S.E. (Etrx & EXTC) sem 4 (Rev.)
Applied Maths IV

07/1/08

Con. 5250-07. CD-5793

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** which is **compulsory**.
(2) Answer any **four** questions from the remaining **six** questions.
(3) If in doubt make **suitable** assumption, **justify** your assumptions and **proceed**.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) State and prove Cauchys-Integral theorem. 5

(b) Evaluate $\int_C \left(z - z^2 \right) dz$ where C is the upper half of the circle $| z - 2 | = 3$. 5

(c) Determine A^{-1}, A^{-2} and A^{-3}. If $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ 5

(d) Prove that $\nabla \times \left[\dfrac{\bar{a} \times \bar{r}}{r^n} \right] = \dfrac{(2-n)}{r^n} \bar{a} + n\, r^{-(n+2)} \left(\bar{a} \cdot \bar{r} \right) \bar{r}$ where $\bar{a}$ is constant vector. 5

2. (a) What is the directional derivative of $f = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 = -4$ at $(-1, 2, 1)$. 6

(b) Find the eigenvalues and eigenvectors of the matrix 6

$$A = \begin{bmatrix} 7 & 2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$$

(c) Expand $f(z) = \dfrac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region 8

(1) $| z | < 1$ (2) $1 < | z | < 4$ (3) $| z | > 4$.

3. (a) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that $A^n = A^{n-2} + A^2 - I$ for every integer $n \geq 3$ and hence find A^{50} 6

(b) Evaluate $\int_C \dfrac{\sin z}{z^2 - iz + 2}\, dz$ where C is 6

(i) $| z + i | = 1$
(ii) the rectangle with vertices at $(1, 0)$, $(1, 3)$, $(-1, 3)$ and $(-1, 0)$.

(c) Verify Greens theorem in plane for 8

$$\oint_C (x^2 - 2xy)\, dx + (x^2 y + 3)\, dy$$

where C is the boundry of the region defined by $y^2 = 8x$ and $x = 2$.

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4. (a) Find b such that the force field 6
$F = (e^x z - bxy)\, i + (1 - bx^2)\, j + (e^x + bz)k$ is conservative. Find the scalar potential ϕ of F, when F is conservative.

(b) Test whether the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory. 6

(c) Evaluate (1) $\int_0^{\infty} \frac{dx}{\left(a^2 + x^2\right)^2}$ 8

(2) $\int_{-\pi}^{\pi} \frac{d\theta}{1+\sin^2\theta}$.

5. (a) If $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ then find (1) 4^A (2) e^A. 6

(b) Find the sum of the residues of the function 6

$f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$.

(c) Verify Divergence theorem for $F = 4x_i - 2y^2 j + z^2 k$ taken over the region bounded by the cylinder 8

$x^2 + y^2 = 4, \quad z = 0, \quad z = 3.$

6. (a) Test whether the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. If yes, find the transforming matrix p and the diagonal matrix D. 6

(b) Define; Singular point, Essential singularity and Removable singularity with one example. 6

(c) Verify Stoke theorem for $F = (x^2 + y^2)\, i - 2xyj$ taken round the rectangle bounded by the lines 8

$x = \pm a,\ y = 0,\ y = b.$

7. (a) Evaluate $\iint_S F \cdot n\, ds$ where $F = (x + y^2)\, i - 2xj + 2yzk$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. 6

(b) (i) Expand the function $f(z) = \frac{\sin z}{z - \pi}$ and $z = \pi$. 6

(ii) Expand cos z in a Taylors series about $z = \frac{\pi}{4}$.

(c) Reduce the given quadratic form to a canonical form by orthogonal transformation and hence find rank index and signature. 8

$Q = 3x^2 + 5y^2 + 3z^2 - 2yz + 2xz - 2xy.$