15109 F.E (Sem II / Rev) All Branches

VT-Mar-09-24(I) Maths - II

11 am to 2 pm

Con. 2507–09. **(REVISED COURSE)** **VR–1020**

Lib.

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six** questions.
(3) **Figures** to the **right** indicates **full** marks.

1. (a) Prove that $\int_0^1 (x\log x)^4 dx = \frac{4!}{5^5}$ **20**

(b) Evaluate $\iint_R xy(x-1)\,dxdy$, where R is the region bounded by $xy = 4$, $y = 0$, $x = 1$ and $x = 4$

(c) Find the volume bounded by the cylinder $y^2 = x$, $x^2 = y$ and the planes $z = 0$ and $x + y + z = 2$

(d) Solve $\frac{dy}{dx} = \frac{-(x^2y^3+2y)}{(2x-2x^3y^2)}$.

2. Solve the following differential equations – **20**

(a) $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$

(b) $\frac{d^2y}{dx^2} + y = \sin x \sin 2x$

(c) $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = e^{ax} + e^{bx}$

(d) $dr + (2r\cot\theta + \sin 2\theta)d\theta = 0$.

3. (a) Show that $\int_0^a \frac{dx}{\sqrt[n]{(a^n - x^n)}} = \frac{\pi}{n}\operatorname{cosec}\left(\frac{\pi}{n}\right)$, Where $n > 1$. **6**

(b) Change the order of integration – **6**

$$\int_0^1 \int_{\sqrt{2x-x^2}}^{1+\sqrt{1-x^2}} f(x,y)\,dydx$$

(c) Use the method of variation of parameters to solve – **8**

$$y'' + 3y' + 2y = e^{e^x}$$

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4. (a) Assuming the validity of DUIS, prove that 6

$$\int_0^{\infty}\left(\frac{e^{-ax}-e^{-bx}}{x}\right)\sin mx\,dx = \tan^{-1}\left(\frac{m}{a}\right)-\tan^{-1}\left(\frac{m}{b}\right).$$

(b) Use spherical polar co-ordinates to evaluate $\iiint xyz\,(x^2+y^2+z^2)\,dxdydz$ over the first octant of the sphere $x^2+y^2+z^2=a^2$. 6

(c) Solve using Taylor's series method, the differential equation $\frac{dy}{dx}=x+y$ numerically. Start from $x=1$, $y=0$ and carry to $x=1{\cdot}2$ with $h=0{\cdot}1$. Compare the final result with the value of the exact solution. 8

5. (a) In a single closed circuit, the current 'i' at any time 't' is given by $Ri+L\frac{di}{dt}=E$, find the current i at any time t, given that $t=0$, $i=0$ and L, R and E are constants. 6

(b) Find the length of the arc of the cardiode $r=a\,(1-\cos\theta)$, which lies outside the circle $r=a\cos\theta$. 6

(c) Use Euler's modified method to find the values of y satisfying the equation $\frac{dy}{dx}=\log(x+y)$, for $x=1{\cdot}2$ and $x=1{\cdot}4$ correct to three decimals by taking $h=0{\cdot}2$ and $y(1)=2$. 8

6. (a) Using Runge-Kutta's fourth order method, find the numerical solution at $x=0{\cdot}6$ for $\frac{dy}{dx}=\sqrt{x+y}$, $y(0{\cdot}4)=0{\cdot}41$ assume step length, $h=0{\cdot}2$. 6

(b) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=xe^{-x}\cos x$. 6

(c) Transform to polar co-ordinates and evaluate $\iint\sqrt{\left(\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}\right)}\,dxdy$ where R is the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 6

7. (a) Find the mass of the leminiscate $r^2=a^2\cos 2\theta$, if the density at any point is proportional to the square of its distance from the pole. 6

(b) Solve $(2x+1)^2\frac{d^2y}{dx^2}-2\,(2x+1)\frac{dy}{dx}-12y=6x$. 6

(c) Sketch the region bounded by the curves $y=x^2$ and $x+y=2$. Express area of this region as a double integral in two ways and evaluate. 8

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