SOLUTION & ANSWER FOR ISAT-2011
SET – A

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART A – PHYSICS

1. A projectile is fired at an angle 60° with some velocity u ------

Ans : No correct answer

Sol: $R = \frac{u^2}{g}\sin 2\theta$

$x = \frac{dR}{R} = 2\cot 2\theta\, d\theta$

$T = \frac{2u\sin\theta}{g}$

$y = \frac{dT}{T} = \cot\theta\, d\theta$

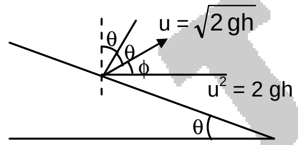
$\frac{x}{y} = \frac{2\cot 2\theta}{\cot\theta} = \frac{2\cot 120^\circ}{\cot 60^\circ} = -2$

$\Rightarrow x = -2y$ (No correct answer)

2. A ball is dropped down vertically from a tall building ------

Ans : $\theta = \frac{1}{4}\sin^{-1}\left(\frac{d}{h}\right)$

Sol:



$\theta + \theta + \phi = 90^\circ \Rightarrow \phi = (90^\circ - 2\theta)$

$d = \frac{1}{2}\frac{u^2}{g}\sin 2\phi = \frac{1}{2}\cdot\frac{2gh}{g}\sin 2\phi$

$= h\sin 2(90^\circ - 2\theta) = h\sin(180^\circ - 4\theta)$

$= h\sin 4\theta \Rightarrow \theta = \frac{1}{4}\sin^{-1}\left(\frac{d}{h}\right)$

3. A photon with an initial frequency 10^{11} Hz ------

Ans : 4×10^3 m s^{-1}

Sol: $E_1 = h\upsilon_1 = 6.63 \times 10^{-34} \times 10^{11}$

$= 6.63 \times 10^{-23}$ J

$E_2 = h\upsilon_2 = 6.63 \times 10^{-34} \times 0.9 \times 10^{11}$

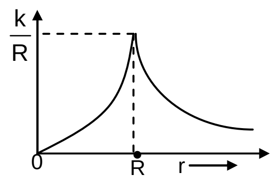
$= 5.967 \times 10^{-23}$ J

$\therefore \Delta E = E_1 - E_2 = 6.63 \times 10^{-24}$ J

$\Delta E = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2\Delta E}{m}} \cong 4 \times 10^3$ m s^{-1}

4. The correct potential energy diagram for ------

Ans :



Sol: For r < R,

$dU = -\overline{F}.dr = \frac{k}{R^3} r\, dr$

$\Rightarrow U = \int_0^r \frac{k}{R^3} r\, dr = \frac{k}{2R^3}r^2$

When r = 0, U = 0

⇒ Only option (a) is correct.

When r = R, $U_{(R)} \Rightarrow \frac{kR^2}{2R^3}$

$= \frac{k}{2R}$

5. Suppose the particle starts from r = ∝ ------

Ans : (3/8) (k/R)

Sol: $KE = U_{(R)} - U_{(R/2)}$

$= \frac{k}{2R} - \frac{k}{8R}$

$= \frac{3k}{8R}$

6. Let a particle have an instantaneous position ----

Ans : $\overline{r}.\overline{v} = 0$; $\overline{a}.\overline{v} > 0$; $\overline{a}.\overline{r} < 0$

Sol: In circular motion,

$\overline{r} \perp \overline{v} \Rightarrow \overline{r}.\overline{v} = 0$

$\overline{a}$ and $\overline{r}$ make angle θ > 90°

⇒  < 0

$\overline{a}$ and $\overline{v}$ may make angle θ between them which is either > 90° or <90° or = 90°

So for general case $\overline{a}.\overline{v} > 0$

7. A large parallel plate capacitor is made of two metal plates of size ------

Ans : $+5 \times 10^5\ \varepsilon_0$ J

Sol: $\Delta C = \dfrac{(K_1 - K_2)\varepsilon_0 A'}{d}$

$= \dfrac{(3-2)\varepsilon_0 \times 0.1 \times 1}{0.1}$

$= \varepsilon_0$

$\Delta U = \dfrac{1}{2}\Delta C V^2 = \dfrac{1}{2} \times \varepsilon_0 \times 10^6$

$= +5 \times 10^5\ \varepsilon_0$ J

8. A current I is flowing in a long straight wire along the z-axis------

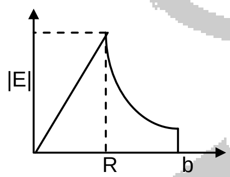
Ans : $v_Z(\Delta t) = v_0$

Sol: $\overline{F}$ on q is along $-\hat{i}$ direction $\Rightarrow$ $\overline{a}$ of q is along $-\hat{i}$ direction $\Rightarrow$ component of velocity in $\hat{k}$ does not change

$\Rightarrow v_Z(\Delta t) = v_0$

9. A non-conducting sphere of radius R has a charge Q distributed ------

Ans :



Sol: From r = 0 to r = R.

$E \propto r \Rightarrow$ straight line, inclined to r.

From r = R to r = b.

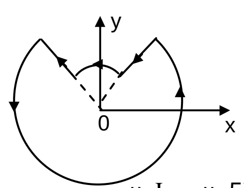
$E \propto \dfrac{1}{r^2}$

At $r \geq b$, E = 0

10. The magnetic field at the centre of a loop carrying ------

Ans : $\dfrac{\mu_0 I}{3R}\dfrac{7}{8}\hat{k}$

Sol:



$B = B_1 + B_2 = \dfrac{\mu_0 I}{12\ R} + \dfrac{\mu_0 5I}{24\ R} = \dfrac{7}{24}\dfrac{\mu_0 I}{R}$;

Direction $\hat{k}$

11. A current I is flowing in a wire of length λ. The total momentum ------

Ans : $\dfrac{mI\lambda}{q}$

Sol: $I = \dfrac{q}{t} \Rightarrow \dfrac{1}{t} = \dfrac{I}{q}$

$v = \dfrac{\lambda}{t} = \dfrac{I\lambda}{q}$

$p = mv = \dfrac{mI\lambda}{q}$

12. In an oil drop experiment, charged oil drops of mass m and charge q------

Ans : $\dfrac{mg4\pi\varepsilon_0(r-R)^2}{q^2}$

Sol: The answer must be dimensionless $\Rightarrow q^2$ is required in denominator to cancel $\varepsilon_0 \Rightarrow$ only (B) can be the answer.

13. Two lenses, one biconvex of focal length f_1 and another ------

Ans : $M = \dfrac{f_1}{f_2}$

Sol: $m_1 = \dfrac{v_1}{u_1} = \dfrac{f_1}{L}$

$m_2 = \dfrac{v_2}{u_2} = \dfrac{L}{f_2}$

$M = m_1 m_2 = \dfrac{f_1}{f_2}$

(L is large $\Rightarrow \infty$)

14. The central fringe in a Young's double slit experiment ------

Ans : $\dfrac{3}{4}$

Sol: Path difference = $(\mu - 1)$ t

$= (1.4 - 1) \times 5 \times 10^{-6}$

$= 2 \times 10^{-6}$ m

Phase difference $\Delta\phi = \dfrac{2 \times 10^{-6} \times 2\pi}{632.8 \times 10^{-9}}$

= 19.858 rad

= $3.16 \times 2\pi$ rad

$\Rightarrow \Delta\phi = 0.16 \times 2\pi = 57.6°$

$\Rightarrow I = I_0 \text{ cm}^2\left(\dfrac{\Delta\phi}{2}\right) = 0.767 I_0 \cong \dfrac{3}{4} I_0$

15. A polarizer is introduced in the path of a beam of unpolarized light incident ------

Ans : $\theta = 30°$ and the polarizer is placed in P (It is assumed that the polarized light is getting completely transmitted.)

Sol: $\tan\phi = \mu$ (Brewster's law)

$\Rightarrow \phi = \tan^{-1}\left(\sqrt{3}\right) = 60°$

on transparent material.

$\Rightarrow \theta = 90° - \phi = 30°$

16. A submarine travelling at 10 m s^{-1} is chasing another one in front of it ------

Ans : 13 m s^{-1}

Sol: $f_1 = f_0 \dfrac{(c-v)}{(c+10)}$

$f_2 = f_1 \dfrac{(c+10)}{(c+v)} = \dfrac{f_0(c-v)(c+10)}{(c-10)(c+v)}$

$= f_0 \dfrac{(c-v)(c+10)}{(c+v)(c-10)}$

$f_0 = 25000$ Hz; $f_2 = 24900$ Hz

$c = 1500$ m s^{-1}; $v = 13$ m s^{-1}

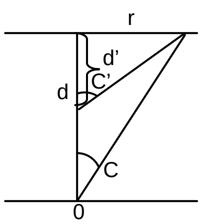
17. When light of intensity I reflects from a surface separating two ------

Ans : $\mu = \sqrt{\mu_1\mu_2}$; $2\mu t = \dfrac{\lambda(2n+1)}{2}$

18. A point object is placed below a wide glass plate of refractive index n. As an ------

Ans : $2\tan^{-1}\dfrac{n}{\sqrt{n^2-1}}$

Sol:



$\theta = 2C'$; $\tan C' = \dfrac{r}{d'} = \dfrac{nr}{d}$

$= n \tan C = \dfrac{n}{\sqrt{n^2-1}} \left(\because \sin C = \dfrac{1}{n}\right)$

$\Rightarrow \theta = 2\tan^{-1}\dfrac{n}{\sqrt{n^2-1}}$

19. A light sensor is fixed at one corner of the bottom of a rectangular tank ------

Ans : A quarter of a circle of radius = $10\sqrt{3}$ m

Sol: $\tan C = \dfrac{1}{\sqrt{n^2-1}} = \dfrac{r}{d}$

$\Rightarrow r = \dfrac{d}{\sqrt{n^2-1}} = 10\sqrt{3}$ m

20. The average pressure on a sphere submerged in water is the pressure ------

Ans : 63 N

Sol: P = pressure at centre = $\rho g H$

$= 1000 \times 10 \times (0.1 + 0.1)$

$= 2000$ N m^{-2}

$F = P \times$ area

$= 2000 \times \pi \times (0.1)^2$

$= 20\pi = 63$ N

21. Laplace correction to the speed of sound is made only for gases and not ------

Ans :Much smaller relative pressure change when the wave is passing through them.

Sol: Knowledge based.

22. Three rods of equal lengths and cross sectional areas are joined ------

Ans : $T_1 = \dfrac{3}{5}T_A + \dfrac{2}{5}T_B$; $T_2 = \dfrac{2}{5}T_A + \dfrac{3}{5}T_B$

Sol: $\dfrac{KA(T_A - T_1)}{L} = \dfrac{2KA(T_1 - T_2)}{L} = \dfrac{KA(T_C - T_B)}{L}$

On solving,

$T_1 = \dfrac{3}{5}T_A + \dfrac{2}{5}T_B$ and

$T_2 = \dfrac{2}{5}T_A + \dfrac{3}{5}T_B$

23. The diameter of a metal wire is measured using a screw gauge, ------

Ans : $1.21 \times 10^{-5}\ \Omega$ m

Sol: Pitch = 0.5 mm

$L.C = \dfrac{\text{Pitch}}{N} = \dfrac{0.5}{50} = 0.01$ mm

$P.S.R = 4 \times 0.5 = 2.0$ mm

$CSR = 20 \times LC = 20 \times 0.01 = 0.2$ mm

$d = 2.2$ mm $= 2.2 \times 10^{-3}$ m

$\rho = \dfrac{RA}{\lambda} \Rightarrow \rho = 1.21 \times 10^{-5}\ \Omega$ m

24. Which of the following quantities has the least number of ------

Ans : 0.08765

Sol: Knowledge based.

25. In an experiment designed to determine the universal gravitational ------

Ans : No correct answer.

Sol: $[G] = M^{-1}L^{3}T^{-2}$

$$= \frac{L^3}{MT^2}$$

$$\frac{dG}{G} = \frac{3\Delta L}{L} + \frac{\Delta M}{M} + \frac{2\Delta T}{T}$$

$$= 3a + b + 2c$$

PART B – CHEMISTRY

26. The relative stability of the octahedral complexes ------

Ans : (i) > (ii) > (iii) > (iv)

Sol: oxygen ligands have high affinity for Fe(III) and affinity of Fe(III) for amines is low

27. Number of isomers that ------

Ans : 3

Sol: The complex is square planar and is of the type M_{abcd}. It has three geometrical isomers.

28. When a metal is in its low oxidation state, ------

Ans : Chloride is a σ donor and the carbon monoxide is both a σ donor as well as π acceptor

Sol: Metal – CO bond is stronger than Metal – Cl bond because CO act as a σ donor as well as π acceptor ligand

29. Freshly prepared, bright blue coloured, ------

Ans : $[e(NH_3)_n]^-$ ('e' is an electron)

Sol: Ammoniated electron brings about the reduction of the functional group

30. The statement that is NOT ------

Ans : silicates are mainly built through 'SiO_2' units

Sol: silicates are built through tetrahedral SiO_4^{4-} units

31. The $(SIO_3^{2-})_n$ ------

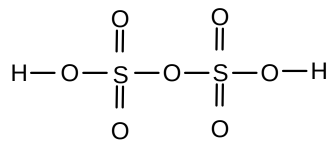
Ans : cyclic silicates

Sol: Linear single chain silicates have empirical formula $[(SiO_3)^{2-}]_n$

32. The oxoacid of sulphur that ------

Ans : pyrosulphuric acid ($H_2S_2O_7$)

Sol: pyrosulphuric acid ($H_2S_2O_7$)

is 

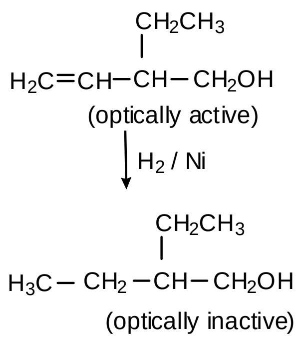
33. The reason for the formation ------

Ans : acidic nature of $B(OH)_3$

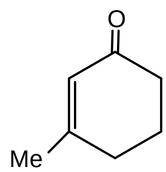
Sol: $B(OH)_3$ is a Lewis acid. It reacts with water to form $[B(OH)_4]^-$ & H^+

34. An optically active alcohol (X) ------

Ans : 2-ethyl-3-buten-1-ol

Sol: 

35. The major product formed in the ------

Ans : 

Sol: Ozonolysis of the given unsaturated compound gives

$CH_3-\underset{\underset{O}{\|}}{C}-(CH_2)_3-\underset{\underset{O}{\|}}{C}-CH_3$

This undergoes intramolecular aldol condensation to form the compound having structure (A)

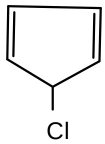
36. The following transformation ------

Ans : NaOH / I_2

Sol: Iodoform reaction will bring about the given conversion

37. Among the following halides, ------

Ans : III

Sol:  does not undergo ionisation

(III)

because the resultant cation is antiaromatic

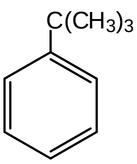
38. Isopropanol can be converted ------

Ans : pyridinium chlorochromate followed by peracetic acid

Sol: $CH_3-CHOH-CH_3 \xrightarrow{PCC}$

$CH_3-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{CH_3CO_3H} CH_3-\underset{\underset{O}{\|}}{C}-OCH_3$

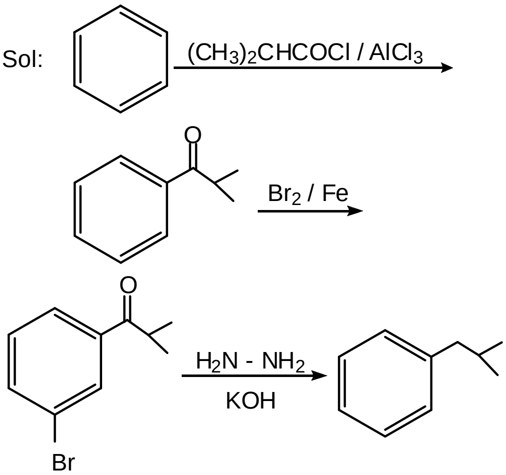
39. Among the isomeric butylbenzenes, the one ------

Ans : 

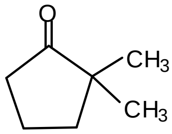
Sol: For the oxidation of the side chain, the carbon attached to the benzene ring must contain at least one hydrogen

40. The following reaction is ------

Ans : i $(CH_3)_2CHCOCl$ / $AlCl_3$:
ii. Br_2 / $FeBr_3$: iii. NH_2. NH_2 / KOH

Sol: 

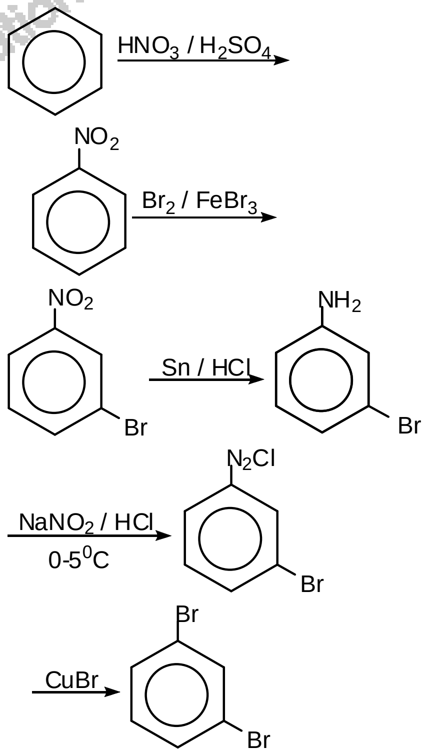
41. The major product of the following------

Ans : 

Sol: Protonation of oxygen followed by cleavage of three membered ring gives a 3° carbocation. This undergoes ring expansion followed by loss of proton gives (B)

42. Conversion of benzene into 1, 3------

Ans : i. HNO_3 / conc. H_2SO_4: ii. Br_2 / $FeBr_3$
iii. Sn / HCl iv. $NaNO_2$ / HCl,0-5°C
v. CuBr

Sol: 

43. Liquid oxygen and liquid nitrogen are ------

Ans : Liquid oxygen will be attracted but liquid nitrogen unaffected

Sol: Oxygen is paramagnetic and nitrogen is diamagnetic

44. The highest transition energy ------

Ans : 27434.25 cm^{-1}

Sol: $\bar{\upsilon} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$= 109737\left[\frac{1}{4} - 0\right]$

$= 27434.25\ cm^{-1}$

45. A one litre glass bulb is evacuated and ------

Ans : 30

Sol: $PV = \frac{W}{M}RT$

$M = \frac{1.2 \times 0.08 \times 312.5}{1 \times 1}$

$= 30$

46. The van der Waals coefficient of the inert ------

Ans : Induced dipole- Induced dipole : increased atomic volume

Sol: 'a' is a measure of attraction between the molecules and 'b' is a measure of the size of the molecules

47. Assuming ΔH^0 and S^0 do not change with ------

Ans : 300 K

Sol: $A_{(\lambda)} \longrightarrow A_{(g)}$

$\Delta H° = 30\ kJ\ mol^{-1}$

$\Delta S° = 100\ J\ mol^{-1}$

$T = \frac{\Delta H^o}{\Delta S^o} = 300\ K$

48. A solution of $CaCl_2$ was prepared by ------

Ans : 0.0006

Sol: $\Delta T_f = i \times K_f \times m$

$= \frac{3 \times 2 \times 0.0112}{112}$

$= 0.0006$

49. Of thr four values of pH given below which ------

Ans : 4.4

Sol: $H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$;

$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}$

$[H^+] = C\alpha = \sqrt{K_{a_1}.C}$ since $K_{a_1} >> K_{a_2}$

$= \sqrt{4 \times 10^{-7} \times 4 \times 10^{-3}}$

$= 4 \times 10^{-5}$

pH = 4.4

50. The Habers's process process for the ------

Ans : Ammonia dissociates spontaneously above 500 K

Sol: $\Delta G° = \Delta H° - T\Delta S°$

At equilibrium $\Delta G° = 0$

$\therefore T = \frac{\Delta H^o}{\Delta S^o} = \frac{-95 \times 10^3}{-190} = 500\,K$

Above 500 K, $\Delta G°$ is +ve

PART C – MATHEMATICS

51. Martin throws two dice simultaneously. ------

Ans : $\frac{1}{20}$

Sol: Let L denote the event of offering lunch

P(L) = P (12). P(L | 12) + P(7) .P(L | 7) + P (others). P(L | others)

$P(12) = \frac{1}{36}; P(7) = \frac{6}{36}\ P(\text{others}) = \frac{29}{36}$

$\therefore P(12 \mid L) = \dfrac{\frac{1}{36} \times \frac{2}{3}}{\frac{1}{36} \times \frac{2}{3} + \frac{6}{36} \times \frac{1}{2} + \frac{29}{36} \times \frac{1}{3}}$

$= \frac{1}{20}$

52. A spices has an initial population 4^{10}. ------

Ans : 20

Sol: $P = 4^{10}$

$P_1 = 4^{10}\ \frac{3}{2}$; $P_2 = 4^{10}\ \frac{3}{2} . \frac{1}{2}$

$P_3 = 4^{10}\left(\frac{3}{2}\right)^2 \frac{1}{2}; P_4 = 4^{10}\left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right)^2$

$\therefore P_t = 4^{10}\left(\frac{3}{2}\right)^{\frac{t}{2}} . \left(\frac{1}{2}\right)^{\frac{t}{2}}$ when t - even

$\therefore 4^{10}\left(\frac{3}{2}\right)^{\frac{t}{2}}\left(\frac{1}{2}\right)^{\frac{t}{2}} = 3^{10}$

$\left(\frac{3}{4}\right)^{\frac{t}{2}} = \left(\frac{3}{4}\right)^{10} \Rightarrow \frac{t}{2} = 10$

$\therefore$ t = 20

53. If 4 squares are chosen at random ------

Ans : $2\frac{{}^{8}C_4}{{}^{64}C_4}$

Sol: Eight squares lie on a main diagonal and there are 2 main diagonals.

$\therefore$ Probability = $2\frac{{}^{8}C_4}{{}^{64}C_4}$

54. A student was calculating the variance ------

Ans : $\frac{825}{100}$

Sol: $\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = V$

$10\sum x_c^2 - \left(\sum x\right)^2$ = 100 V ___(1)

$10\sum x_c^2 - \left(\sum x\right)^2$ = 100 V_c ___(2)

$\sum x = 46$

$\sum x_c = \sum x - 1 + 10 = \sum x + 9$

$\therefore \sum x_c^2 = \sum x^2 - 1^2 + 10^2 = \sum x^2 + 99$

$\therefore$ From (2) $\Rightarrow$ 10 $\left(\sum x^2 + 99\right) - \left(\sum x + 9\right)^2$ = 100 V_c

$\Rightarrow$ $10\left(\sum x^2 - \left(\sum x\right)^2\right) + 990 - 18 + 46 - 81$ = 100 V_c

$\therefore$ 100 V + 81 = 100 V_c

$\therefore V_c = \frac{744 + 81}{100} = \frac{825}{100}$

55. A fair coin is tossed 6 times ------

Ans : $\frac{5}{16}$

Sol: Head appears as the 6th trial for the third time. So in the first 5 trials .Head appeared twice.

$\therefore$ Probability = ${}^{5}P_2 \left(\frac{1}{2}\right)^6 = \frac{5}{16}$

56. The sum of the roots of the equation ------

Ans : $\log_2 11$

Sol: Rewriting

$\log_2 2^x + \log_2 2 - \log_2(2^x + 3)^2 + \log_2(10 - 2^{-x}) = 0$

$\left(\frac{2^x \times 2}{(2^x + 3)^2} \times (10 - 2^{-x})\right) = 1$

Rearranging

$2 \times (10 \times 2^x - 1) = (2^x + 3)^2$

$2(10y - 1) = y^2 + 6y + 9$, taking $y = 2^x$

$Y^2 - 14y + 11 = 0$

$2^{x_1 + x_2} = 11$

$x_1 + x_2 = \log_2 11$

57. Let $z = a\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$ ------

Ans : $\frac{a^{2010}}{1 - z}$

Sol: $z^{2010} = a^{2010}\left(\cos\frac{2010\pi}{5} + i\sin\frac{2010}{5}\pi\right)$

$= a^{2010}(\cos 2\pi + i\sin 2\pi)$

$= a^{2010}$

$\therefore z^{2010} + z^{2011} + z^{2012} + \ldots$

$= z^{2010}\left(\frac{1}{1 - z}\right)$

$= \frac{a^{2010}}{1 - z}$

58. The locus of the point z satisfying arg------

Ans : a single point

Sol: let z = x + iy

Z + 1 = (x + 1) + i y

$\therefore$ arg (z + 1) = $\tan^{-1}\left(\frac{y}{x+1}\right) = \alpha$

arg (z − 1) = $\tan^{-1}\left(\frac{y}{x-1}\right) = \beta$

$\therefore \tan\alpha = \frac{y}{x+1} \quad \tan\beta = \frac{y}{x-1}$

Since $\frac{1}{\tan\alpha} - \frac{1}{\tan\beta} = 2$

$\Rightarrow \frac{x+1}{y} - \frac{(x-1)}{y} = 2$

$\Rightarrow 2 = 2y \Rightarrow y = 1$

line parallel to the x – axis

59. For the equation, sin x + cos x= ------

Ans : there is a solution, for exactly one a> 0

Sol: Equation can be written as

$\frac{1}{\sqrt{2}}\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\left(a + \frac{1}{a}\right)$

$\cos\left(x - \frac{\pi}{4}\right) = a + \frac{1}{a}$

if $a > 0 \Rightarrow a + \frac{1}{a} \geq 2$, equality for a = 1

but $\cos\left(x - \frac{\pi}{4}\right) \leq 1$

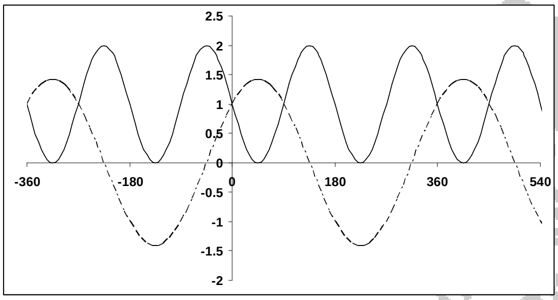
$\therefore$ Equation has one solution

60. The number of solutions of the equation ------

Ans : 6

Sol:

From the following graph it is seen that there are 6 intersection points.



61. Consider the circles $C_1 : x^2 - y^2 = 64$ ------

Ans : $\left(\frac{6}{\sqrt{2}}, \frac{6}{\sqrt{2}}\right)$

Sol: .Let centre of the circle be (α, α)

$\therefore$ Equation is

$x^2 + y^2 - 2\alpha x - 2y\alpha + 2\alpha^2 = 100$ —— (1)

Equation of given circle

$x^2 + y^2 = 64$ —— (2)

$\therefore$ Equation of common chord

$\Rightarrow S_1 - S_2 = 0$

$\Rightarrow x\alpha + y\alpha - \alpha^2 + 18 = 0$ —— (3)

Since length of common chord = 16

which is a diameter of $x^2 + y^2 = 64$

$\Rightarrow \alpha^2 = 18$

$\therefore \alpha = 3\sqrt{2} = \left(\frac{6}{\sqrt{2}}\right)$

$\therefore$ Centre $\left(\frac{6}{\sqrt{2}}, \frac{6}{\sqrt{2}}\right)$

62. A line segment joining (1, 0, 1) and the origin -----

Ans : $x^2 - 2y^2 - z^2 = 0$

Sol: The semi vertical angle α is given by

$\sin \alpha = \frac{x}{r} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ locus of any point on the cone is

$\frac{x}{r} = \frac{1}{\sqrt{2}}$

i.e $\frac{x^2}{x^2 + y^2 + z^2} = \frac{1}{2}$ or $x^2 - y^2 - z^2 = 0$

63. Let (x, y, z) be any point on the line passing ------

Ans : $x^2 - 2y^2 - z^2 = 0$

Sol: (x, y, z) passing through a line which is parallel to that vector i+ j+ k.
Then this vector is perpendicular to the plane passing through (x, y, z)

64. A tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$------

Ans : $\sqrt{82}$

Sol: Equation tangent is

$\frac{x}{5}\cos + \frac{y}{4}\sin\theta = 1$

$\therefore A\left(\frac{5}{\cos\theta}, 0\right)$ and $B\left(0 \frac{4}{\sin\theta}\right)$

sin OAB is isosceles OA = OB $\Rightarrow$

$\frac{5}{\cos\theta} = \frac{4}{\sin\theta} = k$

$\therefore \cos\theta = \frac{5}{k}$ as $\sin\theta = \frac{4}{k} \Rightarrow k = \sqrt{41}$

$\therefore AB = \sqrt{k^2 + k^2} = \sqrt{82}$

65. Let $a_n = \frac{1}{n}\left[(2n+1)(2n+2)...(2n+n)^{1/n}\right]$------

Ans : $\int_2^3 \log(2+x)\,dx$

Sol: log a_n = $\frac{1}{n}\left[\log\left(2+\frac{1}{n}\right)+\log\left(2+\frac{2}{n}\right)+....+\log\left(2+\frac{n}{n}\right)\right]$

$$\lim_{n\to\infty}{}^{(\log a_n)} = \lim_{n\to\infty}\frac{3-2}{n}\sum_{n=1}^{n}\log\left(2+\frac{r}{n}\right)$$

$$= \int_2^3 \log(2+x)dx = L$$

66. The value of $\lim_{n\to x}\frac{1^3+2^3+....+(3n)^3}{3n^4}$ is ------

Ans : 27/4

Sol: $\lim_{n\to\alpha}\frac{\sum_1^{34}(r^3)}{3n^4} = \ = \lim_{n-13}\frac{\left(3n\frac{(3n+1)}{2}\right)^2}{3n^4} = \frac{27}{4}$

67. The value of $\int_0^{\pi/2}\frac{2+\sin x}{1+\cos x}e^{\pi/2}\,dx$ is ------

Ans : $2e^{\frac{\pi}{4}}$

Sol:

$$\int_0^{\frac{\pi}{4}}\frac{2+\sin x}{1+\cos x}e^{\frac{x}{2}}dx = \int_0^{\frac{\pi}{2}}\left(\sec^2\frac{x}{2}+\tan\frac{x}{2}\right)e^{\frac{x}{2}}\ dx$$

$$= 2\int_0^{\frac{\pi}{4}}(\sec u+\tan u)e^u du$$

$$= 2\left(e^u\tan u\right)_0^{\pi/4} = 2e^{\pi/4}$$

68. The differential equation satisfied by------

Ans : $x+3yy^1 = 0$

Sol: $y = \alpha x^3 \Rightarrow y^1 = 3\alpha x^2 \Rightarrow \alpha = \frac{y^1}{3x^2}$

∴ Differential equation of given

come in $y = \frac{y^1x^3}{3x^2} \Rightarrow 3y = xy^1$

∴ Corresponding curve perpendicular to it

ie $3y = x\ \frac{-1}{y^1}$

$\Rightarrow 3yy^1 + x = 0$

69. Let $f(x) = x\ (|x-\pi|)$ ------

Ans : onto but NOT one-one

Sol: $f(x) = x\ |x-\pi|\ \ (2+\cos^2 x)$

F(x) is continuous everywhere. $f(-\infty) = -\infty$, $f((-\infty) = \infty$

∴ f is onto

If $0 < x < \pi$

$f(x) = x(\pi - x)\ (2+\cos^2 x)$

$f(\pi - x) = \pi - x\,(x)\,(2+\cos^2 x)$

∴ f is not one – one

70. The equation $2x^3 - 3x^2 - p = 0$ ------

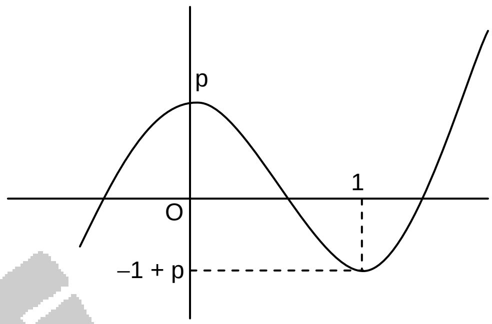
Ans : (0, 1)

Sol: Let $f(x) = 2x^3 - 3x^2 + p$

$f'(x) = 6x(x-1)$;

$f''(0) = -6 < 0$; $f''(1) = 6 > 0$

$f(0) = p$; $f(1) = -1 + p$



f(x) will have 3 distinct real roots if $p > 0$ and $-1 + p < 0$

i.e. $p \in (0, 1)$

71. For a real number x let ------

Ans : continuous at x = 1 but NOT continuous at x = 2

Sol: $f(x) = \{x\}^{[x]}\cos\frac{\pi}{2}x$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0}\{1-h\}^{[1-h]}\cos\frac{\pi}{2}(1-h) = 0$

$\therefore \lim_{x\to 1^+} f(x) = \lim_{h\to 0}\{1+h\}^{[1+h]}\cos\frac{\pi}{2}(1+h) = 0$

f(1) = 0 and f(x) is continuous at x = 1.

Again, $\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$

$= \lim_{h\to 0}\{2-h\}^{[2-h]}\cos\frac{\pi}{2}(2-h)$

$= \lim_{h\to 0}\{2-h\}^{1}\cos\frac{\pi}{2}(2-h)$

$= 1\times\cos\pi = -1$.

$\lim_{x\to 2^+} f(x) = \lim_{h\to 0}\{2+h\}^{2}\cos\frac{\pi}{2}(2+h)$

= 0.

∴ f(x) is not continuous at x = 2.

72. Let f : (0, ∞) → R be------

Ans : 2

Sol: f(x) = $2x^{\sin 2x}\cos 2x$

$\therefore \lim\limits_{x\to 0} f(x) = 2\lim\limits_{x\to 0} x^{\sin 2x}$

$= 2e^{\lim\limits_{x\to 0}\frac{\log x}{\operatorname{cosec} 2x}}$

= 2.

73. The distance of the point (1, 2, 3) ------

Ans : $3\sqrt{3}$

Sol: Equation of line passing through (1, 2, 3) and parallel to $\vec{r} = \left(-3\hat{i} + 2\hat{j}\right) + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right)$ is

$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right)\right)$

$\therefore \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{1} = \lambda$

x = 1 + λ, y = 2 + λ, z = 3 + λ

2x + y + 2z + 5 = 0

2(1 + λ) + (2 + λ) + 2(3 + λ) + 5 = 0

λ = –3

∴ point of intersection is (–2, –1, 0)

Distance from (1, 2, 3) = $\sqrt{3^2 + 3^2 + 3^2} = 3\sqrt{3}$.

74. If the vector------

Ans : 92

Sol: Let $V_1 = \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$

$V_2 = \mu\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$

$V_2 \cdot \left(2\hat{i} - \hat{k}\right) = 0 \Rightarrow c = 2a$

$\therefore V_1 + V_2 = (\lambda + \mu a)\hat{i} + (-\lambda + \mu b)\hat{j} + (\lambda + 2\mu a)\hat{k}$

$\lambda = -1,\ \mu a = 4, \mu b = 3$

$\therefore |V_1|^2 + |V_2|^2 = 3\lambda^2 + 5(\mu a)^2 + (\mu b)^2$

= 3 + 5 × 16 + 9 = 92.

75. A plane H passes through the intersection------

Ans : $\bar{r}\left(3\hat{i} - \hat{j} + 3\hat{k}\right) = 1$

Sol: Point dividing (3, 0, 2) and (0, 3, –1) in the ratio 2:1 internally is (1, 2, 0)

Equation of the required plane is (x + y + z + 3) + λ (x – y +3z – 2) = 0

⇒ 6 + λ (–3) = 0 ⇒ λ = 2

Equation is 3x – y + 3z – 1 = 0, i.e. $\bar{r}\left(3\hat{i} - \hat{j} + 3\hat{k}\right) = 1$