NIMCET 2009 SOLUTIONS

(1 to 40) – MATHEMATICS

1. $\tan^{-1}\left[\frac{1}{1+2}\right]+\tan^{-1}\left[\frac{1}{1+(2)(3)}\right]+.......+\tan^{-1}\left[\frac{1}{1+n(n+1)}\right]=$

Given $\theta = \tan^{-1}\left(\frac{1}{1+2}\right) + \tan^{-1}\left(\frac{1}{1+(2)(3)}\right) + \tan^{-1}\left(\frac{1}{1+(3)(4)}\right)+........+\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)$

$\tan^{-1}\frac{1}{1+2} = \tan^{-1}\left(\frac{2-1}{1+(2)(1)}\right) = \tan^{-1}2 - \tan^{-1}1$

$\tan^{-1}\left(\frac{1}{1+(2)(3)}\right) = \tan^{-1}\frac{3-2}{1+(3)(2)} = \tan^{-1}3 - \tan^{-1}2$

Similarly,

$\tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right) = \tan^{-1}(n+1) - \tan^{-1}n$

$\theta = \tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + ... + \tan^{-1}(n+1) - \tan^{-1}n$

$= \tan^{-1}(n+1) - \tan^{-1}1$

$\theta = \tan^{-1}\left(\frac{n+1-1}{1+(n+1)}\right)$

$= \tan^{-1}\left(\frac{n}{n+2}\right)$ Choice (C)

2. **$d(x, y) = \max(|x|, |y|)$, then locus of (x, y) where $d(x, y) = 1$.**

Given $d(x, y) = \max(|x|, |y|)$.

Given: $d(x, y) = 1$

$\Rightarrow \max(|x|, |y|) = 1$

$\Rightarrow$ (–1, –1), (1, 1), (1, –1), (–1, 1) are the vertices

$|x| = 1, |y| = 1.$

$|x| + |y| = 2$

$\therefore$ It represents a square of side 2 units:

$\therefore$ Area of the square is 4 sq units

Choice (D)

3. $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

$\tan^{-1}\sqrt{x(x+1)}$ is defined only if :

$x(x+1) \geq 0 \Rightarrow x \geq 0$ or $x + 1 \geq 0$

$\Rightarrow x \leq -1$ or $x \geq 0. \rightarrow$ 1

$\sin 1\left(\sqrt{x^2+x+1}\right)$ is defined only if :

$0 \leq x^2 + x + 1 \leq 1$

$-1 \leq x(x+1) \leq 0 \quad \rightarrow \quad 2.$

From (1) and (2), $x = -1$ and $x = 0$ satisfy the equation

$\therefore$ Number of solutions is 2.

Choice (C)

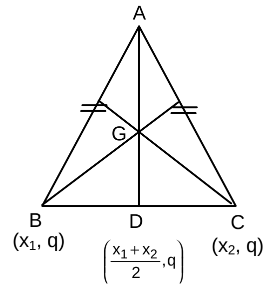
4. **Problem on isosceles triangle. m_1 and m_2 are slopes of medians**

Since BC is parallel to x-axis, slope of BC = 0

Let B = (x_1, q), C = (x_2, q)

Since ABC is isosceles triangle, median AD $\perp$ BC

$\therefore$ Let the coordinates of A be $\left(\frac{x_1+x_2}{2}, p\right)$.



Let centroid G = $\left(\frac{x_1+x_2}{2}, r\right)$

Slope of BG $(m_1) = \left(\frac{r-q}{\frac{x_1+x_2}{2}-x_1}\right) = \frac{2(r-q)}{x_2-x_1}$

Slope of CG$(m_2) = \frac{r-q}{\frac{x_1+x_2}{2}-x_2}$

$\therefore m_1 + m_2 = \frac{2(r-q)}{x_2-x_1} + \frac{2(r-q)}{x_1-x_2} = 0$ Choice (B)

Note: In the problem it is given, that AB = BC; According to AB = BC; we do not get any relation between m_1 and m_2. So we take AB = AC instead of AB = BC.

5. **Problem on system of equations, given $a + b + c \neq 0$**

Given $(b + c)(y + z) - ax = b - c$

$(c + a)(z + x) - by = c - a$

$(a + b)(x + y) - cz = a - b$

The matrix form of the above equations is

$$\begin{bmatrix} -a & b+c & b+c \\ c+a & -b & c+a \\ a+b & a+b & -c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b & -c \\ c & -a \\ a & -b \end{bmatrix}$$

Consider A = $\begin{vmatrix} -a & b+c & b+c \\ c+a & -b & c+a \\ a+b & a+b & -c \end{vmatrix}$

$R_1 \rightarrow R_1 + R_2 + R_3$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ c+a & -b & c+a \\ a+b & a+b & -c \end{vmatrix}$

$= (a + b + c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & -b & c+a \\ a+b & a+b & -c \end{vmatrix}$

$c_2 \rightarrow c_2 - c_1$

$c_3 \rightarrow c_3 - c_1$

$= (a + b + c)\begin{vmatrix} 1 & 0 & 0 \\ c+a & -(a+b+c) & 0 \\ a+b & 0 & -(a+b+c) \end{vmatrix}$

$= (a + b + c)^3\begin{vmatrix} 1 & 0 & 0 \\ c+a & -1 & 0 \\ a+b & 0 & -1 \end{vmatrix} = (a + b + c)^3$ (1)

$|A| \neq 0 \qquad \therefore a + b + c \neq 0$

$\therefore$ The system has a unique solution. Choice (A)

6. $I = \int_0^{\pi} \frac{x\,\text{Sin}x}{1+\text{Cos}^2 x}dx$

$I = \int_0^{\pi} \frac{(\pi - x)\,\text{Sin}(\pi - x)}{1+\text{Cos}^2(\pi - x)}dx$, $(\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(a-x)dx)$

$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx$

$I + I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}dx$

$2I = 2\pi \int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx \quad \left(\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx\right)$

Put cosx = t

$\Rightarrow -\sin dx = dt$

L.L. = t = 1

U.L. = t = 0

$= \pi\int_1^0 \frac{1}{1+t^2}(dt)$

$= \pi\int_0^1 \frac{1}{1+t^2}dt$

$= \pi\left[\tan^{-1}(t)\right]_0^1$

$= \pi\left[\tan^{-1}1 - \tan^{-1}0\right]$

$= \frac{\pi^2}{4}$ Choice (B)

7. Given $\tan^{-1} 2x + \tan^{-1}3x = \frac{\pi}{4}$.

$\tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right) = \frac{\pi}{4} \Rightarrow \frac{5x}{1-6x^2} = 1$

$6x^2 + 5x - 1 = 0.$

$6x^2 + 6x - x - 1 = 0.$

$6x(x + 1) - 1(x + 1) = 0 \Rightarrow (6x - 1)(x + 1) = 0$

$x = -1$; or $x = 1/6 \therefore$ Choice (A)

8. **A = $\cos^2\theta + \sin^4\theta$. Range of A.**

Given $A = \cos^2\theta + \sin^4\theta$

$= \frac{1+\cos^2\theta}{2} + (\sin^2\theta)^2 = \frac{1+\cos^2\theta}{2} + \left(\frac{1-\cos^2\theta}{2}\right)^2$

$= \frac{1+\cos^2\theta}{2} + \left(\frac{1-\cos^2 2\theta - 2\cos 2\theta}{4}\right)^2$

$= \frac{3}{4} + \frac{\cos^2 2\theta}{4}$

The minimum value of $\cos^2 2\theta = 0$

The minimum value of A = $\frac{3}{4}$

The maximum value of $\cos^2 2\theta = 1$

$\therefore$ The maximum value of A = $\frac{3}{4} + \frac{1}{4} = 1.$

$\therefore$ The range is $\frac{3}{4} \le A \le 1$ Choice (D)

9. **Problem based on double headed and double tailed coins.**

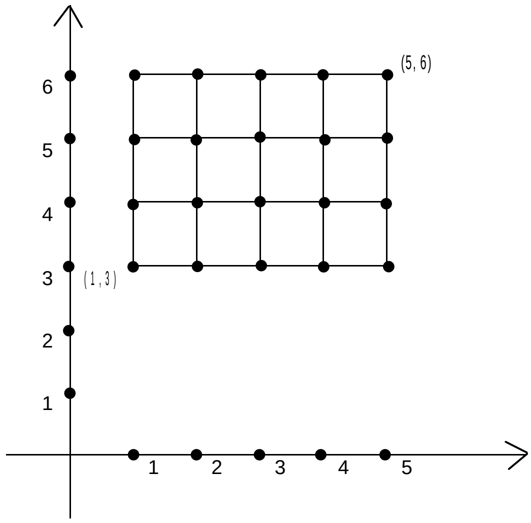
Total number of possibilities is = 10.

favourable possibilities is = 6

$\therefore$ required probability = $\frac{6}{10} = \frac{3}{5}$

Choice (C)

10. **Different paths in xy-plane from (1, 3) to (5, 6).**



The number of paths = $(4 + 3)\,C_3$

$= 7c_3 = 35$ Choice (A)

11. **Problem based on rate of change of water flow into a conical tank.**

Given r = 5 feet and h = 10 feet.

$\therefore r = \frac{h}{2}$

$\frac{dv}{dt} = 2\text{cufeet/min}$

At h = 6 feet, $\frac{dh}{dt} = ?$

Volume of cone = $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\frac{h^2}{4}h$

$V = \frac{1}{12}\pi h^3$; $\frac{dv}{dt} = \frac{1}{12}3\pi h^2\frac{dh}{dv}$

$2 = \frac{\pi}{4}(6)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{2}{9\pi}$ feet/minute. Choice (B)

12. **$\bar{B} = \bar{B}_1$ (parallel to $\bar{A}$) + $\bar{B}_2$ (perpendicular to $\bar{A}$). Find $\bar{B}_1$.**

Given $\bar{B} = 3\bar{i} + 4\bar{k}$

Given B_1 is parallel to the vector $\bar{A} = i + j$

$\therefore \bar{B}_1 = t(\bar{i} + \bar{j})$

B_2 is perpendicular to the vector $\bar{A}$.

$\therefore \bar{B}_2 = s(\bar{i} - j + p\,\bar{k})$

Given $\bar{B} = \bar{B}_1 + \bar{B}_2$

$3i + 4\bar{k} = t(\bar{i} + j) + s(\bar{i} - j + pk)$

Comparing coefficients of $\bar{i}, \bar{j}, \bar{k}$, we get

t + s = 3; t – s = 0

t = s.

$\therefore t = s = \frac{3}{2}$

$\therefore B_1 = \frac{3}{2}(\bar{i} + \bar{j})$ Choice (A)

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13. Probability question based on independent witnesses in a case

The probability that A speaks truth is p (A) = x;
The probability that B speaks truth is p (B) = y

$\therefore$ p($\bar{A}$) = 1 – x; p($\bar{B}$) = 1 – y
$\therefore$ The probability that the statement is true

$$= \frac{p(A)p(B)}{p(A)p(B)+p(\bar{A})p(\bar{B})} = \frac{xy}{xy+(1-x)(1-y)}$$

Choice (A)

14. Triangles formed by joining 10 points, 6 points being collinear.

The number of triangles formed by m points in which n points are collinear is $^mC_3 - {^nC_3}$.
Given m = 10, n =6.
Required number of triangles
= $^{10}C_3 - {^6C_3}$ = 120 – 20 = 100 Choice (A)

15. $\frac{x}{a}-\frac{y}{b}=k$ and $\frac{x}{a}+\frac{y}{b}=1/k$ (k ≠ 0) meeting point / curve.

Given lines are $\frac{x}{a}-\frac{y}{b}=k$ and $\frac{x}{a}+\frac{y}{b}=1/k$

$$\frac{x}{a}+\frac{y}{b}=\frac{1}{\frac{x}{a}-\frac{y}{b}} \Rightarrow \left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2=1$$

$\therefore$ The straight lines meet on a hyperbola. Choice (C)

16. Relations from set A (having 'm' elements) into set A $\times$ A.

The number of elements in A $\times$ A = m^2,
$\therefore$ The number of relations defined on A to A $\times$ A is
$2^{m\times m^2} = 2^{m^3}$ Choice (D)

17. P $\left(\overline{A\cup B}\right)$ = $\frac{1}{6}$; P (A $\cap$ B) = $\frac{1}{4}$, P ($\bar{A}$) = $\frac{1}{4}$

P $\left(\overline{A\cup B}\right)$ = $\frac{1}{6}$; P (A $\cap$ B) = $\frac{1}{4}$ (Given)

P($\bar{A}$) = $\frac{1}{4}$ $\Rightarrow$ P(A) = $\frac{3}{4}$

$\therefore$ P(A $\cup$ B) = 1 – P $\left(\overline{A\cup B}\right)$ = $1-\frac{1}{6}=\frac{5}{6}$.

$\therefore$ P (A$\cup$B) = P(A) + P(B) – P(A$\cap$B)

$\frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$

P(B) = $\frac{5}{6}-\frac{3}{4}+\frac{1}{4} = \frac{10-9+3}{12}=\frac{4}{12}=\frac{1}{3}$

$\therefore$ P(A) P(B) = $\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$ = P(A $\cap$ B)

$\therefore$ The events are independent and not equally likely.
Choice (A)

18. Matrix A given. Find I + A +A^2 + ... + ∞

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

I + A + A^2 + …….. = X
A + A^2 + A^3 + …… = AX
X – I = AX
X (I – A) = I.
XB = I where B = I – A. $\therefore$ X = B^{-1}.

$$B = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -3 & -3 \end{bmatrix}$$

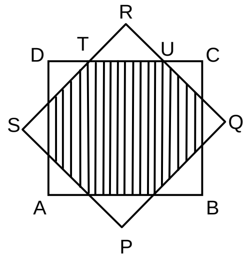
$$B^{-1} = \frac{1}{-6}\begin{bmatrix} -3 & 2 \\ 3 & 0 \end{bmatrix}$$

$$X = B^{-1} = \begin{bmatrix} 1/2 & -1/3 \\ -1/2 & 0 \end{bmatrix}$$

Choice (C)

19. Revolving a square with side 'a', about its centre.

ABCD be the square whose side is 'a', PQRS be the square obtained when the center of ABCD is rotated through 45°.
Area of the shaded region
= area of square ABCD – 4(areas of RTU)



= $a^2 - 4\,\frac{a^2}{4}\left(3-2\sqrt{2}\right)$

= $a^2(1 - 3 + 2\sqrt{2})$

= a^2 (1 – 3 + $2\sqrt{2}$)

= a^2($2\sqrt{2}$ – 2) = $2a^2$($\sqrt{2}$ – 1) sq. mts. Choice (A)

20. Given that,
P = {4^n – 3n – 1, n$\in$ N}
Q = {(9n – 9) /n, (n$\in$N)}
$\Rightarrow$ Q = {0, 9, 18, 27, 36, …….} and
P = {0, 9, 54, …….}
$\therefore$ P $\subseteq$ Q.
P$\cup$Q = Q. Choice (C)

21. Discontinuity of f(x) = [x^2 – 3] in (1, 2).

f(x) = [x^2 – 3]
We know that [x] is discontinuous at all integers.
When x = 1; f(x) = –2
x = 2 $\Rightarrow$ f(x) = 1
$\therefore$ x $\in$ (1, 2) f(x) $\in$ (–2, 1)
The number of integers in between (–2, 1) is 2.
i.e. {–1 and 0}
$\therefore$ At these two points f(x) is discontinuous.
$\therefore$ f(x) is discontinuous at two points in the interval (1, 2)
Choice (B)

22. $\left|\bar{a}-\bar{b}\right|^2+\left|\bar{b}-\bar{c}\right|^2+\left|\bar{c}-\bar{a}\right|^2 \le$ ____

Given $|\bar{a}|=|\bar{b}|=|\bar{c}|=1$.

$$\left|\bar{a}-\bar{b}\right|^2+\left|\bar{b}-\bar{c}\right|^2+\left|\bar{c}-\bar{a}\right|^2$$

$$= |\bar{a}|^2+|\bar{b}|^2-2|\bar{a}\cdot\bar{b}|+|\bar{b}|^2+|\bar{c}|^2-2\,|\bar{b}\cdot\bar{c}|+|\bar{c}|^2+|\bar{a}|^2-2|\bar{c}\cdot\bar{a}|$$

$$= 2\left[|\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2\right] - 2\left(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}\right)$$

$$= 6 - 2\left(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}\right)$$

Since $\bar{a}.\bar{b}$, $\bar{b}.\bar{c}$, $\bar{c}.\bar{a}$ is always positive,
$\therefore$ Maximum value is 6 Choice (D)

23. $2x^4 + x^3 - 11x^2 + x + 2 = 0$. Value of x + $\frac{1}{x}$

Given $2x^4 + x^3 - 11x^2 + x + 2 = 0$
Dividing throughout with x^2 we get
$2x^2 + x - 11 + 1/x + 2/x^2 = 0$
$2(x^2 + 1/x^2) + (x + 1/x) - 11 = 0$.

$$x+\frac{1}{x}=a \Rightarrow x^2+\frac{1}{x^2}=a^2-2$$

$\Rightarrow 2(a^2 - 2) + a - 11 = 0 \Rightarrow 2a^2 + a - 15 = 0$.
$\Rightarrow 2a^2 + 6a - 5a - 15 = 0 \Rightarrow 2a(a + 3) - 5(a + 3) = 0$.
(a + 3) (2a – 5) = 0
a = –3 or 5/2
$\therefore$ x + 1/x = – 3 or 5/2 Choice (A)

24. A is 3×3 matrix. det (A) = 3, det (adj A) =

Given A = 3×3 and det A = 3.
We know that
det (adj. A) = $(\det A)^{n-1}$ (where A is a $n \times n$ matrix)
$\therefore$ det (adj A) = $(\det)^2 = (3)^2 = 9$. Choice (B)

25. $2^{|x+1|} - 2^x = \left|2^x - 1\right| + 1$. Find x

Given x < –1;
$2^{|x+1|} - 2^x = \left|2^x - 1\right| + 1$.

Except in the first option, in all the remaining options, the value of x > – 1 Choice (A)

26. $\sin^{-1}(x) + \cos^{-1}(1 - x) = \sin^{-1}(-x)$.

$\sin^{-1}(x) + \cos^{-1}(1 - x) = \sin^{-1}(-x)$
$\sin^{-1}(x) - \sin^{-1}(-x) + \cos^{-1}(1 - x) = 0$
$2\sin^{-1}(x) + \cos^{-1}(1 - x) = 0$.
Let $\sin^{-1} x = \alpha$; $\cos^{-1}(1 - x) = \beta$.
$\sin\alpha = x$; $\cos\beta = 1 - x$.
$2\alpha + \beta = 0$.
$2\alpha = -\beta$.
$\cos(2x) = \cos(-\beta)$
$1 - 2\sin^2\alpha = \cos\beta$.
$1 - 2x^2 = 1 - x$.
$2x^2 - x = 0$. Choice (D)

27. $\bar{a} \times (\bar{b} \times \bar{c}) = \dfrac{\bar{b} + \bar{c}}{\sqrt{2}}$. Then $(\bar{a}, \bar{b})$ =

Given $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$

$$(\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c} = \frac{\bar{b}}{\sqrt{2}} + \frac{\bar{c}}{\sqrt{2}}$$

$$(\bar{a}.\bar{c})\bar{b} - \frac{\bar{b}}{\sqrt{2}} - (\bar{a}.\bar{b})\bar{c} - \frac{\bar{c}}{\sqrt{2}} = 0$$

$$\left[(\bar{a}.\bar{c}) - \frac{1}{\sqrt{2}}\right]\bar{b} - \left[(\bar{a}.\bar{b}) + \frac{1}{\sqrt{2}}\right]\bar{c} = 0$$

Since $\bar{b}$ and $\bar{c}$ are not collinear vectors;

$\bar{a}.\bar{c} - \frac{1}{\sqrt{2}} = 0$; $\bar{a}.\bar{b} + \frac{1}{\sqrt{2}} = 0$;

$\bar{a}.\bar{b} + \frac{1}{\sqrt{2}} = 0$; $\bar{a}.\bar{b} = -\frac{1}{\sqrt{2}}$

$|\bar{a}||\bar{b}|\cos\theta = -\frac{1}{\sqrt{2}} \Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$

$\theta = 135° = \frac{3\pi}{4}$ Choice (B)

28. $\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$.

Given $\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$.
$\Rightarrow (\sin^2 x)^2 + (\cos^2 x)^2 + \sin 2x + \alpha = 0$.
$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x + \sin 2x + \alpha = 0$.
$\Rightarrow 2 - 4\sin^2 x\cos^2 x + 2\sin 2x + 2\alpha = 0$.
$\Rightarrow \sin^2 2x - 2\sin 2x - 2 - 2\alpha = 0$.

$\Rightarrow \sin 2x = \dfrac{2 \pm \sqrt{4 + 4(1)(2 + 2\alpha)}}{2} = 1 \pm \sqrt{1 + 2 + 2\alpha}$

$\sin x = 1 \pm \sqrt{2\alpha + 3}$

Since $1 + \sqrt{2\alpha + 3} > 1$

$\therefore \sin 2x = 1 - \sqrt{2\alpha + 3}$ $(\because -1 \le \sin 2x \le 1)$

$\Rightarrow -1 \le 1 - \sqrt{2\alpha + 3} \le 1 \Rightarrow -2 \le -\sqrt{2\alpha + 3} \le 0$

$\Rightarrow 0 \le \sqrt{2\alpha + 3} \le 2 \Rightarrow 0 \le 2\alpha + 3 \le 4$

$\Rightarrow -3 \le 2\alpha \le 1 \Rightarrow -3/2 \le \alpha \le 1/2$ Choice (C)

29. Problem on A and B throwing a die in succession to win a bet , with A starting first.

Let p be the success and q be the failure probabilities.
The probability of getting 1 when a dice is rolled is 1/6.
$\therefore$ p = 1/6; q = 5/6.
The probability of A winning the game is
$p + q^2 p + q^4 p + q^6 p + \ldots$..
$= p(1 + q^2 + q^4 + \ldots\ldots.)$

$= \dfrac{p}{1 - q^2}$

$\therefore$ Probability of A winning the game = $\dfrac{1/6}{1 - \frac{25}{36}} = \dfrac{6}{11}$.

Probability of B winning the game = 5/11
Expectation of A = Probability of A winning the game $\times$ Total amount

$= \frac{6}{11} \times 110 = \text{Rs.}60$

Expectation of B = Probability of B winning the game $\times$ Total amount = $\frac{5}{11} \times 110 = \text{Rs.}50$. Choice (B)

30. Problem based on four person – A_1, A_2, A_3, A_4, who are 85 years old.

Of the 4 people, 0, 1, 2, 3 or 4 may die between the ages of 85 and 90. There possibilities and the corresponding probabilities are listed below. The third column gives the probability that A_1 in the first to die.

No. dying between 85 and 90.	No. surviving beyond 90.	Probability	Prob. that A_1 is the first to die (Required prob.)
0	4	$1\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^4$	0
1	3	$4\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3$	$1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = \frac{8}{81}$
2	2	$6\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$	$3\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2\left(\frac{1}{2}\right) = \frac{6}{81}$
3	1	$4\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)$	$3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) = \frac{2}{81}$
4	0	$1\left(\frac{1}{3}\right)^4$	$1\left(\frac{1}{3}\right)^4\frac{1}{4} = \frac{0.25}{81}$

If nobody dies before 90, there are no favourable cases.
If 1 dies before 90, one out of the 4 cases is favourable.
If 2 die before 90, of the 6 ways of selecting 2 out of 4, only 3 are favourable i.e. there are 3 cases in which A_1 is involved (A_1A_2, A_1A_3, A_1A_4). In each of these cases A_1 has 50% chance of dying earlier.
If 3 die before 90, of the 4 ways of selecting 3, A_1 is involved in 3 triplets ($A_1A_2A_3$, $A_1A_2A_4$ and $A_1A_3A_4$) In each case A_1 has 1/3 chance of being first to die.
If all die before 90. A_1 has a 25% chance of being the first to die.

$\therefore$ Required probability = $\dfrac{8 + 6 + 2 + 0.25}{81} = \dfrac{65}{324}$

Choice (C)

31. $x^3 - 6x^2 + kx + 64 = 0$. Roots are in GP.

Given $x^3 - 6x^2 + kx + 64 = 0$.

Let $\frac{\alpha}{\beta}$, α, $\alpha\beta$ be the roots of the given equation

$\therefore$ product of the roots = – 64

$\Rightarrow \frac{\alpha}{\beta}\cdot\alpha\beta = -64$

$\alpha^3 = -64 \Rightarrow \alpha = -4$.

Since 'α' is one of the roots of the equation $x^3 - 6x^2 + kx + 64 = 0$,

$\Rightarrow \therefore \alpha^3 - 6\alpha^2 + k\alpha + 64 = 0$

$-64 - 96 - 4k + 64 = 0$

$4k = -96$

$\Rightarrow k = -24$ Choice (D)

32. Value of $a_2 + a_4 + a_6 + \ldots + a_{12}$.

$(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$

Put x = 1

$0 = 1 + a_1 + a_2 + a_3 + \ldots + a_{12}$ ----- (1)

Put x = –1

$2^6 = 1 - a_1 + a_2 - a_3 + \ldots + a_{12}$ ------(2)

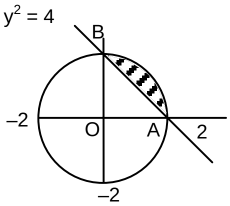
(1) + (2) gives: $2^6 = 2(1 + a_2 + a_4 + \ldots + a_{12})$

$2^5 = 1 + a_2 + a_4 + \ldots + a_{12}$

$a_2 + a_4 + \ldots + a_{12} = 31$ Choice (D)

33. Smaller of areas bound by $y = 2 - x$ and $x^2 + y^2 = 4$.

$y = 2 - x$; $x^2 + y^2 = 4$



Area of the circle = $\pi r^2 = \pi(2)^2 = 4\pi$

Area of the circle in first quadrant = π

Area of the triangle OAB = $\frac{1}{2}OA \times OB = \frac{1}{2} \times 2 \times 2$

= 2 sq units

$\therefore$area of the shaded region = area of the circle in first quadrant – area of the triangle OAB

$=(\pi - 2)$ square units Choice (B)

34. Value of 'a' satisfying $2^{2a} - 3.(2^{a+2}) + 2^5 = 0$

$2^{2a} - 3.(2^{a+2}) + 2^5 = 0$

Let $2^a = x$.

$\Rightarrow x^2 - 3x.2^2 + 32 = 0 \Rightarrow x^2 - 12x + 32 = 0$

$\Rightarrow x^2 - 8x - 4x + 32 = 0 \Rightarrow x(x - 8) - 4(x - 8) = 0$

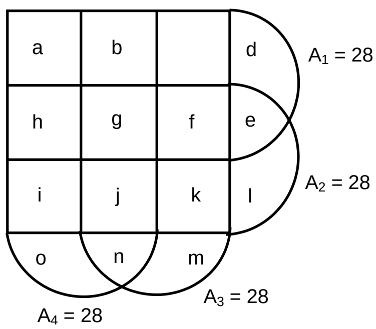
$\Rightarrow (x - 8)(x - 4) = 0 \Rightarrow x = 8; x = 4$

$\Rightarrow 2^a = 8; 2^a = 4$

a = 3 or 2

Number of values of 'a' are two. Choice (C)

35. Let us represent the given sets as follows:



Given, intersection of all 4, i.e., g = 1.

Intersections of any three sets is 5.

$\therefore$ j + g = f + g = h + g = b + g = 5.

$\therefore$ j = f = h = b = 4.

Intersection of any two of the subsets is 12.

$\therefore$ a + b + h + g = c + b + f + g = k + j + f + g = j + l + h + g = e + f + g + h = n + j + g + b = 12.

$\therefore$ a = c = k = i = e = n = 3.

d = A – (a + b + c + e + f + g + h) = 6

Similarly, e = m = o = 6.

$\therefore$ The number of elements that belong to none of the four subsets = $\mu - (A_1 \cup A_2 \cup A_3 \cup A_4)$

= 75 – 59 = 16. Choice (C)

36. Problem on vene diagram, where 50 students take exam in Maths, Physics and Chemistry.

Given $n(M \cup P \cup C) = 50$

n (M) = 37, n (P) = 24; n (C) = 43

$n(M \cap P) \le 19$, $n(M \cap C) \le 29$;

$n(P \cap C) \le 20$, $n(M \cap P \cap C) = ?$

We know that $n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - (P \cap C) - n(M \cap C) + n(M \cap P \cap C)$

$50 = 37 + 24 + 43 - n(M \cap P) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$

$n(M \cap P \cap C) = n(M \cap P) + n(P \cap C) + n(M \cap C) - 54$

$M \cap P \cap C$ is maximum, when $M \cap P$, $P \cap C$, $M \cap C$ are maximum

$n(M \cap P \cap C) = 19 + 29 + 20 - 54 = 14$ Choice (D)

37. y = f(x) (odd + differentiable). $f^1(3) = -2$, $f^1(-3) =$

Given y= f(x) is an odd function

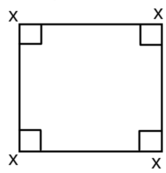
$\therefore \frac{dy}{dx} = f^1(x)$ is an even function

Given $f^1(3) = -2$

$\therefore f^1(-3) = -2$ ($\because f^1(x)$ is even) Choice (C)

38. Problem on a piece of cardboard, from which an open top box is to be made.

Let x be the side of the square that is cutoff from all the corners of the given square



$\therefore$ Measurements of the cuboid is 6 – 2x, 6 – 2x, x.

Volume (V) = $(6 - 2x)^2$. x.

$\frac{dV}{dx} = (6 - 2x)^2 + x(6 - 2x)(2 . - 2)$

For maximum, $\frac{dV}{dx} = 0$; $(6 - 2x)^2 - 4x(6 - 2x) = 0$

(6 – 2x) (6 – 2x – 4x) = 0

$\therefore$ x = 1 or 3; When x = 1 $\frac{d^2V}{dx^2} < 0$

$\therefore$ V is maximum when x = 1 Choice (D)

39. Problem on probability, where it is given that an anti aircraft gun can take a maximum of 4 shots.

Let A, B, C, D be the events of the gun hitting the plane.

P(A) = 0.4, P(B) = 0.3, P(C) = 0.2 and P (D) = 0.1

$P(\bar{A}) = 0.6$, $P(\bar{B}) = 0.7$, $P(\bar{C}) = 0.8$, $P(\bar{D}) = 0.9$

$P(A \cup B \cup C \cup D) = 1 - P\left(\overline{A \cup B \cup C \cup D}\right)$

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}) = 1 - P(\bar{A})\ P(\bar{B})\ P(\bar{C})\ P(\bar{D})$

= 1. – 0.6 . 0.7 . 0.8 .0.9

= 1 – 0.3024 = 0.6976 Choice (C)

40. A set has (2n + 1) elements.

The number of subsets that contain at most n elements = 4096

$^{(2n+1)}C_0 + {}^{(2n+1)}C_1 + ... + {}^{(2n+1)}C_n = 4096$ ----- (1)

($\because$ the set contain 2n+1 elements)

Since $^{(2n+1)}C_{n-1} = {}^{(2n+1)}C_{n+2}$ $^{(2n+1)}C_{n+1} = {}^{(2n+1)}C_n$

$^{2n+1}C_o = {}^{2n+1}C_{2n+1}$

$\therefore {}^{(2n+1)}C_0 + {}^{(2n+1)}C_1 + ... + {}^{(2n+1)}C_n = 4096$ ----- (2)

$\therefore$(1) + (2)

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_{2n+1} = 8192$

$\Rightarrow 2^{2n+1} = 8192$ ($\because n_{co} + n + . . n_{cn} = 2^n$)

$\Rightarrow 2^{2n} = 4096 \Rightarrow 2^{2n} = 2^{12}$

$\Rightarrow 2n = 12 \Rightarrow n = 6$ Choice (D)

(41– 95) – ANALYTICAL ABILITY AND LOGICAL REASONING

41. Problem based on Bala and distribution of chocolates among his three sons.

Let the total number of chocolates that Bala initially had be K.

Chocolates Given		Remaining Chocolates
Eldest son	$\frac{K}{2}+3$	$\frac{K}{2}-3$
2nd eldest son	$\frac{1}{3}\frac{(k-3)}{2}+4$	$\frac{2}{3}\left[\frac{(k-3)}{2}\right]-4$
Youngest son	$\frac{1}{4}\left[\frac{2}{3}\frac{(k-3)}{2}-4\right]+4$	$\frac{3}{4}\left[\frac{2}{3}\frac{(k-3)}{2}-4\right]-4$

Now, $\frac{3}{4}\left[\frac{2}{3}\frac{(k-3)}{2}-4\right]-4=11$ or $\frac{3}{4}\left[\frac{2}{3}\frac{(k-3)}{2}-4\right]=15$

or $\frac{3}{4}\left[\frac{2}{3}\frac{(k-3)}{2}\right]=24$

or, $\frac{k}{2}-3=36$

or, k = 78.

Therefore, initially he had 78 chocolates with him.

Choice (B)

42. Problem based on counting 1 billion orally.

Since he gets 1 day off every 4 years, we can consider 4 years to consist of 4(365) days or 1460 days (since every 4 years there will be a leap year with 366 days).

In a day there are 24 × 60 minutes i.e 1440 minutes. Since he counts 200 numbers in 1 minute, so to count 1 billion, he will take

$\frac{1,000,000,000}{200}$ minutes $= 5 \times 10^6$ minutes

No. of days taken = 1440)5000000(3472

4320

6800
5760

10400
10080

3200
2880

320

3472 days and 320 minutes.

i.e 9(365) days + 187 days + 320 minutes

i.e 9 years, 187 days, 5 hours and 20 minutes

Choice (C)

43. $\left(\frac{1}{2^{\log_x 4}}\right)\left(\frac{1}{2^{\log_x 16}}\right)\left(\frac{1}{2^{\log_x 256}}\right)$**… α = 2. Find x.**

$\left(\frac{1}{2^{\log_x 4}}\right)\left(\frac{1}{2^{\log_x 16}}\right)\left(\frac{1}{2^{\log_x 256}}\right) \ldots \alpha = 2$

Let $\log_x 4 = y$

$\therefore \log_x 16 = 2\log_x 4 = 2y.$

$\text{Log}_x 256 = 2\log_x 16 = 4y.$

Thus we get

$\left(2^{1/y}\right)\left(2^{1/2y}\right)\left(2^{1/4y}\right) . . . \alpha = 2$

$2^{1/y\left(1+\frac{1}{2}+\frac{1}{4}+\ldots\alpha\right)} \Rightarrow 2^{1/y\left(\frac{1}{1-1/2}\right)} = 2$

$\Rightarrow 2^{2/y} = 2^1$ or $2/y = 1$ or $y = 2 \Rightarrow \log_x 4 = 2$

$\Rightarrow \log_x 2 = 1$

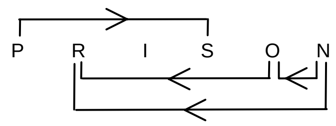
$\Rightarrow x = 2$ Choice (A)

44. Units digit in $(13687)^{3265}$

Units digit of $(13687)^{3265}$ = unit's digit of (___ 7)$^{3264 + 1}$

$7^{4x816+1} = 7^1$ i.e 7. Choice (C)

45. Problem based on pairs of letters in the word 'PRISON'.



There are four such pairs, PS, NO, OR and NR.

Choice (C)

46. Twelve villages in a district are divided into 3 zones with 4 villages per zone.

Number of direct lines between villages in the same zone $= 3(^4C_2 \times 3) = 54$ direct lines.

(There are 3 zones. Now a pair of villages in the same zone can be selected in $^4C_2$ or 6 ways and there are 3 direct lines connecting them.

Number of direct lines between villages in different zones $= {}^3C_2\,(^4C_1 \times {}^4C_1 \times 2) = 96$ direct lines.

(Two zones can be selected in $^3C_2$ ways and a village from each of the two zones can be selected in $^4C_1$ ways and there are 2 direct lines connecting these two villages).

Therefore total number of direct lines = 54 + 96

=150 direct lines Choice (D)

47. Cars are safer than planes – Problem based on strong and weak arguments.

The statement talks in terms of percentages. It does not reflect the absolute values. If it can be shown that the number of car accidents is more than the number of plane accidents, than we can weaken the argument that cars are safer than planes.

Choice (A): Planes are inspected more often than cars, does not imply that cars are not road worthy. Hence, (A) does not weaken the argument.

Choice (B): It is stated that the number of car accidents is several hundred thousands higher than the number of plane accidents. Hence, one percent of car accidents would be more than 50 % of plane accidents. Hence, (B) weakens the argument.

Choice (C): This choice has no relevance to number of accidents.

Choice (D): This gives reason for plane accidents. This has no relevance to the argument

Choice (B)

48. Sum of all 3 digit numbers (no digit being zero), such that all digits are perfect squares.

We have 3 single digits which are perfect squares they are 1, 4 and 9.
Now, we will have 27 numbers in all.
Let us find the number of times the digit 9 comes in the unit's place. Fixing 9 in the units place, the remaining two places can be filled in 3.3 or 9 ways,
Similarly the digits 4 and 1 comes 9 times each in the units place.
Again the digits 1, 4 and 9 each comes 9 times in the tens place and 9 times in the hundreds place. Therefore the sum of all the three digit numbers
= Sum of the digits in the hundreds place $\times$ 100 + sum of the digits in the tens place $\times$ 10 + sum of the digits in the units place.
= 9(1 + 4 + 9) (100 + 10 + 1) = 9(14)(111)
= 13986. Choice (C)

49. A teacher gave a student the task of adding 'N' natural numbers, starting from 1.

The sum of N natural numbers = $\frac{N(N+1)}{2}$

When N = 36, the sum is $\frac{36\times37}{2}=666$.

When N = 37, the sum is $\frac{37\times38}{2}=703$.

Therefore the student was supposed to get the sum as 666, but due to his adding a number twice, he got it as 700. Thus the number that he added twice was 700 – 666 = 34.
Sum of the digits of 34 is 7. Choice (C)

50. Problem on 3 computers A, B and C, where each takes certain time to process an input.

In an hour, computer A will process $\frac{60}{3}=20$ inputs

In an hour, computer B will process $\frac{60}{5}=12$ inputs

In an hour A, B and C together process = 14 $\times$ 3 inputs.
Therefore C alone processes 42 – (20+12) = 10 inputs in 1 hr.

Thus C takes $\frac{60}{10}$ = 6 minutes to process one input

Choice (C)

51. Arranging of statements P, Q, R and S in a logical sequence to make a sensible paragraph.

Bank rate cut takes place after RBI decides to do it. This means S follows Q. R explains why RBI is doing this, hence, R follows QS. QSR present in (C).
The correct answer is (C) QSRP. The logic of this, incidentally, can be explained in a simpler manner. The statement P talks about 'the logic' being 'two-fold'. There is no explanation of this 'two-fold' logic in any statement. This means that statement P can come only at the end. The only option with P at the end is(C). Hence, the correct answer. Choice (C)

52. A + B = C + D, B + D = 2A, D + E > A+ B, C + D > A + E.

It is given that,
A + B = C + D -------- (1)
B + D = 2A -------- (2)
D + E > A + B -------- (3)
C + D > A + E --------- (4)

D + E > A + B

$\therefore$D + E > C + D ($\because$ A + B = C + D)

$\therefore$E > C
C + D > A + E

$\therefore$D > A ($\because$ E > C)

B + D = A + A.

$\therefore$B < A ($\because$ D > A)

Again C + D > A + E

A + B > A + E ($\because$ A + B = C + D)

$\therefore$ B > E
Therefore we have D > A > B > E > C Choice (D)
Alternate solution:

Since B + D = 2 A, therefore A cannot be the highest.
Thus we can rule out option (B) and option (C).
Again in option (A) it is given as D > B > A which is not possible as A must lie between B and D.
Thus only option (D) can be the answer Choice (D)

53. Problem based on school reunion of Reena and Natasha.

Let P be the age of Preeti and R the age of Rahul.
According to Rahul $\rightarrow$ $P^3 + R^2 = 7148$ ---- (1)
According to Preeti $\rightarrow$ $P^2 + R^3 = 5274$ -----(2)
Here we can proceed from the answer choices. We need not go for actual calculation. By observing the units places we can eliminate the answer choices.

Choice (A): $P^2 + R^3 = 23^2 + 14^3$
We get 3 in the unit place

Choice (B): $P^2 + R^3 = 18^2 + 16^3$
Here we get zero in the unit place.

Choice (C): $P^2 + R^3 = 21^2 + 19^2$
Here we get zero in the unit place.

Choice (D): $P^2 + R^3 = 19^2 + 17^3 = 5274$
$P^3 + R^2 = 19^3 + 17^2 = 7148$

$\therefore$ Choice (D) satisfies. Choice (D)

54. The given series follows the pattern given below:
$12^{\times 2-2}$, $22^{\times 3+3}$, $69^{\times 4-4}$, $272^{\times 5+5}$, $1365^{\times 6-6}$, 8184.
$\therefore$8184 is the next number in the series Choice (D)

55. Rs. 1074 to be divided into a number of bags.

We need to divide the number in such a way that any amount between Rs.1 and Rs.1074 can be obtained by combinations of one or more of those divided parts. This is possible when the parts are.
1, 2, 4, 8, 16, 32, 64, 128, 256, 512 and 51
i.e considering all the powers of 2 from 2^0 to 2^9 we get 1023.
Since the amount to be divided is 1074, we need one bag containing Rs. (1074 – 1023) i.e Rs. 51.
Thus we would require a minimum of 11 bags.
Choice (D)

56. By taking eight in the place of question mark, the sum of all the numbers in the square (27), becomes equal to the sum of numbers outside the square (27) Choice (D)

57. Remainder, when x = 1! + 2! + 3! +. . . . + 100! Is divided by 240.

The remainder when x is divided by 240 is the same as the remainder, when 1! + 2! + 3! + 4! + 5! is divided by 240 (Since 6! onwards all the numbers till 100! will be multiples of 240 and thus would be divisible by 240)
i.e 1! + 2! + 3! + 4! + 5! = 153
153 when divided by 240 will leave a remainder of 153
Choice (A)

58. Sum of numbers formed using digits 1, 5, 2, 8.

Let us consider the sum of all the numbers in the units place. Each of the 4 digits will appear in the units place 3! or 6 times. That is fixing a digit in the units place, the remaining 3 places can be filled by the remaining 3 digits in 3! Ways. Therefore sum of all the digits in the units place
= 6 (1 + 2 + 5 + 8) = 96.
Similarly we will get 96 as the sum of the digits in tens, hundreds and thousands places as well.
Therefore sum of all the 4 digit numbers
= 96(1000 + 100 + 10 + 1) = 106656.
Now 106656 lies between 100000 and 150000
Choice (D)

59. Sum of numbers from 1 to 100, which are not divisible by 3 and 5.

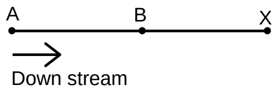
The sum of the numbers from 1 to 100, which are not divisible by 3 and 5 = Sum of all the numbers from 1 to 100 – Sum of all the numbers from 1 to 100 which are divisible by 3 and 5.

$\frac{100(101)}{2}$ – (Sum of nos. divisible by 3 + Sum of nos. divisible 5 – Sum of nos. divisible by both 3 as well as by 5)

$= 5050 - \left\{\frac{33}{2}(3+99)+\frac{20}{2}(5+100)-\frac{6}{2}(15+90)\right\}$
$= 5050 - \{1683 + 1050 - 315\}$
$= 5050 - 2418 = 2632$
Thus sum of the numbers from 1 to 100 which are divisible neither by 3 nor by 5 is 2632 Choice (C)

60. Motorboat going down stream, crosses a raft.

Let the speed of the boat in still water and the speed of the stream be v km/hr and s km/hr respectively.



The boat, in 1 hr, traveled from A to X
AX = 1(v + s).
Let the boat take 't' time to travel from X to B.
$\therefore$BX = t (v – s).
AX – BX = v + s – t (v – s)
AB = v + s – t (v – s)
In the time of 1 hr + t hr, the raft travelled s(1 + t) = AB.
Since AB = 6.
$\therefore$ v + s – t (v – s) = 6
v – v t + st + s = 6
v – v t + 6 = 6 or, v = v t or, t = 1 hr
$\therefore$s(1 + t) = 6
s (1 + 1) = 6
s = $6/2$ = 3 km/hr.

Alternate solution:

The boat with respect to the raft is always travelling at a speed of V km/hr.

Therefore it will take the same time it had taken to travel away from the raft to get back to the raft once more.
Thus it travels for 2 hours.

In 2 hours the raft covers 6 km. Therefore the speed of the water current is $6/2$ km / hr or 3 km / hr. Choice (B)

61. Problem based on 3 types of rabbits in the land of logic.

According to the given information, blue rabbit is truth teller and red rabbit is liar. A truth teller does not say that "I always lie", because it will be a false statement for it. A liar does not say that "I always lie", because in such case it becomes a true statement.

Since, the green rabbit some times tells truth and some times lies, it has the possibility of making such statement. Choice (C)

62. Problem based on the argument about doctors and patients.

In the passage, the author concludes that as it is wrong on the part of mechanics, carpenters etc. (service providers) to lie to their customers, similarly it is wrong on the part of the doctors to lie to their patients regarding their health condition.

Choice(A) states that by lying, the doctors violate good faith. This does not specify that doing such thing is wrong, which is the essence of the passage.

The passage is not just about lying but how it reflects on the professional ethics. Hence, neither B nor D expresses the conclusion of the argument.

Choice (C) states that it is wrong on the part of the doctors to lie about patient's illness. This is the conclusion of the argument. Choice (C)

63. Rank of the 58th word, when letters of INDIA are permuted.

Words starting with A.
A - - - -

With the alphabets INDI, we can form $\frac{4!}{2!}$ = 12 words.

Words starting with D, we can form $\frac{4!}{2!}$ or 12 words with the remaining 4 alphabets.

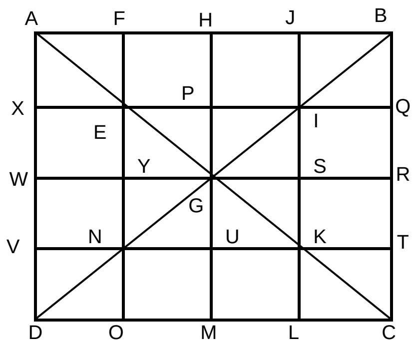
Words starting with I, with the remaining 4 alphabets we can form 4! Words (i.e with A, D, I and N).
Next we have words starting with N. with NA as the first two alphabets we can form $\frac{3!}{2!}$ or 3 words
Again with ND as the first two alphabets we can form $\frac{3!}{2!}$ or 3 words.

So we have 12 + 12 + 24 + 3 + 3 = 54 words.
With NIA as the first 3 alphabets we can form 2 words.
With NID as the first 3 alphabets we can form 2 words.
Thus the 58th word is NIDIA.

Alternate solution:

With the given alphabets we can form a total of $\frac{5!}{2}$ = 60 words
60th word is NIIDA.
59th word is NIIAD
58th word is NIDIA. Choice (C)

64. Number of triangles in the given 4×4 block.



Diagonal AC forms four triangles, one with each of the vertical lines EF, GH, KJ and CB.
Diagonal EC forms three triangles with the vertical lines GP, KI and CQ.
Diagonal GC forms two triangles with the vertical lines KS and CR.
Diagonal KC forms one triangle with the vertical line CT.
Hence, along the diagonal AC ten triangles are formed on one side of it and another ten on the other side.
Similarly for the diagonal DB there are 20 triangles.
There are another eight triangles AGB, AGD, BGC, CGD, NGK, KGI, IGE and EGN.
$\therefore$ In all, there are 48 triangles.
Choice (B)

65. "Each time Sachin is the captain, India loses". Pair of statements which are logically consistent with this statement.

Each time Sachin is the captain India loses.
Let "Sachin is the captain" be "A" and "India loses" be "B".
$\therefore$ Each time "A" then "B". The possible implications are

(i) $A \Rightarrow B$

(II) $\sim B \Rightarrow \sim A$

(A) $PS = A \Rightarrow \sim B$

(B) $SR = \sim B \Rightarrow \sim A$ (i.e ii)

(C) $SP = \sim B \Rightarrow A$

(D) $RP = \sim A \Rightarrow A$

$\therefore$ Only (B) i.e SR is the correct ordered pair.
Choice (B)

66. All 6's replaced by 9's.

There are 10 numbers with 6 appearing in the units place and 10 numbers with 6 appearing in the tens place. If all the 6's are replaced by 9's, than the sum of all the numbers from 1 to 100 will vary by $(9 - 6)\,10 + (9 - 6)\,10 \times 10$
i.e by 3(10) + 30(10) or by 330 Choice (A)

67. Problem based on escalator of a tube station in Bangalore.

Let the speed of the escalator be 's' steps per seconds. Therefore in 18 seconds, the escalator takes 18s steps where as in 30 seconds the escalator takes 30s steps.
In either case, the total distance is the same i.e the total number of steps exposed to view on the escalator.

$18s + 34 = 30s + 26 \Rightarrow 12s = 8 \Rightarrow s = 2/3$

Therefore the number of steps exposed to view on the escalator $= 34 + 18\left(2/3\right) = 46$

Therefore the height of the stairway, in steps, is 46
Choice (B)

68. Reversing the digits of the number 13, the number increases by 18.

The value of the two digit number xy is 10x + y.
The value of the reverse of that number yx = 10y + x
yx – xy = 18
(10y + x) – (10x + y) = 18
9(y – x) = 18 or y – x = 2
The two digit numbers which satisfy this condition are 13, 24, 35, 46, 57, 68 and 79. Therefore there are 6 other 2 digit numbers which increase by 18 when reversed.
Choice (B)

69. Problem based on letters in the word REASONING.

Given : R E A S O N I N G
1st letter $\to$ R
2nd letter $\to$ E
4th letter $\to$ S
5th letter $\to$ O
6th letter $\to$ N
New word using the above mentioned letters is "SNORE"
Hence, the first letter is 'S' and the last letter is 'E'.
Choice (A)

70. Problem based on Hameed and his butcher shop.

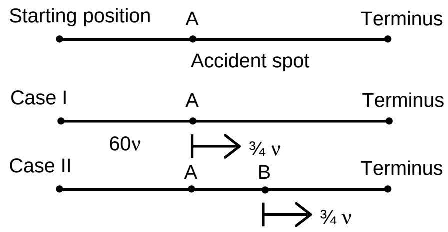
Each shopkeeper has given two descriptions about the dog, which are as follows.

	Shopkeeper 1	Shopkeeper 2	Shopkeeper 3
(i)	Black hair	Short tail	White hair
(ii)	Long tail	Wore a collar	No collar

Let us assume that (i) of shopkeeper 1 is true, then (ii) is false, $\therefore$ (i) of shopkeeper 2 is true and as only one part is true in each statement, (ii) of shopkeeper 2 is false and hence (i) of shopkeeper is false and (ii) is true. Based on the above assumption the description of the dog is (Black hair, Short tail and no collar) but this is not among the given answer choices.

So, let us assume that (i) of shopkeeper 1 is false. It implies (ii) is true. i.e., the dog has long tail. Hence, (i) of shopkeeper 2 is false and (ii) is true (ii) of shop keeper 3 is false and (i) is true.
$\therefore$ The dog has a long tail, wore a collar and has white hair.
Choice (B)

71. A train, after traveling 60 km, meets with an accident and proceeds at $3/4$ of its former speed.



Let the train travel at ν till it reaches A, From there, the train travels at $3/4\,\nu$.

Let the time taken by the train to travel from A to the terminus at speed ν be t hrs and the time taken when it travels at $3/4$ of its former rate over the same distance be T hours.

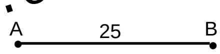
$3/4\,\nu\ T = \nu t.$

$T = \frac{4}{3}t = t + \frac{1}{3}t.$

So $1/3\,t = 40$ minutes.

t = 120 minutes or t = 2 hrs.

Now at $3/4\,\nu$ over a distance of 25 km, the train takes an extra 10 minutes.



At ν if the train takes time t_1 over 25 km at $3/4\,\nu$ it will take $\frac{4}{3}t_1$ over the same distance.

Now $\frac{4}{3}t_1 = t_1 + 1/3\,t_1$

It is given that $1/3\,t_1 = 10$ minutes

$\therefore t_1 = 30$ minutes.

Therefore the train takes 30 minutes to cover 25 km, its speed $= \frac{25}{1/2} = 50$ km/hr.

In 2 hrs at 50 km/hr, the train covers $50 \times 2 = 100$ km.
Therefore distance from A to the terminus is 100 km. Thus the total distance is 60 + 100 = 160 km. Choice (C)

(72 to 75): Questions based on talent searching for an administrative assistant to the president of a college. Five applicants A, B, C, D, E finalized.

These questions can be answered by elemination process.
The conditions to be applied while selecting the persons who would attend the interviews, as given in the data, are:
(2) 3 candidates in a week.
(3) Each candidate must appear at least once.
(4) Not more than one candidate can be interviewed in two consecutive weeks.
(5) Both A and B attend in a week or A may attend without B or neither A nor B attends in a week.
(6) C can be called only once.

72. Choice (A): Violates (5).
Choice (B): Violates (4).
Choice (C); Does not violate any condition.
Choice (D); Violates (5). Choice (C)

73. Choice (A): Violates (5).
Choice (B): Does not violate any condition.
Choice (C); Violates (4).
Choice (D); Violates (4). Choice (B)

74. By applying the given conditions, in two weeks of interviews, each applicant appears at least once. Hence, I correctly states the procedure.
Since, there are five applicants and by applying conditions (2), (3) and (4), it is not the intention of the company to interview each applicant twice.
The interviews are conducted in a third session also the combination of applicants can be ABC; ADE; ABE. Here, D is not interviewed twice.
∴ Only I correctly states the procedure. Choice (A)

75. If A, B and D appear at the interview and D is called for an additional interview the following week, then only C and E can be asked to appear with D. Choice (B)

76. Counting the number of 5's in the given sequence

The given number series is 456 656 455 455 654 456 456 5454.
The underlined '5' are the only two 5's, which are immediately followed by 4 but not immediately preceded by 6. Choice (D)

(77 – 81): Questions based on the given two pie charts.

77. As the price of car is Rs. 1,00,000 and the margin is 10%, the cost of car is Rs. 90,000. As transmission cost is 20% of 1,00,000, the cost of transmission is Rs. 20000. If a 20% increase happens in the price of transmission, the cost would increase by Rs. 4,000 and so the profit would decrease by Rs. 4,000 Choice (B)

78. Price of tyres is 15% of transmission cost, i.e, 3% of total, or Rs. 3,000. If it increases by 25%, i.e, Rs. 750, the sale price should be increased by Rs. 750 to maintain the profit. Choice (A)

79. Assume the sale price is Rs. 100
Total cost = Rs. 90
Transmission cost per unit = Rs. 20
New transmission cost = Rs. 22
Current engine cost = Rs. 30
New engine cost = Rs. 36

The required percentage $= \frac{22}{989} \times 100 = 22.44\%$

Choice (B)

80. Let the selling price be Rs.100
The cost price = Rs. 90
Current profit = Rs. 10
The cost price after increase = Rs. 99
The new profit = Rs. 1

∴ The reduction in profit $= \frac{10-1}{10} \times 100 = 90\%$

Choice (B)

81. The profit percentage $= \frac{10}{90} \times 100 = 11.11\%$

Choice (C)

(82 – 84): Set based on – Rajita's unique way of attempting the question paper having 50 questions

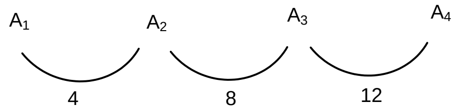
Rajita starts answering from the first question and answers every third question which is in A.P with a common difference.
Forward direction: 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, and 49.
Now she starts in the reverse direction with the first unanswered question i.e 50.
Reverse direction: 50, 47, 44, 41, 38, 35, 32, 29, 26, 23, 20, 17, 14, 11, 8, 5 and 2.
Now she changes the direction and starts with the first unanswered question i.e. 3
Forward direction: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45 and 48.
With this she completes answering all the questions.

82. Going by the question, she reverses her direction twice. Once after reaching question 49 and second time after reaching question 3. But such answer choice is not given.

83. The last question that she attempts is 48. Choice (C)

84. The 20th question that she answers is question number 44. Choice (D)

85. A_1 has 1 element.
A_2 has 3 elements.
A_3 has 5 elements.
Therefore A_{20} will have 2(20)–1 or 39 elements.
Average of A_1 is 3, that of A_2 is 7, that of A_3 is 15 and that of A_4 is 27.



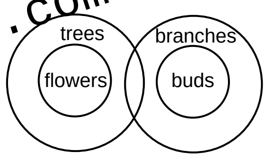
Therefore the average of A_{20} will be.
3 + (4 + 8 + 12 + . . . 19 terms)
= 3 + 4 (1 + 2 + 3 + . . . till 19 terms)

$= 3 + 4\frac{(19)\ (20)}{2} = 3 + 760$

= 763 Choice (B)

(86 – 89): Questions based on Deductions

86. The basic diagram is as follows.



From the above diagram
Conclusion I, affirmative, follows.
Conclusion II, affirmative, follows.
Conclusion III, affirmative, does not follow.
∴ Only I and II follow. Choice (C)

87. The basic diagram is as follows.

drinks
eatables pots banana

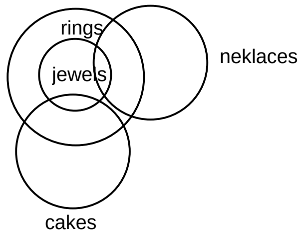
From the above diagram
Conclusion I, affirmative, follows.
Conclusion II, affirmative, doesnot follow.
Conclusion III, affirmative, follows.
∴ Only I and III follow Choice (D)

88. The basic diagram is as follows.

rings
jewels neklaces

cakes

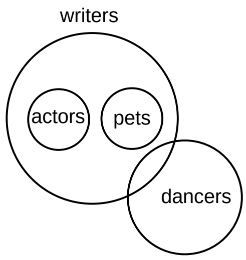
Conclusion I, affirmative, does not follow.
Conclusion II, affirmative, follows.
Conclusion III, negative, follows.
As the negative conclusion is true, the alternative diagram to disprove this is as follows.



Conclusion I follows.
Conclusion II follows.
Conclusion III does not follow.
Conclusion I and III are contrary, hence, only II and either I or III follows.

Choice (A)

89. The basic diagram is as follows.



Conclusion I, affirmative, does not follow.
Conclusion II, affirmative, follows.
Conclusion III, affirmative, does not follow.
Only II follows, but II is derived from only one statement. Hence, none follows.

Choice (A)

(90 – 94): Questions based on five houses A, B, C, D and E and these houses having different coloured roofs and chimneys.

It is given that five houses A, B, C, D, and E are built in a row next to each other in the same order. Each of these houses have coloured roofs and chimneys. The roof and chimney of each house is painted as follows.
From (1), the roof must be painted with Green, Red or Yellow.
From (2), the chimney must be painted with Black, White or Red.
From (3), no house may have the same colour chimney as the colour of roof.
From (4), no two adjacent houses are of the same colour.
From (5), E – Green roof
From (6), B – Red roof, Black chimney.

Roof →		Red			Green
	A	B	C	D	E
Chimney →		Black			

As B's chimney is Black and roof is Red, A and C's chimney cannot be painted either Red or Black. Hence A and C's chimneys are painted White.
D's Chimney can be either Black or Red, E can paint with any one of the three colours.
D's roof cannot be Green, hence it can be painted in Yellow or Red. A and C's roofs are painted in Green or Yellow.

Roof →	Green Yellow	Red	Green Yellow	Red Yellow	Green
	A	B	C	D	E
Chimney →	White	Black	White	Black Red	Black Red White

90. (A) Can be true.
(B) Is definitely false
(C) Can be true.
(D) Can be true. Choice (B)

91. (A) may or may not be true.
(B) may or may not be true.
(C) Is definitely true.
(D) may or may not be true. Choice (C)

92. The maximum total number of Green houses can be three i.e., A, C and E. Choice (C)

93. A house can have Red roof and Black chimney → I is possible.
A house can have Yellow roof and Red chimney → II is possible.
A house can have Yellow roof and Black chimney → III is possible.
Hence, I and II and III are possible. Choice (A)

94. If house C has a Yellow roof then the order is as follows

Green Yellow	Red	Yellow	Red	Green
A	B	C	D	E
White	Black	White	Black	White

(A) is true.
(B) is false.
(C) is false.
(D) is false. Choice (A)

95. Minimum number of weighing required to find an odd ball (of different weight) in a set of 13 identical balls.

Among the 13 balls, except one all others have equal weight. We do not know whether the one which is of different weight is heavier or lighter. The 13 balls are separated into four groups as 1, 4, 4 and 4. Let us name them as groups a, b, c and d respectively.

Weigh group b against group c. We have three possible cases

	(i)	(ii)	(iii)
1st Weighing	c = b	c > b	c < b

Consider Case (i), c = b
In this case the wrong ball is in group a or d. Weigh d against c. Because we are looking for minimum number of weighings in which we can surely find the wrong ball, we always assume worst case.
∴ If d > c, it implies that the wrong ball is heavier and it is in group d.
If d < c, it implies that the wrong ball is lighter and it is in group d.
With the second weighing we come to know whether the wrong ball is heavier or lighter. Let us say it is heavier.

Now, divide group d further into two groups of two balls each. Let the groups be e and f. Weigh e against f. They will be of different weights. The heavier one contains the wrong ball. Let us say it is in e. Weigh the two balls in group e against each other and we find the heavier one.
Thus we need four weighing.
Even in cases (ii) and (iii) also we need four weighings.

Choice (D)

(96 – 105) – COMPUTER AWARENESS

96. On receiving an interrupt from an I/O device, the CPU branch off to interrupt service subroutine immediately to find out from which input/ output device interrupt signal has come. Choice (B)

97. In each location in control memory, a program for, say addition, subtraction, multiplication etc is written. Depending upon the address of control memory it performs the corresponding operation. This concept is called Micro Programming. It always refers to memory. On the other hand, in hard-wired, there is no need to refer to memory. But there is no flexibility. Therefore Micro Programming is slower than hard-wired and easy to implement new instructions. Choice (D)

98. In the CPU of a computer, an index register is used for indirect addressing where, an immediate constant (i.e, which is part of the instruction itself) is added to the contents of a register to form the address to the actual operand or data. Choice (B)

99. Virtual memory is used whenever data size is more than RAM capacity. It is a part of hard disk created by OS for processing. Therefore the address space of CPU must be larger than the physical memory (RAM) and smaller than the secondary storage size. Choice (C)

100. Given Boolean function is
f (A,B,C,D) = $\sum$ (1,4,5,9,11,12)
Binary equivalents and fundamental products for given decimals are as below.

1→	0001→	$\bar{A}\,\bar{B}\,\bar{C}\,D$	9→	1001	$A\,\bar{B}\,\bar{C}\,D$
4→	0100→	$\bar{A}\,B\,\bar{C}\,\bar{D}$	11→	1011	$A\,\bar{B}\,C\,D$
5→	0101→	$\bar{A}\,B\,\bar{C}\,D$	12→	1100	$A\,B\,\bar{C}\,\bar{D}$

The k – map for given Boolean expression is

	$\bar{C}\,\bar{D}$	$\bar{C}\,D$	$C\,D$	$C\,\bar{D}$
$\bar{A}\,\bar{B}$	0	1	0	0
$\bar{A}\,B$	1	1	0	0
$A\,B$	1	0	0	0
$A\,\bar{B}$	0	1	1	0

Using pairs simplification method, the Boolean expression is $B\,\bar{C}\,\bar{D} + \bar{A}\,\bar{C}\,D + A\,\bar{B}\,D$

Choice (A)

101. 2's complement of given number 11111011 is 00000101 which is equivalent to 5 in decimal.
The 2's complement and equivalent decimal numbers of given choices are as follows.

	Given	2's complement	Decimal
(A)	11100100	00011100	28
(B)	11010111	00101001	41
(C)	11011011	00100101	37
(D)	00000110	11111010	250

Among the given choices 250 is divisible by 5
Choice (D)

102. In digital electronics, a multiplexer is a device that performs multiplexing i.e it selects one of many input signals and forwards the selected input into a single line.
A multiplexer of 2^n inputs has n select bits, which are used to select which input line to send to the output.
Choice (C)

103. Range of integers that can be represented in 2's complement using n bit registers is -2^{n-1} to $2^{n-1}-1$
Therefore with 32 bits range is -2^{31} to $2^{31}-1$
Choice (C)

104. Upper case letter 'A' in ASCII is 65
Lower case letter 'a' in ASCII is 97
The difference is 32 In binary, 0100000.
$\therefore$ to change upper case to lower case in ASCII correct mark and operation should be 0100000 and OR
Choice (C)

105. Consider the words width and speed given in the question. If the width in more, the processing speed is also greater. This is directly given in choice (c). Choice (C)

(106 – 120) – GENERAL ENGLISH

106. The correct answer is Choice (D). Although Choices (C) and (D) are correct passive voice of the verb in the given sentence. The word 'known' is followed by only 'to' and not 'by'. Choice (D)

107. Something can be 'impossible to determine' only when it is hidden or inactive. So, for the first blank, we can have dormant in choice (A) and latent (hidden) in choice (D). (We eliminate choices (B) and (C)). Now for the second blank, postulate is absurd as it does make any sense with 'dormant'. However, observation in choice (D) makes sense with 'latent'. Choice (D)

108. 'Quibble' means 'complain'. Choice (D)

109. 'Gung ho' means 'enthusiastic'. Choice (C)

110. To 'disparage' is to belittle and the correct antonym is choice (D) applaud (to praise). Choice (D)

111. The blank requires the verb persuade. The phrase that means 'persuade' is choice (B), 'prevail upon'.
Choice (B)

112. We 'have' a difficulty, we do not 'get' it or 'be getting' it. Hence, choices (B) and (D) are wrong. Also, we have difficulty 'in' doing something not 'on' doing something.
Choice (A)

113. The word 'object' can be used as both noun and verb. As a noun it means 'a thing' and as a verb it means 'to take exception'. Hence, we can replace the underlined parts with choice (A), object. The other words don't have the same meanings in any of their forms.

Note: In statement I, the article before the underlined word (thing) should have been given as a/an, since 'a' would make the student wonder if the word can be replaced by one beginning with a vowel. Choice (A)

114. The meaning of the given idiom is choice (D), 'to express disbelief'. Choice (D)

115. Plagiarism is an act of misusing other's ideas, words etc., and similarly misappropriation is an act of misusing someone else's money or property. Hence, the correct answer is choice (B). Theft in choice (C) can be the stealing of anything, not necessarily 'gold' and so choice (C) is wrong. Choice (B)

116. Abstemious means temperate (not allowing yourself much food or alcohol). Choice (B)

117. The first sentence (S) says Tagore had 'no blue print'. This is apt by followed by R which says he did not believe in any 'ism'. So choices (A) and (B) can be ruled out. Between choices (C) and (D), the latter is better since 'He merely emphasised certain basic truths……' carries forward the idea in R (did not believe in any 'ism'). While S can also follow R, P following R, (with 'therefore') is incorrect. P correctly follows Q (he emphasized 'basic truths' and 'therefore never be out of date'). Choice (D)

118. We cannot say 'can be able to' as both 'can' and 'able to' mean the same. One of the two is redundant. 'be able' is given as an option and so is the answer. The other parts are correct.
Note: The question should have underlined 'be able to' and choice (A) should include 'to' since the sentence would not read right with 'to'. Choice (A)

119. Sentence 1 of paragraph 2 clearly states that 'the least controversial assertion about the pterosaurs is that they were reptiles'. From this we can 'infer' that the answer is choice (D). Choice (D)

120. From 2nd, 3rd and 4th statements in 2nd paragraph, we can understand that the correct answer is choice (C).
Choice (C)