N.B.(1) Question No. **1** is **compulsory**.

(2) Attempt any **four** questions from remaining **six** questions.

1. (a) Prove that— **20**

$$\int_0^\infty x\, e^{-x^4}\, dx \times \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\, dx = \frac{\pi}{4\sqrt{2}} .$$

(b) Use the rule of D.U.I.S. to prove that—

$$\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)}\, dx = \frac{\sqrt{\pi}}{2}\, e^{-2a}$$

$a > 0$

given $\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$.

(c) Solve—

$$\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}\left(x + xy^2\right)dy = 0 .$$

(d) Use Taylor's series method to find y at x = 0·2 given

$$\frac{dy}{dx} = 1 + y^2 \quad \text{with } y = 0 \text{ at } x = 0.$$

2. (a) Solve— **6**

$(D^3 + D^2 + D + 1)\ y = \sin^2 x$

(b) Solve— **7**

$(D^2 + 2)\ y = e^x \cos x + x^2 e^{3x}$

(c) Evaluate $\iint_R \frac{2xy^5}{\sqrt{1 + x^2y^2 - y^4}}\, dx\, dy$ **7**

where R is a triangle whose vertices are (0, 0), (1, 1) and (0, 1).

3. (a) Find the mass of Lamina in the form of cardioide $r = a(1 + \cos\theta)$ if the density at any point varies as it's distance from the pole of cardioide. **6**

(b) Change the order of integration $\int_0^a \int_{\sqrt{a^2 - x^2}}^{x+3a} f(x \cdot y)\ dx\ dy.$ **7**

(c) Solve using Runge-Kutta method of order 4. **7**

$$\frac{dy}{dx} = \frac{y}{x} \quad x_0 = 1,\ y_0 = 1, \quad \text{for } x = 1{\cdot}2 \text{ with } h = 0{\cdot}1.$$

4. (a) Prove that— **6**

$$\int_0^1 \frac{x^3 - 2x^4 + x^5}{(1+x)^7}\, dx = \frac{1}{960} .$$

(b) Solve $\frac{dy}{dx} = x + 3y$, $x_0 = 0$, $y_0 = 1$ by Eular's modified method for x = 0·1 in one step. **7**

Compare the answer with exact value.

(c) Solve : **7**

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos(\log(1+x))$$

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5. (a) Solve— **6**
$y\,dx + x\,(1 - 3x^2y^2)\,dy = 0$

(b) Use polar co-ordinates to evaluate— **7**

$$\iint_R \frac{x^2+y^2}{x^2y^2}\,dx\,dy$$

where R is area common to circles
$x^2 + y^2 = ax$ and $x^2 + y^2 = by$
$a, b > 0$.

(c) Find total length of astroid curve $x^{2/3} + y^{2/3} = a^{2/3}$. **7**

Also prove that the line $\theta = \dfrac{\pi}{3}$ divides the arc of astroid in + ve quadrant in the ratio 1:3.

6. (a) Evaluate— **6**

$$\iiint_V x^2\,dx\,dy\,dz$$ over volume of tetrahedran bounded by

$x = 0,\ y = 0,\ z = 0$, and $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$.

(b) Change the order of integration and evaluate.

$$\int_0^2 \int_{\sqrt{2y}}^2 \frac{x^2\,dx\,dy}{\sqrt{x^4 - 4y^2}}$$

(c) Solve by the method of variation of parameters. **7**

$$(D^2 - 6D + 9)\,y = \frac{e^{3x}}{x^2}.$$

7. (a) Find the volume of solid bounded by cylinder **6**
$x^2 + y^2 = 2ay$, the paraboloid $x^2 + y^2 = az$ and the plane $z = 0$.

(b) The radial displacement in a rotating disc at a distance r from the axis is given by **7**

$$\frac{d^2y}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} + Kr = 0$$

Find the displacement if $u = 0$ at $r = 0$ and at $r = a$.

(c) Prove that—

(i) $$\beta(m,m) \times \beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\pi}{m}\,2^{1-4m}$$

(ii) $$\int_0^\infty \frac{x^4\left(1+x^5\right)}{(1+x)^{15}}\,dx = \frac{1}{5005}.$$