1,5,9,13) ……………………………….. (8,12,16,20) = 8 ways
(1,6,11,16) ……………………………….. (5,10,15,20) = 5 ways
(1,7,13,19) ……………………………….. (2,8,14,20) = 2 ways

required arability = $\frac{(17+14+11+8+5+2)4!}{{}^{20}C_4\ 4!}$

$$= \frac{57\ 4!}{20.19.18.17} = \frac{3.4.3.2.1}{20.18.17.}$$

$$= \frac{1}{85}$$

S : 1 is true.
S : 2
possible cases of common difference are
$[\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6]$
S:2 is false
Ans. (3)

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j, S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement – 1 : $S_3 = 55 \times 2^9$.

Statement – 2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is ture.

Sol. $S_1 = \sum_{j=1}^{10} j(j-1)\frac{10!}{j(j-1)(j-2)!(10-j)!} = 90\sum_{j=2}^{10}\frac{8!}{(j-2)!(8-(j-2))!} = 90\times 2^8$

$S_2 = \sum_{j=1}^{10} j\frac{10!}{j(j-1)!(9-(j-1))!} = 10\sum_{j=1}^{10}\frac{9!}{(j-1)!(9-(j-1))!} = 10\times 2^9$

$S_3 = \sum_{j=1}^{10}[j(j-1)+j]\frac{10!}{j(10-j)!} = \sum_{j=1}^{10}(j-1)\,{}^{10}C_j = \sum_{j=1}^{10}(j)\,{}^{10}C_j = 90.2^8 + 10.2^9 = 110.2^8 = 55.2^9$

Hence statement 1 is true, statement 2 is false

Ans. (3)

88. **Statement – 1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane x – y + z = 5.

Statement – 2 : The plane x – y + z = 5 bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is ture.

Sol. Mid point of A(3, 1, 6) and B(1, 3, 4) should lie in the plane
mid point : (2, 2, 5) it satisfies the plane x – y + z = 5.
Also AB $\perp$ to plane. Hence Dr's of AB are < 1, – 1, 1 >
statement 1 and 2 are true

Ans. (1)

89. Let f : R $\to$ R be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$

Statement – 1 : $f(c) = \frac{1}{3}$, for some $c \in$ R.

Statement – 2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all x $\in$ R.

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is ture.

Sol. AM $\ge$ GM

$$\frac{e^x + \frac{2}{e^x}}{2} \ge \sqrt{(e^x)\left(\frac{2}{e^x}\right)}$$

$e^x + \frac{2}{e^x} \geq 2\sqrt{2}$ (1)

$\because e^x > 0 \Rightarrow e^x + \frac{2}{e^x} > 0$ (2)

$0 < \frac{1}{e^x + \frac{2}{e^x}} \leq \frac{1}{2\sqrt{2}}$

also f(c) = 1/3 for c = 0
so statement 1 : is true
statement 2 : is also true with correct explanation
Ans. (1)

90. Let A be a 2 × 2 matrix with non zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define

Tr(A) = sum of diagonal elements of A and

|A| = determinant of matrix A.

Statement – 1 : Tr(A) = 0

Statement – 2 : |A| = 1

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is ture.

Sol. let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = I$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a^2 + bc & ab + bd \\ ac + dc & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

ab + bd = 0
b (a + d) =0
$b \neq 0$
so, a = – d

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ a + d = 0

$T_r(A) = 0$

But $|A| \neq 1$.

So, statement I is true and statement 2 is false.

Ans. (3)