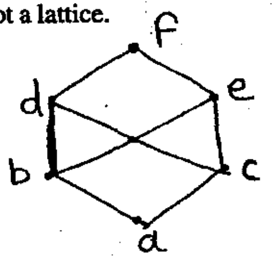
X. (a) Define Lattice. Show that the poset with the Hasse diagram given below is not a lattice.



(b) Show that the posets $(A, \leq)$ and $(A', \leq')$ whose Hasse diagrams are shown below are not isomorphic.

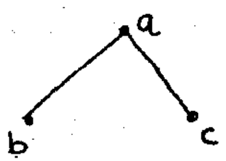


Fig. 1

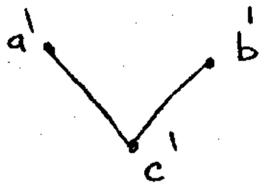


Fig. 2

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B.Tech. Degree III Semester Examination
November 2002

IT/CS 303 DISCRETE MATHEMATICAL STRUCTURES

(1999 Admissions onwards)

Time: 3 Hours Maximum Marks: 100

(All questiosn carry ***EQUAL*** marks)

I. Prove the following:

(i) $\sim(p \leftrightarrow q) \equiv ((p \wedge \sim q) \vee (q \wedge - p))$

(ii) $\sim(p \rightarrow q) \equiv (p \wedge \sim q)$

(iii) $1+2^n < 3^n$, For $n \geq 2$.

OR

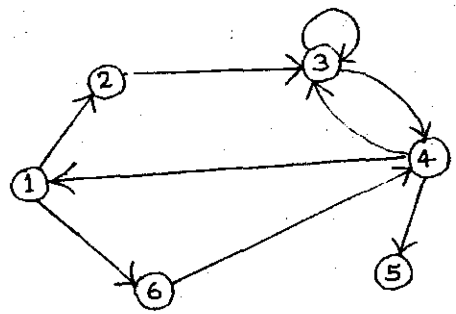
II. (a) An Urn contains 15 balls, 8 of which are red and 7 are black. In how many ways can 5 balls be chosen that

(i) 2 are red and 3 are black?

(ii) All 5 are red.

(b) A woman has five pairs of gloves in a drawer. If she selects two gloves at random what is the probability that the gloves will be a matching pair?

III. (a) Let R be a relation whose digraph is as shown below:



Find (i) MR^2 and (ii) MR^∞

(Turn over)

(b) Let $A=\{1,2,3,4\}$. Determine whether the relation R whose matrix M_R is given is an equivalence relation.

$$M_R=\begin{bmatrix}0&1&0&1\\1&0&1&1\\0&1&0&0\\1&1&0&0\end{bmatrix}$$

OR

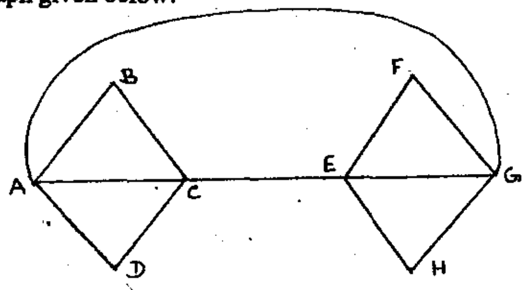
IV. (a) Let $f:R\to R$ and $g:R\to R$ where R is the set of real numbers. Find $f\circ g$ and $g\circ f$ where $f(x)=x^2-2$ and $g(x)=x+4$.

(b) Let $f:R\to R$ be given by $f(x)=x^3-2$, find f^{-1}.

(c) Let $A=B=R$, the set of real numbers. Let $f:A\to B$ be given by the formula $f(x)=2x^3-1$ and let $g:B\to A$ given by $g(y)=\sqrt[3]{\frac{1}{2}y+\frac{1}{2}}$ show that f is a bijection between A and B.

V. (a) Define the following and illustrate them through examples: Euler path, Euler circuit and Hamiltonian path.

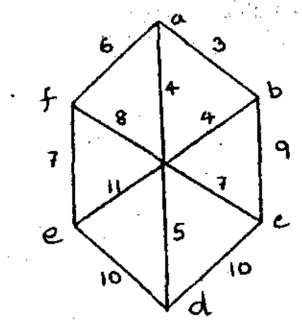
(b) Use Fleury's algorithm to construct an Euler circuit for the graph given below:



OR

Contd.............3.

VI. (a) Find a minimal spanning tree for the connected graph given below using Kruskal's method.



(b) Define proper coloring and chromatic number of a graph G.

VII. (a) Define a group. Give an example of a group which is not abelian.

(b) Let $(A, *)$ be a group and B a subset of A. If B is a finite set, then prove that $(B, *)$ is a subgroup of $(A, *)$ if '$*$' is closed operation on B.

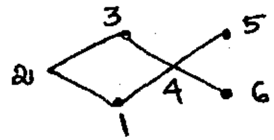
OR

VIII. (a) Define a Monoid. Give an example.

(b) If F is a homomorphism from a commutative semigroup $(S, *)$ onto a semigroup $(T, *')$, then $(T, *')$ is also commutative.

IX. (a) Let $S=\{a,b,c\}$ and $A=P(S)$. Draw the Hasse diagram of the poset A with the partial order $\subseteq$ (set inclusion).

(b) Determine all maximal and minimal elements of the poset given below:



OR

Contd.............4.