AIEEE2010 Mathematics Solution

PART C - Mathematics

61. Consider the following relations :
$R = \{ (x, y) \mid y$ are real numbers and $x = wy$ for some rational number $w)$

$S = \{(\frac{m}{n}, \frac{p}{q}) \mid$ m, n, p and q are integers such that $n, q \neq 0$ and $qm = pn\}$

Then
(1) R is an equivalence relation but S is not an equivalence relation
(2) neither R nor S is an equivalence relation
(3) S is an equivalence relation but R is not an equivalence relation.
(4) R and S both are equivalence relations.

61. **(3)** Here, R is not a symmetric because $(0,y) \in R$ but $(y, 0) \notin R$
$\Rightarrow R$ is not an equivalence relation
But S is an equivalence relation. For reflexive : as $nm = mn \Rightarrow (m / n, m / n) \in S$,
For Symmetric : As $(m / n, p / q) \in S \Rightarrow qm = pn \Rightarrow np = mq \Rightarrow (p / q, m / n) \in S$,
For transitive : As $(m / n, p / q) \in$ S and $(p / q, x / y) \in S \Rightarrow qm = pn$ and $yp = xq$
$\Rightarrow n / y = m / x \Rightarrow ym = xn \Rightarrow (m / n, x / y) \in S$

62. The number of complex numbers z such that
$|z - 1| = |z + 1| = |z - i|$ equals
(1) 0 (2) 1 (3) 2 (4) ∞

62. **(2)** Since, (1, 0), (– 1, 0) and (0, 1) are non-collinear, hence unique value of z would satisfy the given equation i.e. the circumcentre of triangle formed by three points.

63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$
(1) –2 (2) –1 (3) 1 (4) 2

63. **(3)** Since, $\alpha = -\omega$ and $\beta = -\omega^2 \Rightarrow$ required expression = 1

64. Consider the sysytem of linear equations :
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x + x_3 = 3$
$3x_1 + 5x_2 + 2x = 1$
The system has
(1) infinite number of solutions (2) exactly 3 solutions
(3) a unique solution (4) no solution

64. **(4)** Since, $\Delta = 0$ and $\Delta_1 \neq 0 \Rightarrow$ Using Cramer's rule system is inconsistent. Hence no solution.

65. There are two urns. Urn *A* has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then tranferred to the other. The number of ways in which this can be done is

(1) 3 (2) 36 (3) 66 (4) 108

65. **(4)** $^9C_2 \times {}^3C_2 = 108$

66. Let $f : (-1, 1) \to \mathbf{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

(1) 4 (2) –4 (3) 0 (4) –2

66. **(2)** $g'(x) = 2(f(2f(x) + 2)).f'(2f(x) + 2).2f'(x)$
put x = 0, we get $g'(0) = -4$

67. Let $f : \mathbf{R} \to \mathbf{R}$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$

Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$

(1) 1 (2) 2 / 3 (3) 3 / 2 (4) 3

67. **(1)** Let $\lim_{x\to\infty} \frac{f(2x)}{f(x)} = t$, then given limit

$$\lim_{x\to\infty} \frac{f(3x)}{f(2x)} \times \frac{f(2x)}{f(x)} = 1 \Rightarrow t^2 = 1 \Rightarrow t = 1,$$ since function is positive.

68. Let p(x) be a function defined on **R** such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$

(1) $\sqrt{41}$ (2) 21 (3) 41 (4) 42

68. **(2)** $\because p'(x) = p'(1 - x)$. Integrate, $p(x) = -p(1 - x) + k$. put, $x = 0$ then $k = 42$

$$\therefore \int_0^1 p(x)dx = \int_0^1 \frac{p(x) + P(1-x)}{2}dx = \int_0^1 \frac{42}{2}dx = 21$$

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference – 2, then the time taken by him to count all not is

(1) 24 minutes (2) 34 minutes (3) 125 minutes (4) 135 minutes

69. **(2)** $a_1 = a_2 = a_3 = ... a_{10} = 150$
Currency counted in 1st 10 min = 1500
remaining currency = 3000.
$a_{11}, a_{12}, ... \equiv 148, 146, ...$ (sum should be 3000)
$3000 = (n / 2) \{ 2.148 + (n - 1). (-2)\}$
$\therefore$ n = 24 or 125 (125 neglected)
$\therefore$ Rest 3000 can be counted in next 24 min
$\therefore$ total time taken = 34 min.

70. The euqation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x -axis, is

(1) y = 0 (2) y = 1 (3) y = 2 (4) y = 3

70. **(4)** $y = x + (4 / x^2)$ .... (i) $\Rightarrow y' = 1 - (8 / x^3) = 0$ [since parallel to x-axis] $\therefore$ x = 2
$\therefore$ y = 2 + (4 / 4) = 3 (putting in eqn (i))
$\therefore$ Eq.of the tangent parallel to x -axis, $y = 3$

71. The area bounded by the curves y = cos x and y = sin x between the ordination x = 0 and and x = $\frac{3\pi}{2}$

(1) $4\sqrt{2} - 2$ (2) $4\sqrt{2} + 2$ (3) $4\sqrt{2} - 1$ (4) $4\sqrt{2} + 1$

71. **(1)** Required Area = $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x) dx$

$= 4\sqrt{2} - 2$

72. Solution of the differential equation

cos x dy = y (sin x – y) dx, $0 < x < \frac{\pi}{2}$ is

(1) sec x = (tan x + c) y (2) y sec x = tan x + c
(3) y tan x = sec x + c (4) tan x = (sec x + c) y

72. **(1)** $\because \frac{dy}{dx} - y \tan x = -y^2 \sec x \Rightarrow \frac{dt}{dx} + t \tan x = \sec x$, [put, $1 / y = t$]
$\therefore$ I.F. = $e^{\int \tan x dx} = \sec x$ *Hence, Solution is :* $t \sec x = \int \sec^2 x dx + c$
$\Rightarrow \sec x = (\tan x + c) y$

73. Let $\vec{a} = \vec{j} - \vec{k}$ *and* $c = \vec{i} - \vec{j} - \vec{k}$. Then the vector $\vec{b}$ satisasfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ *and* $\vec{a}.\ \vec{b} = 3$

(1) $-\hat{i} + \hat{j} = 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$ (3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

73. **(1)** let $\mathbf{b} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ $\mathbf{a} \times \mathbf{b} = (a_3 + a_2)\,\mathbf{i} - a_1 \mathbf{j} - a_1 \mathbf{k}$

Now $\mathbf{a} \times \mathbf{b} + \mathbf{c} = 0$ $\Rightarrow (a_3 + a_2 + 1)\,\mathbf{i} - (a_1 + 1)\,\mathbf{j} - (a_1 + 1)\,\mathbf{k} = 0$

$\therefore a_1 = -1$ & $a_3 + a_2 = -1$

Again $\mathbf{a} \cdot \mathbf{b} = 3 \Rightarrow a_2 - a_3 = 3$ $\therefore a_2 = 1$ $a_3 = -2$ $\therefore \mathbf{b} = -\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

74 . If the vectors $\vec{a} = \vec{i} - \vec{j} + 2\vec{k},\ \vec{b} = 2\vec{i} + 4\vec{j} + \vec{k}$ and $\vec{c} = \lambda\vec{i} + \vec{j} + \mu\vec{k}$ are matually orthogonal, then $(\lambda, \mu) =$

(1) (-3,2) (2) (2,-3) (3) (-2,3) (4) 3,-2)

74. **(1)** $\mathbf{a}.\ \mathbf{c} = 0 \Rightarrow \lambda - 1 + 2\mu = 0$ also $\mathbf{b}.\ \mathbf{c} = 0$ $2\lambda + 4 + \mu = 0$ $\therefore \lambda = -3,\ \mu = 2.$

75. If two tangents dranw from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) x = 1 (2) 2x + 1 = 0 (3) x = –1 (4) 2x – 1 = 0

75. **(3)** Locus will be directrix, x + 1 = 0

76. The lines L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$ (3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

76. **(4)** First line passes through (13, 32) $\Rightarrow b = -20$, line becomes $4x - y - 20 = 0$

$\therefore -3 / C = 4$ $\Rightarrow C = -3/4$

Second line becomes $4x - y + 3 = 0$ So, perpendicular distance = $(20 + 3 / \sqrt{(4^2 + 1^2)})$ $= 23 / \sqrt{17}$.

77. A line AB is three-dimensional space makes agnels 45° and 120° with positive x-axis and the positive y -axis respectively. If AB makes an acute angle θ with the positive *z*-axis, then θ euals

(1) 90° (2) 45° (3) 60° (4) 75°

77. **(3)** $\cos^2 45° + \cos^2 120° + \cos^2 \theta = 1$ $\therefore \theta = 60°$

78. Let S be a non-empty subset of **R**. Consider the following statement:

P **:** There is a rational number $x \in S$. Such that $x > 0$
Which of the following statements is the negation of the statement **P**?
(1) There is a rational number $x \in S$ such that $x \leq 0$.
(2) There is a no rational number $x \in S$ such that $x \leq 0$.
(3) Every ratioanl number $x \in S$ satisfies $x \leq 0$.
(4) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational

78. **(3)** Negation of *P* is ~ *P*

79. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq 0\ \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) 25 / 10 (2) 56 / 33 (3) 19 / 12 (4) 20 / 7

79. **(2)** $\tan(\alpha + \beta) = (3/4)$, $\tan(\alpha + \beta) = (5/12)$
$\tan 2\alpha = \tan((\alpha + \beta) + (\alpha - \beta)) = 56/33$.

80. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if
(1) $-85 < m < -35$ (2) $-35 < m < 15$
(3) $15 < m < 65$ (4) $35 < m < 85$

80. **(2)** Solving given circle with the line
$25x^2 - x(6m + 160) + m^2 + 32m - 80 = 0$
Since $D > 0$, gives $-35 < m < 15$.

81. For two data sets, each size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4 respectively. The variance of the combined data set is
(1) 5 / 2 (2) 11 / 2 (3) 6 (4) 13 / 2

81. **(2)** Since, combined mean, $a = \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2}\ \frac{5 \times 2 + 5 \times 4}{10} = \frac{30}{10} = 3$

Hence, variance, $\sigma^2 = \frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}$

[where , $d_1 = m_1 - a = -1$, $d_2 = m_2 - a = 1$]

$\therefore \sigma^2 = \frac{5(4+1) + 5(5+1)}{10} = \frac{11}{2}$

82. An run contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have difference colours is
(1) 1 /3 (2) 2 / 7 (3) 1 / 21 (4) 2 / 23

82. **(2)** $n(s) = 9C_3$ $n(E) = 3C_1 . 4C_1 . 2C_1$
$\therefore p(E) = (3C_1 . 4C_1 . 2C_1) / 9C_3 = (2/7)$

83. For a regular polygon, let r and R be the radii of the inscribed and that circumscribed circles. A ***false*** statements among the following is

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

83. **(3)** Since, $\cos(\pi / n) = (r / R)$, for n-sided regular polygon,
Where $n \in N$ and $n \geq 3$. Only option (3) is not possible.

84. The number of 3 × 3 non-singular matrices, with four entries as 1 and 2 all other entires as 0, is

(1) less than 4 (2) 5 (3) 6 (4) at least 7

84. **(4)** In R_1 place two 1' s & one 0 : 3 ways
In R_2 place 1 any where : 3 ways
In R_3, place 1 below 0 of R_1 if R_2's 1 is not below zero, else anywhere **:** At least 1 way
Non- singular since column with 0 in first row has one 1 & expanding using it, submatrix has 3 one's and is non-zero.
∴ At least 3 × 3 = 9 ways.

85. Let f **:** **R** → **R** be defined by

$$f(x) = \begin{cases} k - 2x, & if \quad x \leq -1 \\ 2x + 3, & if \quad x > -1 \end{cases}$$

If f has a local minimum at x = – 1, then a possible values of k is

(1) 1 (2) 0 (3) $-\frac{1}{2}$ (4) – 1

Directions: Questions number 86 to 90 are Assertion-Reason type questions. Each of these questions contains two statements.

Statement -1 : (Assertion) and

Statement-2 : (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. you have to select the correct choice.

85. **(4)** For continuity $k + 2 = 2(-1) + 3 \Rightarrow k = -1$.

86. Four numbers are chosen at random(without replacement) from the set {1, 2, 3, ...20}.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an AP is (1 **/** 85).

Statement-2 : If the four chosen numbers form an AP, then the set of all possible values of {±1, ±2, ±3, ±4, ±5}.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Staement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1.

(3) Statement-1 is true, Staement-2 is false;

(4) Statement-1 is false, Staement-2 is true;

86. **(3)** Four numbers from { 1, 2, ... 20} can be chosen with common difference ±6 also i.e. numbers can be 1, 7,13, 19
$\therefore$ statement - 2 false.

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$, $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement-1 : S_3 **= 55 × 2^{9}.**

Statement-2 : S_1 **= 90 × 2^{8}** and S_2 **= 10 × 2^{8}.**

(1) Statement-1 is true, Statement-2 is true; Staement-2 is a correct explanation for Staement-1.

(2) Statement-1 is true, Statement-2 is true; Staement-2 is not a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false;

(4) Statement-1 is false, Staement-2 is true;

87. **(3)** $S_1 = 10 \times 9 \times 2^8$, $S_2 = 10 \times 2^9$ since $S_3 = S_1 + S_2 = 55 \times 2^9$.

88. **Statement-1 :** The point A(3, 1, 6) is the mirroe image of the point B(1, 3, 4) in the plane x **–** y **+** z = 5.

Statement-2 : The plane x **–** y **+** z = 5 bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement-1 is true, Staement-2 is true; Staement-2 is a correct explanation for Staement-1.

(2) Statement-1 is true, Staement-2 is true; Staement-2 is not a correct explanation for Staement- 1.

(3) Statement-1 is true, Staement-2 is false;

(4) Statement-1 is false, Staement-2 is true;

88. **(2)** Since d.r. of AB is (2**,** –2**,** 2) which is parallel to (1, –1 , 1) i.e. the d.r.of normal to the plane, and mid-point of AB lies on x – y + z = 5. Hence, statement 1 & 2 are both correct but not the proper explanation because only bisection is not sufficient for image, AB must be perpendicular to plane.

89. Let $f : R \to R$ be a continuous function defined by $f(x) = \frac{1}{e^{x} + 2e^{-x}}$

Statement-1 : $f(c) = (1 / 3)$, for some $c \in R$.
Statement-2 : $0 < f(x) \leq (1 / 2\sqrt{2})$, for all $x \in R$.

(1) Statement-1 is true, Staement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Staement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(3) Statement-1 is true, Staement-2 is false;
(4) Statement-1 is false, Staement-2 is true;

89. **(1)** $f(0) = 1 / 3$ Hence, statement -1 is correct. Let $e^{x} = t \in R^{+}$ for all x .
$y = t / (t^2 + 2) \Rightarrow t^2 y - t + 2y = 0$ since $t \in R \Rightarrow \Delta \geq 0$.
$\Rightarrow 1 - 8y^2 \geq 0 \Rightarrow 0 < y \leq 1 / (2\sqrt{2})$. Hence, statement -2 is correct and
$0 < (1 / 3) < (1 / 2\sqrt{2})$, Statement -2 is properly explaining statement -1.

90. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = i$, where I is 2 × 2 identity matrix. Define
Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A.
Statement-1 : Tr (A) = 0
Statement-2 : |A| = 1
(1) Statement-1 is true, Statement-2 is true; Staement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false;
(4) Statement-1 is false, Statement-2 is true;

90. **(3)** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, Since, $A^2 = I \Rightarrow bc + a^2 = 1$, $ab + bd = 0$; $ca + cd = 0$ and $d^2 + bc = 1$
if $a = -d$ we get $|A| = ad - bc = -1$
$\therefore$ Tr(A) $= 0$ but $|A| \neq 1$.