B.Tech. Degree III Semester Examination, December 2006

IT/CS 303 DISCRETE MATHEMATICAL STRUCTURES

(1999 Admissions onwards)

Time: 3 Hours Maximum Marks: 100

I a) Define the characteristic function f_A of a subset A of the universal set U. Show that $f_{AUB} = f_A + f_B - f_A f_B$. (5)

b) Construct truth tables to determine whether following statement is a tautology, a contingency or an absurdity

(i) $q \vee (\sim q \wedge q)$

(ii) $q \Rightarrow (q \Rightarrow p)$ (5)

c) Let $A_1, A_2, \ldots\ldots A_n$ be n sets. Show by mathematical induction that $\left(\bigcup_{i=1}^{\bar{n}} Ai\right) = \bigcap_{i=1}^{n} \bar{A}i$ (10)

OR

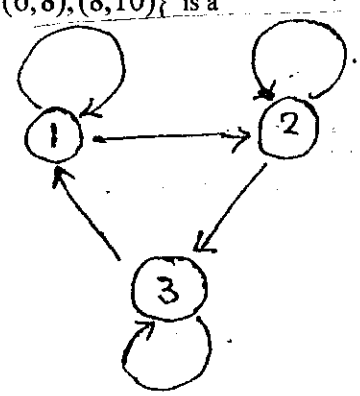
II a) How many 'words' of three distinct letters can be formed from the letters of the word MAST? (5)

b) How many different seven person committees can be formed each containing three women from an available set of 20 women and four men from an available set of 30 men? (5)

c) A die is tossed three times and the resulting sequence of numbers is recorded. What is the probability of the event E that either all three numbers are equal or none of them is 4? (10)

III a) Let $A = \{4,6,8,10\}$ and R $= \{(4,4)(4,10),(6,6),(6,8),(8,10)\}$ is a relation on A. Determine transitive closure of R. (10)

b) Determine whether the relation whose digraph is as follows is an equivalence relation:



(5)

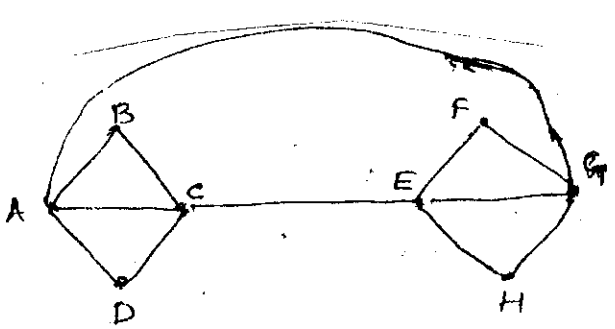
c) If R and S equivalence relations on a set show that $R \cap S$ is also an equivalence relation. (5)

OR

IV a) Let A$= \{1,2,3,4\}$ and R$= \{(2,1),(2,3),(3,2),(3,3),(2,2),(4,2)\}$. Find RoR. (10)

b) Find all partitions of S$= \{1,2,3\}$. (10)

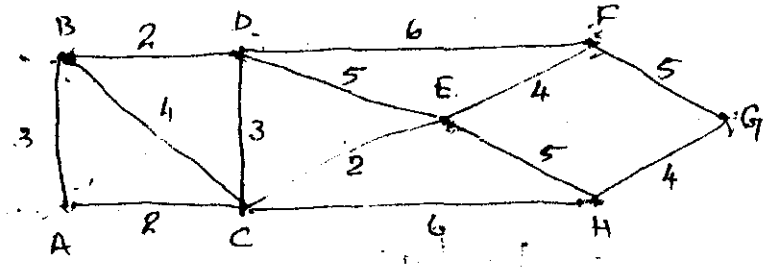
V a) Use Fleury's algorithm to construct an Euler circuit for the following graph.



(10)

(Turn Over)

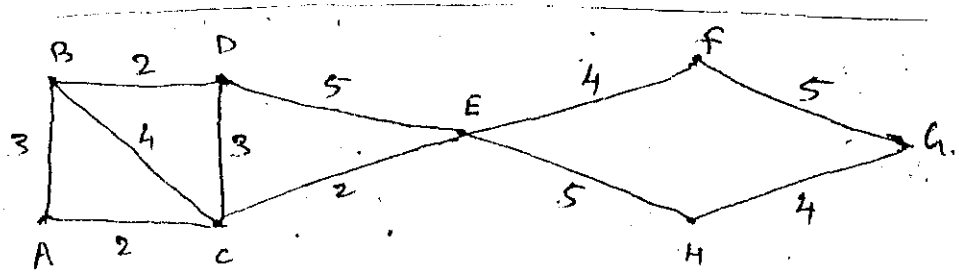
b) Define Hamiltonian circuit. Find a Hamiltonian circuit for the graph given below:



(10)

OR

VI a) Explain Prim's algorithm for finding a minimal spanning tree. Use this algorithm to find a minimal spanning tree for:



(10)

b) Define chromatic number of a graph G. Show that every tree with two or more vertices is 2-chromatic. (10)

VII a) Define a semi group. Let A be a non empty set, explain what is the free semi group F_A generated by A. Is F_A a monoid? (10)

b) Let A=$\{a,b\}$. Does the following table define a semi group?

*	a	b
a	a	b
b	a	a

(10)

OR

VIII a) Show that the set of all permutations of three elements $\{x_1, x_2, x_3\}$ under the operation of composition is a group. (10)

b) Let G and G' be two groups. Define a homomorphism $f: G \to G'$. What is the kernel of f? (10)

IX Show that D_6 the set of all positive divisors of 6 and P(s) where $S = \{a,b\}$, the set of all subsets of S are lattices. Are they isomorphic? (20)

OR

X Let L be a lattice. Then prove the following:

(i) $a \vee a = a$

(ii) $a \vee (b \vee c) = (a \vee b) \vee c$

(iii) $a \vee (a \wedge b) = a$ (20)

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