

PART - I (CHEMISTRY)

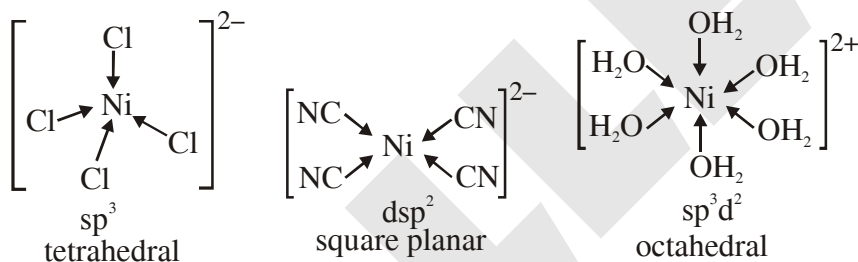
SECTION - I (TOTAL MARKS : 21)

(Single Correct Answer Type)

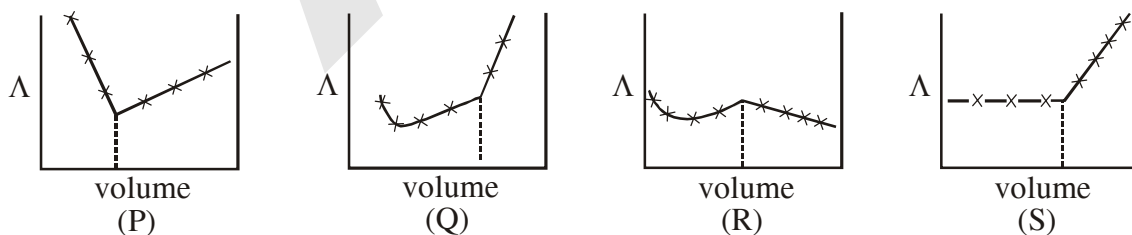
This section contains 7 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Geometrical shapes of the complexes formed by the reaction of  $\text{Ni}^{2+}$  with  $\text{Cl}^-$ ,  $\text{CN}^-$  and  $\text{H}_2\text{O}$  respectively, are -
- (A) octahedral, tetrahedral and square planar  
(B) tetrahedral, square planar and octahedral  
(C) square planar, tetrahedral and octahedral  
(D) octahedral, square planar and octahedral

1. Ans.(B)



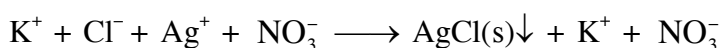
2.  $\text{AgNO}_3$  (aq.) was added to an aqueous  $\text{KCl}$  solution gradually and the conductivity of the solution was measured. the plot of conductance ( $\Lambda$ ) versus the volume of  $\text{AgNO}_3$  is -



- (A) (P)                      (B) (Q)                      (C) (R)                      (D) (S)

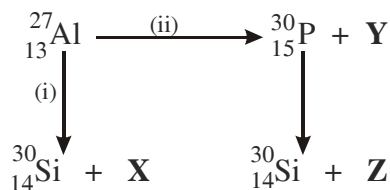
2. Ans.(D)

As  $\text{AgNO}_3$  is added



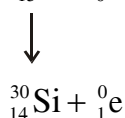
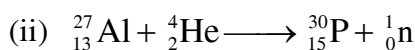
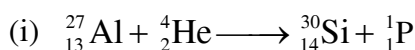
Number of ions are approximately constant, when  $\text{AgNO}_3$  is added till equivalence point. After equivalence point number of ions, increases and hence conductivity.

3. Bombardment of aluminium by  $\alpha$ -particle leads to its artificial disintegration in two ways, (i) and (ii) as shown. Products X, Y and Z respectively are :



- (A) proton, neutron, positron  
(B) neutron, positron, proton  
(C) proton, positron, neutron  
(D) positron, proton, neutron

3. Ans.(A)



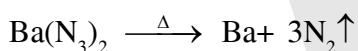
X  $\rightarrow$  proton, Y  $\rightarrow$  neutron, Z  $\rightarrow$  positron

4. Extra pure  $\text{N}_2$  can be obtained by heating

- (A)  $\text{NH}_3$  with  $\text{CuO}$   
(B)  $\text{NH}_4\text{NO}_3$   
(C)  $(\text{NH}_4)_2\text{Cr}_2\text{O}_7$   
(D)  $\text{Ba}(\text{N}_3)_2$

4. Ans.(D)

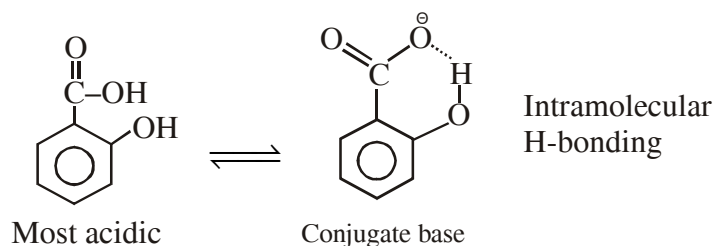
Extra pure  $\text{N}_2$  can be obtained by heating of  $\text{Ba}(\text{N}_3)_2$



5. Among the following compounds, the most acidic is

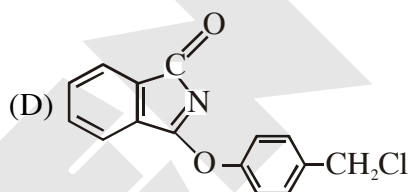
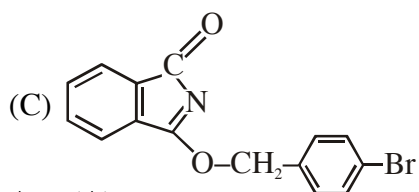
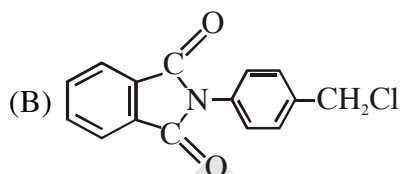
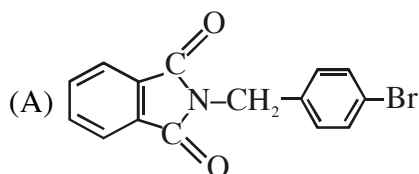
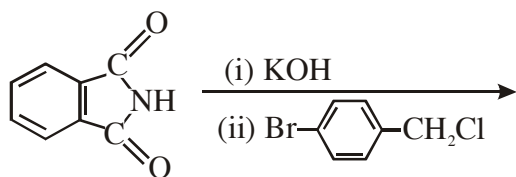
- (A) p-nitrophenol  
(B) p-hydroxybenzoic acid  
(C) o-hydroxybenzoic acid  
(D) p-toluic acid

5. Ans.(C)

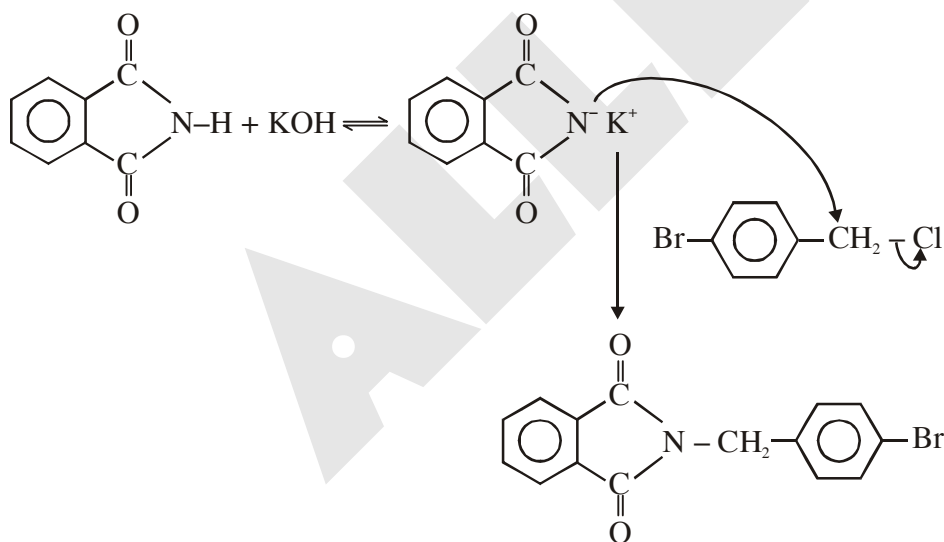


Due to ortho effect and chelation in its conjugate base, o-hydroxy benzoic acid is most acidic.

6. The major product of the following reaction is



6. Ans.(A)



7. Dissolving 120 g of urea (mol. wt. 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is

- (A) 1.78 M                      (B) 2.00 M                      (C) 2.05 M                      (D) 2.22 M

7. Ans.(C)

Mass of solute (urea) = 120 g

Mass of solvent (water) = 1000 g

Total mass of solution = (mass of solute + mass of solvent)

$$= 120 + 1000$$

$$= 1120 \text{ g}$$

$$M = \frac{2}{1120/1.15} \times 1000 = 2.05 \text{ M}$$

**SECTION - II (TOTAL MARKS : 16)**

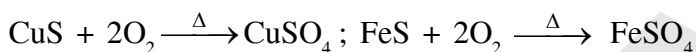
**(Multiple Correct Answers Type)**

This section contains 4 **multiple choice questions**. Each questions has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

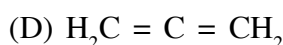
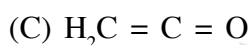
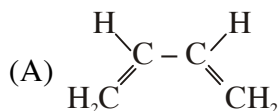
8. Extraction of metal from the ore cassiterite involves
- (A) carbon reduction of an oxide ore                      (B) self-reduction of a sulphide ore
- (C) removal of copper impurity                              (D) removal of iron impurity

8. **Ans.(A,C,D)**

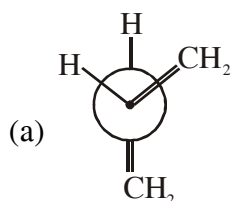
Extraction of tin from cassiterite ( $\text{SnO}_2$ ) involves reduction of ore (oxide) by carbon. When the concentrated tin stone ore  $\text{SnO}_2$  (ore of Sn) is heated strongly in a free supply of air (roasting) the impurities of CuS and FeS present in the ore are converted into  $\text{CuSO}_4$  and  $\text{FeSO}_4$  **respectively**. The  $\text{CuSO}_4$  and  $\text{FeSO}_4$  are water soluble which are leached out by hot water.



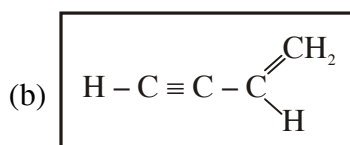
9. Amongst the given option, the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is (are) -



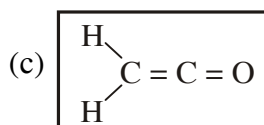
9. **Ans.(B,C)**



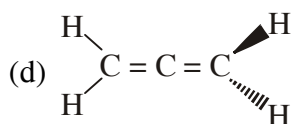
Different conformations don't have all atoms in the same plane.



Molecular plane



Molecular plane



All atoms are not present in the same plane.

Ans. is (B) and (C)

10. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is (are) -
- (A) Adsorption is always exothermic  
 (B) Physisorption may transform into chemisorption at high temperature  
 (C) Physisorption increases with increasing temperature but chemisorption decreases with increasing temperature  
 (D) Chemisorption is more exothermic than physisorption, however it is very slow due to higher energy of activation

10. **Ans.(A,B,D)**

**Note : Option (A) is correct as per NCERT book but few examples are known where  $\Delta H_{\text{adsorption}}$  is even endothermic.**

11. According to kinetic theory of gases
- (A) collisions are always elastic  
 (B) heavier molecules transfer more momentum to the wall of the container  
 (C) only a small number of molecules have very high velocity  
 (D) between collisions, the molecules move in straight lines with constant velocities.

11. **Ans.(A,B,C,D)**

**SECTION - III (TOTAL MARKS : 15)**

**(Paragraph Type)**

This section contains **2 paragraphs**. Based upon one of the paragraph, **3 multiple choice questions** and based on the other paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

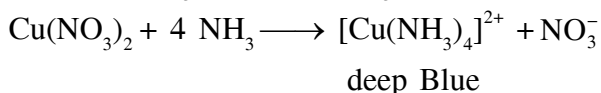
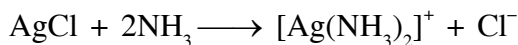
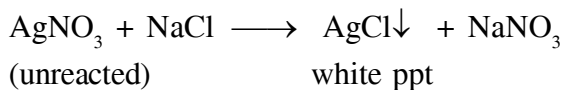
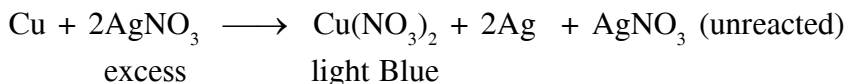
**Paragraph for Questions Nos. 12 to 14**

When a metal rod M is dipped into an aqueous colourless concentrated solution of compound N, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives a white precipitate O. Addition of aqueous  $\text{NH}_3$  dissolves O and gives in intense blue solution.

12. The metal rod M is -
- (A) Fe                      (B) Cu                      (C) Ni                      (D) Co
13. The compound N is -
- (A)  $\text{AgNO}_3$               (B)  $\text{Zn}(\text{NO}_3)_2$               (C)  $\text{Al}(\text{NO}_3)_3$               (D)  $\text{Pb}(\text{NO}_3)_2$
14. The final solution contains -
- (A)  $[\text{Pb}(\text{NH}_3)_4]^{2+}$  and  $[\text{CoCl}_4]^{2-}$               (B)  $[\text{Al}(\text{NH}_3)_4]^{3+}$  and  $[\text{Cu}(\text{NH}_3)_4]^{2+}$   
 (C)  $[\text{Ag}(\text{NH}_3)_2]^+$  and  $[\text{Cu}(\text{NH}_3)_4]^{2+}$               (D)  $[\text{Ag}(\text{NH}_3)_2]^+$  and  $[\text{Ni}(\text{NH}_3)_6]^{2+}$

Sol. of 12 to 14

M - Cu, N - AgNO<sub>3</sub>, O - AgCl



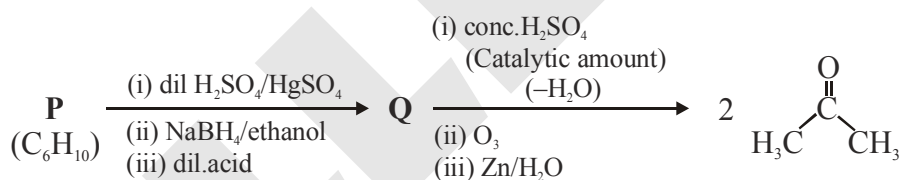
12. Ans.(B)

13. Ans.(A)

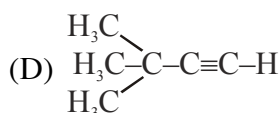
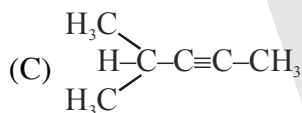
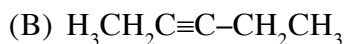
14. Ans.(C)

**Paragraph for Questions Nos. 15 to 16**

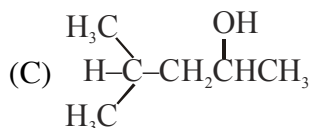
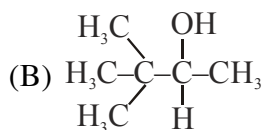
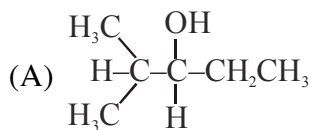
An acyclic hydrocarbon P, having molecular formula C<sub>6</sub>H<sub>10</sub>, gave acetone as the only organic product through the following sequence of reactions, in the which Q is an intermediate organic compound.



15. The structure of compound P is -



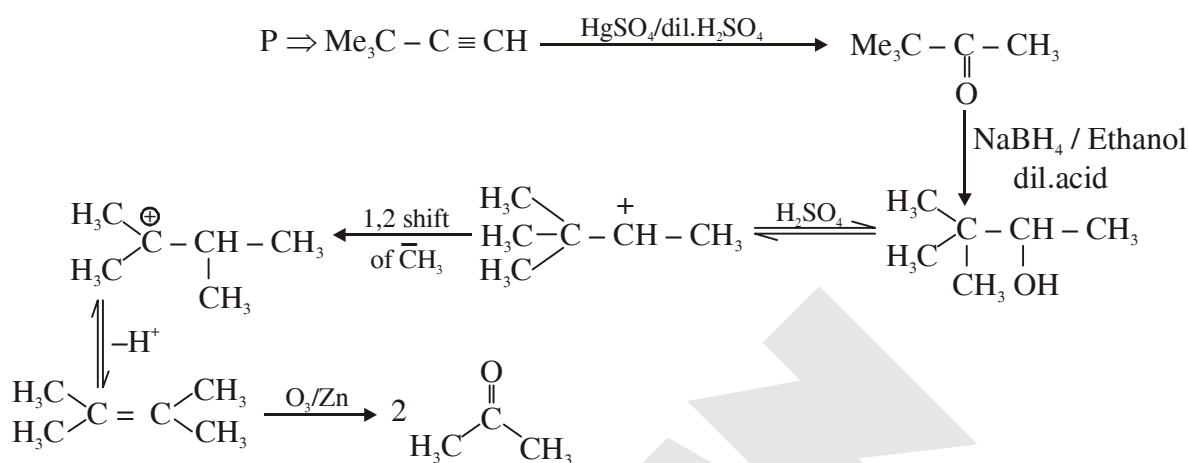
16. The structure of the compound Q is -



Sol. of 15 to 16

15. Ans.(D)

16. Ans.(B)



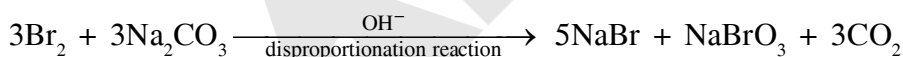
SECTION - IV (TOTAL MARKS : 28)

(Integer Answer Type)

This section contains 7 questions. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

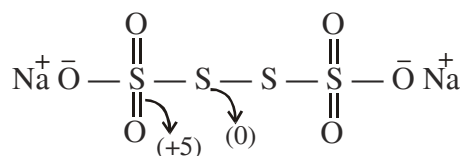
17. Reaction of  $\text{Br}_2$  with  $\text{Na}_2\text{CO}_3$  in aqueous solution gives sodium bromide and sodium bromate with evolution of  $\text{CO}_2$  gas. The number of sodium bromide molecules involved in the balanced chemical equation is.

17. Ans.(5)



18. The difference in the oxidation numbers of the two types of sulphur atoms in  $\text{Na}_2\text{S}_4\text{O}_6$  is.

18. Ans.(5)



Hence, the difference in oxidation states of 'S' atom =  $5 - 0 = 5$

19. The maximum number of electrons that can have principal quantum number,  $n=3$ , and spin quantum number,  $m_s = -1/2$ , is

19. Ans.(9)

No. of  $e^-$  having  $m_s = -1/2$

$n = 3$ ,	$l = 0$	1
	$l = 1$	3
	$l = 2$	5

Total No. of  $e^- = 1 + 3 + 5 = 9$

20. A decapeptide (Mol. Wt. 796) on complete hydrolysis gives glycine (Mol. Wt. 75), alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is

20. Ans.(6)

No. of peptide linkage = No. of water molecules added for complete hydrolysis.

$$= n - 1$$

So, number of molecules of H<sub>2</sub>O added = 9

So total wt. of the product = Mol. wt. of polypeptide + total wt. of H<sub>2</sub>O added.

$$= 796 + (9 \times 16)$$

$$= 796 + 162$$

$$= 958$$

$$\therefore \text{wt. of glycine obtained} = 958 \times \frac{47}{100} \approx 450$$

$$\text{No. of units of glycine} = \frac{450}{75} = 6 \text{ units}$$

21. To an evacuated vessel with movable piston under external pressure of 1 atm., 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. at 0°C) are introduced. Considering the ideal gas behaviour, the total volume (in litre) of the gases at 0°C is close to

21. Ans.(7)

Since external pressure is 1 atm

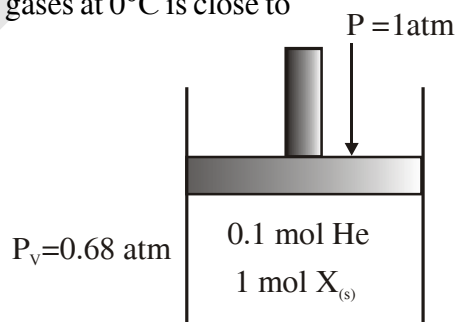
$$P_v + P_{\text{He}} = 1$$

$$\Rightarrow P_{\text{He}} = 1 - 0.68 = 0.32 \text{ atm}$$

Now from ideal gas equation  $\Rightarrow PV = nRT$

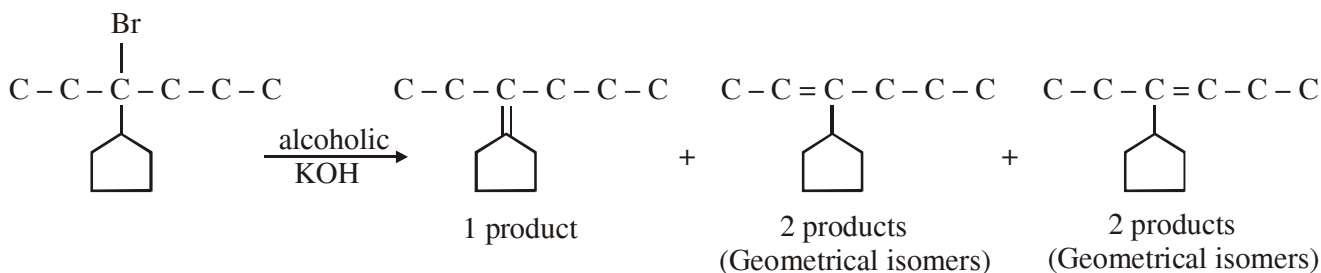
$$\Rightarrow 0.32 \times V = 0.1 \times (R \times 273)$$

$$\Rightarrow V = 7 \text{ litre.}$$



22. The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is

22. Ans.(5)





23. The work function ( $\phi$ ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is : :-

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
$\phi$ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

23. **Ans.(4)**

$$\lambda = 300 \text{ nm}$$

$$E = \frac{1240}{\lambda \text{ in nm}} \text{ eV}$$

$$= \frac{1240}{300} \text{ eV}$$

$$= 4.13 \text{ eV}$$

To show photoelectric effect  $E \geq \phi$ .

Total no. of metals that show photoelectric effect will be 4.

**PART - II (PHYSICS)**

**SECTION-I : (Total Marks : 21)**

**(Single Correct Answer Type)**

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

- 24.** 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be  $T_1$ , the work done in the process is

(A)  $\frac{9}{8}RT_1$                       (B)  $\frac{3}{2}RT_1$                       (C)  $\frac{15}{8}RT_1$                       (D)  $\frac{9}{2}RT_1$

**Ans. (A)**

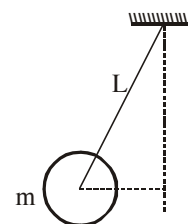
No. of moles =  $n = \frac{5.6}{22.4} = \frac{1}{4}$

$TV^{\gamma-1} = \text{constant} \Rightarrow T_1(5.6)^{2/3} = T_2(0.7)^{2/3} \Rightarrow T_1(8)^{2/3} = T_2 \Rightarrow 4T_1 = T_2$

$W = \frac{-nR\Delta T}{\gamma - 1} = -\frac{1R(3T_1) \times 3}{4 \times 2} = -\frac{9}{8}RT_1$ . Therefore  $W_{\text{external}} = \frac{9}{8}RT_1$

- 25.** A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is

(A) 9                                      (B) 18                                      (C) 27                                      (D) 36



**Ans. (D)**

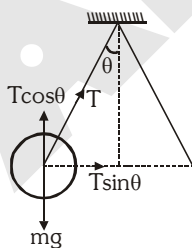
$T \sin \theta = m\omega^2 r$

$T \sin \theta = m\omega^2 L \sin \theta$

$T = m\omega^2 L$

$324 = \frac{1}{2}(\omega^2) \frac{1}{2}$

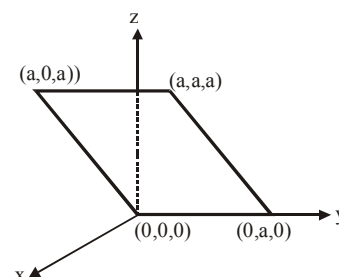
Therefore  $\omega = 36$



- 26.** Consider an electric field  $\vec{E} = E_0 \hat{x}$ , where  $E_0$  is a constant. The flux through the shaded area (as shown in the figure) due to this field is

(A)  $2E_0 a^2$                                       (B)  $\sqrt{2}E_0 a^2$

(C)  $E_0 a^2$                                       (D)  $\frac{E_0 a^2}{\sqrt{2}}$



**Ans. (C)**

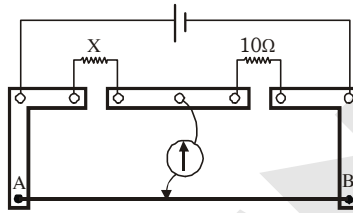
$\phi = \int \vec{E} \cdot \vec{dS} = E_x \text{ projected area perpendicular to } E \text{ (x-axis)} = E \times a^2$

27. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is  
 (A) 8.50 kHz                      (B) 8.25 kHz                      (C) 7.75 kHz                      (D) 7.50 kHz

Ans. (A)

$$f' = \left( \frac{v}{v - v_s} \right) \left( \frac{v + v_0}{v} \right) f \Rightarrow f' = \left( \frac{320}{320 - 10} \right) \left( \frac{320 + 10}{320} \right) \times 8 \Rightarrow f' \approx 8.50 \text{ kHz}$$

28. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is

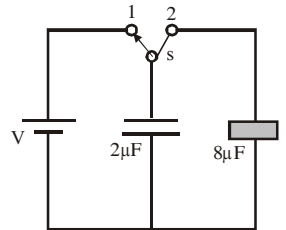


- (A) 10.2 ohm                      (B) 10.6 ohm                      (C) 10.8 ohm                      (D) 11.1 ohm

Ans. (B)

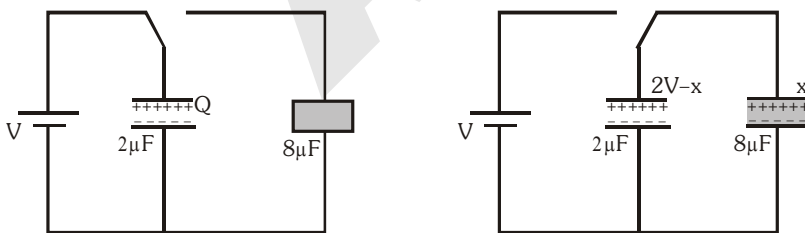
Apply condition of wheatstone bridge,  $\frac{x}{52+1} = \frac{10}{48+2} \Rightarrow x = \frac{10}{50} \times 53 \Rightarrow x = 10.6 \Omega$

29. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



- (A) 0%                                      (B) 20%  
 (C) 75%                                      (D) 80%

Ans. (D)



$$Q_1 = CV, Q_1 = 2V, \frac{2V-x}{2} = \frac{x}{8}, x = \frac{8V}{5} \Rightarrow V_i = \frac{1}{2} \times (2)V^2 = V^2; U_f = \left( \frac{8V}{5} \right)^2 + \frac{\left( \frac{2V}{5} \right)^2}{2 \times 2} = \frac{4V^2}{5}$$

$$\text{Loss} = \frac{4V^2}{5} \Rightarrow \% \text{ loss} = \frac{\frac{4V^2}{5} \times 100}{V^2} = 80\%$$

30. The wavelength of the first spectral line in the Balmer series of hydrogen atom is  $6561 \text{ \AA}$ . The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is  
 (A)  $1215 \text{ \AA}$                       (B)  $1640 \text{ \AA}$                       (C)  $2430 \text{ \AA}$                       (D)  $4687 \text{ \AA}$

Ans. (A)

$$\frac{1}{\lambda} = Rz^2 \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$

$$\text{First line of Balmer of Hydrogen : } \frac{1}{6561} = R(1)^2 \left( \frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\text{Second line of Balmer of single ionized He : } \frac{1}{\lambda} = R(z^2) \left( \frac{1}{2^2} - \frac{1}{4^2} \right)$$

$$\text{Dividing : } \lambda = 6561 \times \frac{5}{9 \times 3} = 1215 \text{ \AA}$$

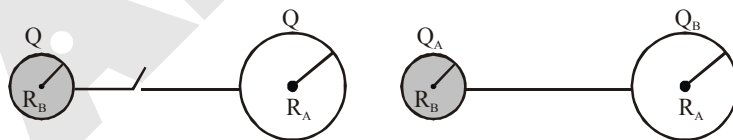
**SECTION-II : (Total Marks : 16)**  
**(Multiple Correct Answer Type)**

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or MORE** may be correct.

31. A spherical metal shell A of radius  $R_A$  and a solid metal sphere B of radius  $R_B (< R_A)$  are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then

- (A)  $E_A^{\text{inside}} = 0$                       (B)  $Q_A > Q_B$                       (C)  $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$                       (D)  $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

Ans. (ABCD)



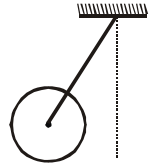
$E_A^{\text{inside}} = 0$  (because of electrostatic condition) So, A option is true.

$$\Rightarrow v_A = v_B \Rightarrow \frac{kQ_A}{R_A} = \frac{kQ_B}{R_B} \Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B} \Rightarrow R_B < R_A \text{ So, } Q_B < Q_A, \text{ so B is true}$$

$$\Rightarrow \frac{\sigma_A 4\pi R_A^2}{\sigma_B 4\pi R_B^2} = \frac{R_A}{R_B} \Rightarrow \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}, \text{ So C is true}$$

$$E_{\text{near surface}} = \sigma \times \frac{1}{R}. \text{ So, D is also true}$$

32. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disk system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is(are) true?
- (A) Restoring torque in case A = Restoring torque in case B  
 (B) Restoring torque in case A < Restoring torque in case B  
 (C) Angular frequency for case A > Angular frequency for case B  
 (D) Angular frequency for case A < Angular frequency for case B



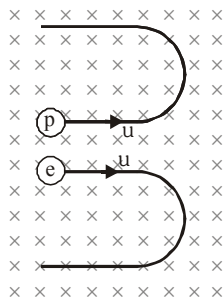
Ans. (AD)

Torque for both the arrangement is same.

Since in case B disc is not rotating, there is no speed of the pendulum at equilibrium in case (B).

33. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?
- (A) they will never come out of the magnetic field region  
 (B) they will come out travelling along parallel paths  
 (C) they will come out of the same time  
 (D) they will come out at different times

Ans. (BD)



By diagram B is true.

$$T = \frac{2\pi m}{qB}$$

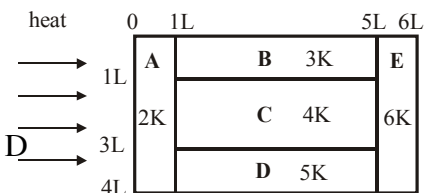
$$T \propto m$$

$$m_p > m_e$$

$$T_p > T_e$$

So, D is also true.

34. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state
- (A) heat flow through A and E slabs are same  
 (B) heat flow through slab E is maximum  
 (C) temperature difference across slab E is smallest  
 (D) heat flow through C = heat flow through B + Heat flow through D



Ans. (ABCD)

- In steady state : heat in = heat out. So, A is true
- Option B is also true because total heat is flowing through E.

$$Q = \frac{\Delta T}{R}$$

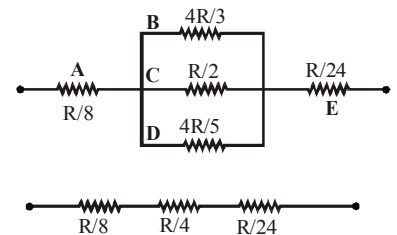
Q = same

$R_E$  is minimum. So,  $\Delta T$  is minimum

So option C is true

$$Q_B = \frac{\Delta T}{4R/3}, Q_C = \frac{\Delta T}{4R/2}, Q_D = \frac{\Delta T}{4R/5}, \text{ So, } Q_B + Q_D = Q_C.$$

Hence D is also true.



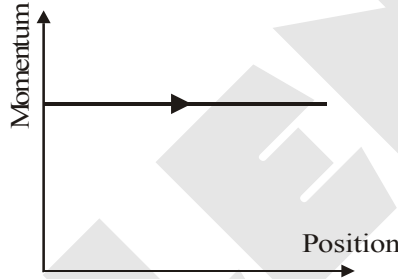
**SECTION-III : (Total Marks : 15)**

**(Paragraph Type)**

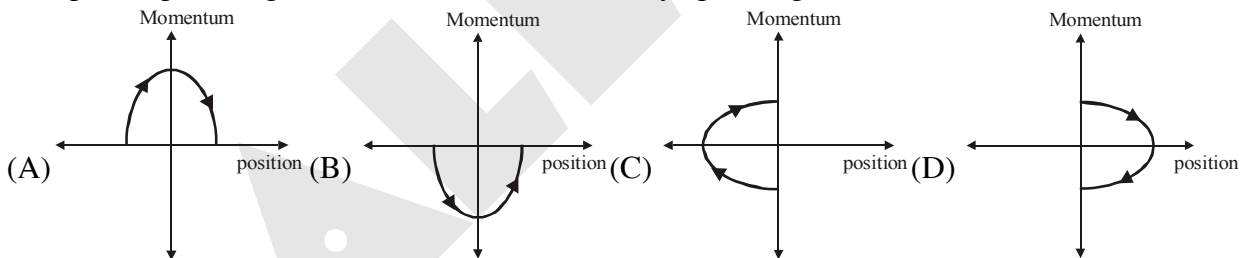
This section contains **2 paragraphs**. Based upon one of the paragraph, **3 multiple choice questions** and based on the other paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

**Paragraph for Questions Nos. 35 to 37**

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is  $x(t)$  vs.  $p(t)$  curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.



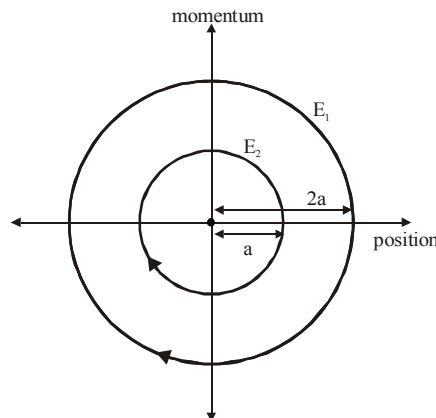
35. The phase space diagram for a ball thrown vertically up from ground is



Ans. (D)

Initial momentum was positive and final momentum negative. So option (D) is correct.

36. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and  $E_1$  and  $E_2$  are the total mechanical energies respectively. Then



(A)  $E_1 = \sqrt{2}E_2$

(B)  $E_1 = 2E_2$

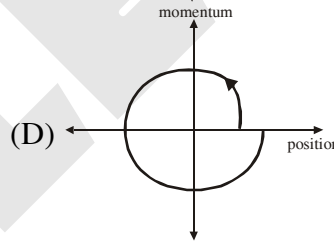
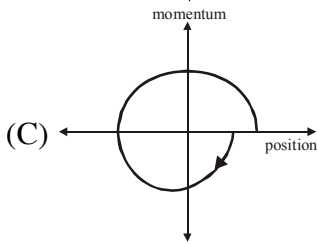
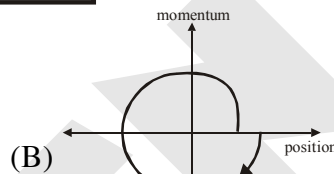
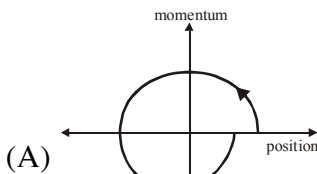
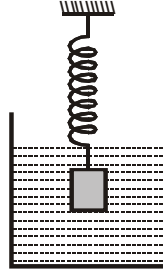
(C)  $E_1 = 4E_2$

(D)  $E_1 = 16E_2$

Ans. (C)

$$E \propto (\text{amplitude})^2 \Rightarrow \text{so } \frac{E_2}{E_1} = \left(\frac{a}{2a}\right)^2 \Rightarrow E_1 = 4E_2$$

37. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



Ans. (B)

Since at start time position was positive

**Paragraph for Question Nos. 38 and 39**

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ' $\omega_p$ ', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency  $\omega$ , where a part of the energy is absorbed and a part of it is reflected. As  $\omega$  approaches  $\omega_p$ , all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

38. Taking the electronic charge as 'e' and the permittivity as ' $\epsilon_0$ ', use dimensional analysis to determine the correct expression for  $\omega_p$ .

(A)  $\sqrt{\frac{Ne}{m\epsilon_0}}$       (B)  $\sqrt{\frac{m\epsilon_0}{Ne}}$       (C)  $\sqrt{\frac{Ne^2}{m\epsilon_0}}$       (D)  $\sqrt{\frac{m\epsilon_0}{Ne^2}}$

Ans. (C)

$$\left[ \sqrt{\frac{Ne^2}{m\epsilon_0}} \right] = \sqrt{\frac{\left(\frac{1}{L^3}\right)(C^2)}{(M)\left(\frac{C^2T^2}{L^3M}\right)}} = \frac{1}{T} = [\omega]$$

39. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons  $N \approx 4 \times 10^{27} m^{-3}$ . Take  $\epsilon_0 \approx 10^{-11}$  and  $m \approx 10^{-30}$ , where these quantities are in proper SI units

(A) 800 nm      (B) 600 nm      (C) 300 nm      (D) 200 nm

Ans. (B)

$$\omega = 2\pi f = \frac{2\pi c}{\lambda} = \sqrt{\frac{Ne^2}{m\epsilon_0}} \Rightarrow \lambda = 2\pi c \sqrt{\frac{m\epsilon_0}{Ne^2}}$$

$$\lambda = \frac{2 \times 3.14 \times 3 \times 10^8}{1.6 \times 10^{-19}} \sqrt{\frac{(10^{-30})(10^{-11})}{(4 \times 10^{27})}} = \frac{9.42}{1.6} \times 10^{27} \times 10^{-34} = 6 \times 10^{-7} m = 600 nm$$

SECTION-IV : (Total Marks : 28)

(Integer Answer Type)

This Section contains **7 questions**. The answer to each of the question is a **single digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

40. A block is moving on an inclined plane making an angle  $45^\circ$  with the horizontal and the coefficient of friction is  $\mu$ . The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define  $N = 10\mu$ , then N is

Ans. (5)

Force to just prevent it from sliding =  $mg\sin\theta - \mu mg\cos\theta$

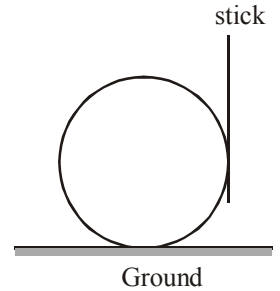
Force to just push up the plane =  $mg\sin\theta + \mu mg\cos\theta$

$$mg\sin\theta + \mu mg\cos\theta = 3(mg\sin\theta - \mu mg\cos\theta)$$

$$\frac{1}{\sqrt{2}} + \mu \frac{1}{\sqrt{2}} = 3\left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right) \Rightarrow \mu = \frac{1}{2} \Rightarrow N = 10 \mu = 5$$



41. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of  $0.3 \text{ m/s}^2$ . The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is  $(P/10)$ . The value of P is



Ans. (4)

$$N_1 = 2 \text{ N}$$

$$N_1 - f = ma \dots(i)$$

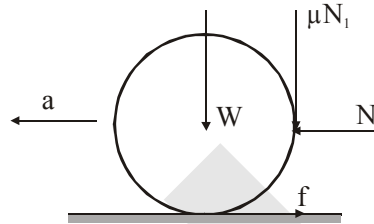
$$(f - \mu N_1)R = mR^2\alpha = ma \dots(ii)$$

From equation (i) and (ii) we get

$$N_1 (1 - \mu) = 2ma$$

$$2 (1 - \mu) = 2 \times 2 \times 0.3$$

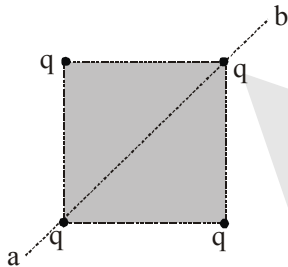
$$1 - \mu = 0.6 \Rightarrow \mu = 0.4$$



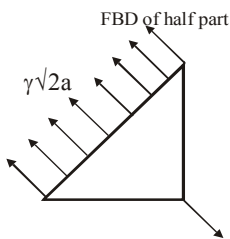
42. Four point charges, each of  $+q$ , are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is  $\gamma$ . The system of charges and planar film are in equilibrium,

and  $a = k \left[ \frac{q^2}{\gamma} \right]^{1/N}$ , where 'k' is a constant. Then N is

Ans. (3)



Line ab divides the soap film into two equal parts.



$$\frac{Kq^2}{a^2} \left[ \sqrt{2} + \frac{1}{2} \right] \text{ where } K = \frac{1}{4\pi \epsilon_0}$$

$$\gamma \sqrt{2} a = \frac{Kq^2}{a^2} \left( \sqrt{2} + \frac{1}{2} \right); a^3 = \frac{Kq^2}{\gamma} \left( \sqrt{2} + \frac{1}{2} \right); a = \left[ \frac{q^2}{\gamma} \right]^{1/3} \left[ K \left( \sqrt{2} + \frac{1}{2} \right) \right]^{1/3} \Rightarrow N=3$$

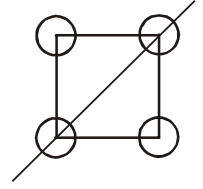
where  $\left[ K \left( \sqrt{2} + \frac{1}{2} \right) \right]^{1/3} = k$

43. Four solid spheres each of diameter  $\sqrt{5}$  cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is  $N \times 10^{-4}$  kg-m<sup>2</sup>, then N is

Ans. (9)

$$I = \frac{2}{5} mR^2 + \frac{2}{5} mR^2 + \frac{2}{5} mR^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 + \frac{2}{5} mR^2 + m\left(\frac{a}{\sqrt{2}}\right)^2$$

$$I = \frac{8}{5} mR^2 + ma^2 = \left[ \frac{8}{5} \times 0.5 \times \frac{5}{4} + 0.5 \times 4^2 \right] \times 10^{-4} = (1+8) \times 10^{-4} = N \times 10^{-4} \Rightarrow N=9$$



44. The activity of a freshly prepared radioactive sample is  $10^{10}$  disintegrations per second, whose mean life is  $10^9$  s. The mass of an atom of this radioisotope is  $10^{-25}$  kg. The mass (in mg) of the radioactive sample is

Ans. (1)

$$A = \lambda N \Rightarrow 10^{10} = \lambda N \Rightarrow N = \frac{10^{10}}{\lambda} = (10^{10}) \tau = 10^{10} \times 10^9 = 10^{19}$$

$$M = Nm = (10^{19})(10^{-25}) = 10^{-6} \text{ kg} = 1 \text{ mg}$$

45. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as  $I = I_0 \cos(300t)$  where  $I_0$  is constant. If the magnetic moment of the loop is  $N\mu_0 I_0 \sin(300t)$ , then 'N' is

Ans. (6)

$$\phi = B\pi r^2 = \left(\frac{\mu_0 I}{L}\right) \pi r^2 = \mu_0 I_0 \frac{\pi r^2}{L} \cos 300t \Rightarrow \varepsilon_1 = \frac{d\phi}{dt} = \left(\frac{\mu_0 I_0 \pi r^2}{L}\right) 300 \sin 300t$$

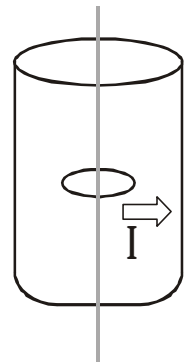
$$i = \frac{\varepsilon}{R} = (\mu_0 I_0 \sin 300t) \left[ \frac{\pi r^2 (300)}{LR} \right] \Rightarrow M = i\pi r^2 = \left[ \frac{\pi^2 r^4 (300)}{LR} \right] \mu_0 I_0 \sin 300t$$

46. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40° to 30° C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is  $10^{-5}/^\circ\text{C}$ , Young's modulus of steel is  $10^{11}$  N/m<sup>2</sup> and radius of the wire is 1 mm. Assume that  $L \gg$  diameter of the wire. Then the value of 'm' in kg is nearly

Ans. (3)

$$\frac{\Delta x}{L} = \frac{YA}{mg} = \alpha \Delta \theta \Rightarrow m = \frac{YA}{g\alpha \Delta \theta}$$

$$m = \frac{(10^{11})(3.14)(10^{-6})}{(10)(10^{-5})(10)} \Rightarrow m = 3.14 \text{ kg} \Rightarrow m = 3$$



**PART - III (MATHEMATICS)**

**SECTION-I : (Total Marks : 21)**

**(Single Correct Answer Type)**

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

47. The value of  $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$  is

- (A)  $\frac{1}{4} \ln \frac{3}{2}$                       (B)  $\frac{1}{2} \ln \frac{3}{2}$                       (C)  $\ln \frac{3}{2}$                       (D)  $\frac{1}{6} \ln \frac{3}{2}$

47. **Ans.(A)**

$$I = \int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx; \text{ put } x^2 = t \Rightarrow 2x dx = dt$$

$$\Rightarrow I = \frac{1}{2} \int_{\ln 2}^{\ln 3} \frac{\sin t}{\sin t + \sin(\ln 6 - t)} dt \quad \dots(i)$$

$$\Rightarrow I = \frac{1}{2} \int_{\ln 2}^{\ln 3} \frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t) + \sin t} dt \quad \dots(ii)$$

Adding equation (i) & (ii)

$$\Rightarrow 2I = \frac{1}{2} \int_{\ln 2}^{\ln 3} dt \Rightarrow I = \frac{1}{4} \ln \left( \frac{3}{2} \right)$$

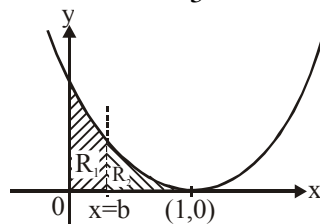
48. Let the straight line  $x = b$  divide the area enclosed by  $y = (1 - x)^2$ ,  $y = 0$  and  $x = 0$  into two parts  $R_1$  ( $0 \leq x \leq b$ ) and  $R_2$  ( $b \leq x \leq 1$ ) such that  $R_1 - R_2 = \frac{1}{4}$ . Then  $b$  equals

- (A)  $\frac{3}{4}$                       (B)  $\frac{1}{2}$                       (C)  $\frac{1}{3}$                       (D)  $\frac{1}{4}$

48. **Ans.(B)**

$$\therefore R_1 - R_2 = \frac{1}{4}$$

$$\Rightarrow \int_0^b (1-x)^2 dx - \int_b^1 (1-x)^2 dx = \frac{1}{4}$$



$$\Rightarrow -\left(\frac{(1-x)^3}{3}\right)_0^b + \left(\frac{(1-x)^3}{3}\right)_b^1 = \frac{1}{4} \Rightarrow -\left\{\frac{(1-b)^3}{3} - \frac{1}{3}\right\} - \frac{(1-b)^3}{3} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{3} - \frac{2}{3}(1-b)^3 = \frac{1}{4} \Rightarrow \frac{2}{3}(1-b)^3 = \frac{1}{12}$$

$$\Rightarrow (1-b)^3 = \frac{1}{8} \Rightarrow 1-b = \frac{1}{2} \Rightarrow b = \frac{1}{2}$$

49. Let  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$  and  $\vec{c} = \hat{i} - \hat{j} - \hat{k}$  be three vectors. A vector  $\vec{v}$  in the plane of  $\vec{a}$  and  $\vec{b}$ , whose projection on  $\vec{c}$  is  $\frac{1}{\sqrt{3}}$ , is given by

- (A)  $\hat{i} - 3\hat{j} + 3\hat{k}$       (B)  $-3\hat{i} - 3\hat{j} - \hat{k}$       (C)  $3\hat{i} - \hat{j} + 3\hat{k}$       (D)  $\hat{i} + 3\hat{j} - 3\hat{k}$

49. **Ans.(C)**

$$\vec{v} = x\vec{a} + y\vec{b}$$

$$= \hat{i}(x+y) + \hat{j}(x-y) + \hat{k}(x+y) \quad \dots(i)$$

Given,  $\vec{v} \cdot \hat{c} = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{x+y-x+y-x-y}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \begin{aligned} y-x &= 1 \\ x-y &= -1 \quad \dots(ii) \end{aligned}$$

using (ii) in (i) we get  $\vec{v} = (x+y)\hat{i} - \hat{j} + (x+y)\hat{k}$

50. Let  $(x_0, y_0)$  be the solution of the following equations

$$\begin{aligned} (2x)^{\ln 2} &= (3y)^{\ln 3} \\ 3^{\ln x} &= 2^{\ln y} \end{aligned}$$

Then  $x_0$  is

- (A)  $\frac{1}{6}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{2}$       (D) 6

50. **Ans.(C)**

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$

$$\Rightarrow \ln 2 (\ln 2 + \ln x) = \ln 3 (\ln 3 + \ln y) \quad \dots(i)$$

$$3^{\ln x} = 2^{\ln y}$$

$$\Rightarrow (\ln x) (\ln 3) = (\ln y) (\ln 2) \quad \dots(ii)$$

using (ii) in (i)

$$\Rightarrow \ln 2 (\ln 2 + \ln x) = \ln 3 \left( \ln 3 + \frac{(\ln x)(\ln 3)}{\ln 2} \right)$$

$$\Rightarrow \ln^2 2 - \ln^2 3 = \ln x \left\{ \frac{\ln^2 3}{\ln 2} - \ln 2 \right\}$$

$$\Rightarrow \ln x = -\ln 2$$

$$\Rightarrow x = \frac{1}{2}$$

51. Let  $\alpha$  and  $\beta$  be the roots of  $x^2 - 6x - 2 = 0$ , with  $\alpha > \beta$ . If  $a_n = \alpha^n - \beta^n$  for  $n \geq 1$ , then the value of  $\frac{a_{10} - 2a_8}{2a_9}$  is

- (A) 1      (B) 2      (C) 3      (D) 4

51. Ans. (C)

$\alpha, \beta$  are roots of  $x^2 - 6x - 2 = 0$   
 $\Rightarrow \alpha^2 - 6\alpha - 2 = 0$  &  $\beta^2 - 6\beta - 2 = 0$

$$\frac{a_0 - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^9 - \beta^9)} = 3$$

52. A straight line L through the point (3, -2) is inclined at an angle  $60^\circ$  to the line  $\sqrt{3}x + y = 1$ . If L also intersect the x-axis, then the equation of L is

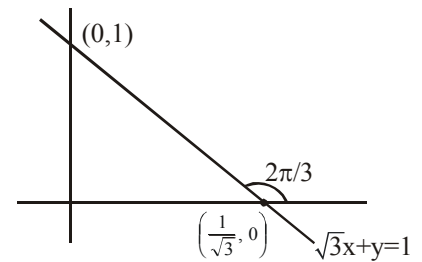
- (A)  $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$                       (B)  $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$   
 (C)  $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$                       (D)  $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

52. Ans. (B)

Line L has two possible slopes with inclination;  $\theta = \frac{\pi}{3}, \theta = 0$

$\therefore$  equation of line L when  $\theta = \frac{\pi}{3}$ ,  $y + 2 = \sqrt{3}(x - 3)$   
 $\Rightarrow y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$   
 equation of line L when  $\theta = 0$ ,  $y = -2$  (rejected)

$\therefore$  required line L is  $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$



53. Let  $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2} \cos\theta\}$  and  $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2} \sin\theta\}$  be two sets. Then

- (A)  $P \subset Q$  and  $Q - P \neq \emptyset$                       (B)  $Q \not\subset P$   
 (C)  $P \not\subset Q$     (D)  $P = Q$

53. Ans.(D)

$P = \{\theta : \sin\theta - \cos\theta = \sqrt{2} \cos\theta\}$   
 $\Rightarrow \tan\theta = \sqrt{2} + 1$                       ... (i)

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2} \sin\theta\}$   
 $\Rightarrow \tan\theta = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$                       ... (ii)

from (i) & (ii)

$\Rightarrow P = Q$

**SECTION-II : (Total Marks : 16)**
**(Multiple Correct Answer Type)**

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or MORE** may be correct.

54. The vector(s) which is/are coplanar with vectors  $\hat{i} + \hat{j} + 2\hat{k}$  and  $\hat{i} + 2\hat{j} + \hat{k}$ , and perpendicular to the vector  $\hat{i} + \hat{j} + \hat{k}$  is/are

(A)  $\hat{j} - \hat{k}$                       (B)  $-\hat{i} + \hat{j}$                       (C)  $\hat{i} - \hat{j}$                       (D)  $-\hat{j} + \hat{k}$

**Sol. Ans. (A,D)**

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k} \quad \vec{b} = \hat{i} + 2\hat{j} + \hat{k} \quad \vec{c} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{v} = \lambda((\vec{a} \times \vec{b}) \times \vec{c}) = \lambda((\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a})$$

$$\vec{v} = \lambda[4(\hat{i} + 2\hat{j} + \hat{k}) - 4(\hat{i} + \hat{j} + 2\hat{k})]$$

$$\vec{v} = 4\lambda(\hat{j} - \hat{k})$$

55. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function such that

$$f(x + y) = f(x) + f(y), \quad \forall x, y \in \mathbb{R}.$$

If  $f(x)$  is differentiable at  $x = 0$ , then

- (A)  $f(x)$  is differentiable only in a finite interval containing zero  
 (B)  $f(x)$  is continuous  $\forall x \in \mathbb{R}$   
 (C)  $f'(x)$  is constant  $\forall x \in \mathbb{R}$   
 (D)  $f(x)$  is differentiable except at finitely many points

**Sol. Ans. (B,C)**

$$f(x + y) = f(x) + f(y)$$

$$f(0) = 0$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{f(x) + f(h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(h)}{h} = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$f'(x) = f'(0) = k \quad (k \text{ is constant})$$

$$\Rightarrow f(x) = kx, \text{ hence } f(x) \text{ is continuous and } f'(x) \text{ is constant } \forall x \in \mathbb{R}$$

56. Let  $M$  and  $N$  be two  $3 \times 3$  non-singular skew-symmetric matrices such that  $MN = NM$ . If  $P^T$  denotes the transpose of  $P$ , then  $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$  is equal to -

(A)  $M^2$                       (B)  $-N^2$                       (C)  $-M^2$                       (D)  $MN$

**Sol. Ans. (C)**

*(Comment : Although  $3 \times 3$  skew symmetric matrices can never be non-singular. Therefore the information given in question is wrong. Now if we consider only non singular skew symmetric matrices  $M$  &  $N$ , then the solution is-)*

Given  $M^T = -M$

$$N^T = -N$$

$$MN = NM$$

$$\begin{aligned}
 & \text{according to question } M^2 N^2 (M^T N)^{-1} (M N^{-1})^T \\
 & = M^2 N^2 N^{-1} (M^T)^{-1} (N^{-1})^T M^T \\
 & = M^2 N^2 N^{-1} (-M)^{-1} (N^T)^{-1} (-M) \\
 & = -M^2 N M^{-1} N^{-1} M \\
 & = -M^2 N N^{-1} M^{-1} M = -M^2
 \end{aligned}
 \quad \left[ \begin{array}{l} MN = NM \\ (MN)^{-1} = (NM)^{-1} \\ N^{-1} M^{-1} = M^{-1} N^{-1} \end{array} \right.$$

57. Let the eccentricity of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  be reciprocal to that of the ellipse  $x^2 + 4y^2 = 4$ . If the hyperbola passes through a focus of the ellipse, then -

- (A) the equation of the hyperbola is  $\frac{x^2}{3} - \frac{y^2}{2} = 1$
- (B) a focus of the hyperbola is (2,0)
- (C) the eccentricity of the hyperbola is  $\sqrt{\frac{5}{3}}$
- (D) the equation of the hyperbola is  $x^2 - 3y^2 = 3$

Sol. Ans. (B,D)

Given hyperbola is  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

ellipse is  $\frac{x^2}{2^2} + \frac{y^2}{1} = 1$

eccentricity of ellipse =  $\sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

eccentricity of hyperbola =  $\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{4}{3}}$

$\Rightarrow \frac{b^2}{a^2} = \frac{1}{3} \Rightarrow 3b^2 = a^2 \dots\dots\dots(1)$

also hyperbola passes through foci of ellipse  $(\pm\sqrt{3}, 0)$

$\frac{3}{a^2} = 1 \Rightarrow a^2 = 3 \dots\dots\dots(2)$

from (1) & (2)  
 $b^2 = 1$

equation of hyperbola is  $\frac{x^2}{3} - \frac{y^2}{1} = 1 \Rightarrow x^2 - 3y^2 = 3$

eccentricity of hyperbola =  $\sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}}$

focus of hyperbola =  $\left( \pm\sqrt{3} \cdot \frac{2}{\sqrt{3}}, 0 \right) \equiv (\pm 2, 0)$

**SECTION-III : (Total Marks : 15)**
**(Paragraph Type)**

This section contains **2 paragraphs**. Based upon one of the paragraph, **3 multiple choice questions** and based on the other paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

**Paragraph for Question 58 and 60**

Let a, b and c be three real numbers satisfying

$$[a \ b \ c] \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0 \ 0 \ 0] \quad \dots(E)$$

- 58.** If the point P(a,b,c), with reference to (E), lies on the plane  $2x + y + z = 1$ , then the value of  $7a+b+c$  is  
 (A) 0 (B) 12 (C) 7 (D) 6
- 59.** Let  $\omega$  be a solution of  $x^3 - 1 = 0$  with  $\text{Im}(\omega) > 0$ . If  $a = 2$  with b and c satisfying (E), then the value of  $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$  is equal to -  
 (A) -2 (B) 2 (C) 3 (D) -3
- 60.** Let  $b = 6$ , with a and c satisfying (E). If  $\alpha$  and  $\beta$  are the roots of the quadratic equation  $ax^2 + bx + c = 0$ , then  $\sum_{n=0}^{\infty} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right)^n$  is-  
 (A) 6 (B) 7 (C)  $\frac{6}{7}$  (D)  $\infty$

**Sol.**

**Paragraph for Question 58 to 60**

$$a + 8b + 7c = 0$$

$$9a + 2b + 3c = 0$$

$$7a + 7b + 7c = 0$$

$$\Rightarrow a = K, b = 6K, c = -7K$$

**58. Ans. (D)**

$$(K, 6K, -7K)$$

$$2x + y + z = 1$$

$$2K + 6K - 7K = 1 \quad (\because \text{point lies on the plane})$$

$$\Rightarrow K = 1$$

$$\Rightarrow 7a + b + c = 7K + 6K - 7K = 6$$

**59. Ans. (A)**

$$x^3 - 1 = 0$$

$$\Rightarrow x = 1, \omega, \omega^2$$

$$\omega = -\frac{1}{2} + \frac{i\sqrt{3}}{2} \quad \text{since } \text{Im}(\omega) > 0$$

$$\text{If } a = 2 = K \Rightarrow b = 12 \ \& \ c = -14$$



$$\text{Hence } \frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{-14}} = 3\omega + 1 + 3\omega^2$$

$$= -3 + 1 = -2$$

**60. Ans. (B)**

$$\begin{aligned} \because b = 6 &\Rightarrow 6K = 6 \Rightarrow K = 1 \\ \Rightarrow a = 1, & \quad b = 6 \quad \& \quad c = -7 \\ x^2 + 6x - 7 &= 0 \\ \Rightarrow \alpha + \beta &= -6, \quad \alpha\beta = -7 \\ \Rightarrow \sum_{n=0}^{\infty} \left( \frac{\alpha + \beta}{\alpha\beta} \right)^n &= \sum_{n=0}^{\infty} \left( \frac{6}{7} \right)^n = \frac{1}{1 - \frac{6}{7}} = 7 \end{aligned}$$

**Sol.**

**Paragraph for Question 61 and 62**

Let  $U_1$  and  $U_2$  be two urns such that  $U_1$  contains 3 white and 2 red balls, and  $U_2$  contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from  $U_1$  and put into  $U_2$ . However, if tail appears then 2 balls are drawn at random from  $U_1$  and put into  $U_2$ . Now 1 ball is drawn at random from  $U_2$ .

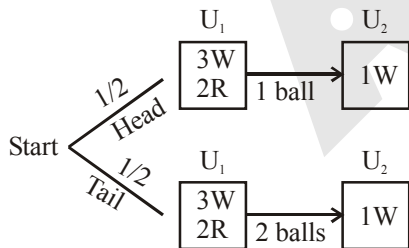
**61.** The probability of the drawn ball from  $U_2$  being white is -

- (A)  $\frac{13}{30}$                       (B)  $\frac{23}{30}$                       (C)  $\frac{19}{30}$                       (D)  $\frac{11}{30}$

**62.** Given that the drawn ball from  $U_2$  is white, the probability that head appeared on the coin is -

- (A)  $\frac{17}{23}$                       (B)  $\frac{11}{23}$                       (C)  $\frac{15}{23}$                       (D)  $\frac{12}{23}$

**Paragraph for Question 61 and 62**



**61. Ans. (B)**

Required probability

$$\begin{aligned} &= \frac{1}{2} \left( \frac{3}{5} \cdot 1 + \frac{2}{5} \cdot \frac{1}{2} \right) + \frac{1}{2} \left( \frac{{}^3C_2}{{}^5C_2} \cdot 1 + \frac{{}^2C_2}{{}^5C_2} \cdot \frac{1}{3} + \frac{{}^3C_1 {}^2C_1}{{}^5C_2} \cdot \frac{2}{3} \right) \\ &= \frac{1}{2} \left( \frac{4}{5} \right) + \frac{1}{2} \left( \frac{3}{10} + \frac{1}{30} + \frac{2}{5} \right) = \frac{2}{5} + \frac{11}{30} = \frac{23}{30} \end{aligned}$$

**62. Ans. (D)**

Required probability

$$\begin{aligned} &= \frac{2/5}{2/5 + 11/30} \quad (\text{using Baye's theorem}) \\ &= \frac{12}{23} \end{aligned}$$

**SECTION-IV : (Total Marks : 28)**

**(Integer Answer Type)**

This Section contains **7 questions**. The answer to each of the question is a **single digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

63. Let  $a_1, a_2, a_3, \dots, a_{100}$  be an arithmetic progression with  $a_1 = 3$  and  $S_p = \sum_{i=1}^p a_i, 1 \leq p \leq 100$ . For any integer  $n$  with  $1 \leq n \leq 20$ , let  $m = 5n$ . If  $\frac{S_m}{S_n}$  does not depend on  $n$ , then  $a_2$  is

**Sol. Ans. 9 or 3**

*(Comment : The information about the common difference i.e. zero or non-zero is not given in the question. Hence there are two possible answers)*

Consider  $d \neq 0$  the solution is

$a_1, a_2, a_3, \dots, a_{100} \rightarrow AP$

$$a_1 = 3 ; S_p = \sum_{i=1}^p a_i \quad 1 \leq n \leq 20$$

$$m = 5n$$

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a_1 + (m-1)d]}{\frac{n}{2}[2a_1 + (n-1)d]}$$

$$\frac{S_m}{S_n} = \frac{5[(2a_1 - d) + 5nd]}{[(2a_1 - d) + nd]}$$

for  $\frac{S_m}{S_n}$  to be independent of  $n$

$$\therefore 2a_1 - d = 0 \Rightarrow d = 2a_1 \Rightarrow d = 6 \Rightarrow a_2 = 9$$

$$\text{If } d = 0 \Rightarrow a_2 = a_1 = 3$$

64. Consider the parabola  $y^2 = 8x$ . Let  $\Delta_1$  be the area of the triangle formed by the end points of its latus rectum and the point  $P\left(\frac{1}{2}, 2\right)$  on the parabola, and  $\Delta_2$  be the area of the triangle formed by

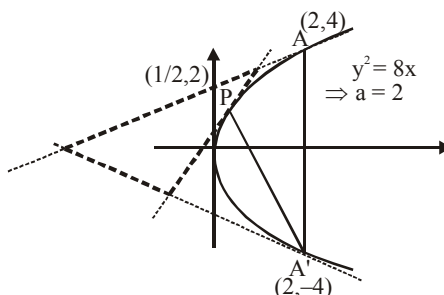
drawing tangents at  $P$  and at the end points of the latus rectum. Then  $\frac{\Delta_1}{\Delta_2}$  is

**Sol. Ans. 2**

$$\Delta_1 = \text{area of } \triangle PAA'$$

$$= \frac{1}{2} \cdot 8 \cdot \frac{3}{2} = 6$$

$$\Delta_2 = \frac{1}{2} (\Delta_1)$$



(Using property : Area of triangle formed by tangents is always half of original triangle)

$$\Rightarrow \frac{\Delta_1}{\Delta_2} = 2$$

65. The positive integer value of  $n > 3$  satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)} \text{ is}$$

Sol. Ans. 7

$$\frac{1}{\sin \frac{\pi}{n}} = \frac{1}{\sin \frac{2\pi}{n}} + \frac{1}{\sin \frac{3\pi}{n}} \Rightarrow \frac{1}{\sin \frac{\pi}{n}} - \frac{1}{\sin \frac{3\pi}{n}} = \frac{1}{\sin \frac{2\pi}{n}}$$

$$\Rightarrow \frac{\sin \frac{3\pi}{n} - \sin \frac{\pi}{n}}{\sin \frac{\pi}{n} \sin \frac{3\pi}{n}} = \frac{1}{\sin \frac{2\pi}{n}} \Rightarrow \frac{2 \cos \frac{2\pi}{n} \sin \frac{\pi}{n}}{\sin \frac{\pi}{n} \sin \frac{3\pi}{n}} = \frac{1}{\sin \frac{2\pi}{n}}$$

$$\Rightarrow 2 \cos \frac{2\pi}{n} \sin \frac{2\pi}{n} = \sin \frac{3\pi}{n}$$

$$\Rightarrow \sin \frac{4\pi}{n} = \sin \frac{3\pi}{n} \Rightarrow \frac{4\pi}{n} = K\pi + (-1)^K \frac{3\pi}{n}$$

$$\text{If } K = 2m \Rightarrow \frac{\pi}{n} = 2m\pi \Rightarrow n = \frac{1}{2m} \Rightarrow n = \frac{1}{2}, \frac{1}{4}, \frac{1}{6} \dots\dots$$

$$\text{If } K = 2m + 1 \Rightarrow \frac{7\pi}{n} = (2m + 1)\pi \Rightarrow n = \frac{7}{2m + 1} \Rightarrow n = 7, \frac{7}{3}, \frac{7}{5} \dots\dots$$

Possible value of  $n$  is 7

66. Let  $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin \theta}{\sqrt{\cos 2\theta}}\right)\right)$ , where  $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ . Then the value of  $\frac{d}{d(\tan \theta)}(f(\theta))$  is

Sol. Ans. 1

$$\text{Let } f(\theta) = \sin \alpha \text{ where } \alpha = \tan^{-1}\left(\frac{\sin \theta}{\sqrt{\cos 2\theta}}\right)$$

$$\Rightarrow \tan \alpha = \frac{\sin \theta}{\sqrt{\cos 2\theta}}$$

$$\Rightarrow \sin \alpha = \frac{\sin \theta}{\cos \theta} = \tan \theta \quad \left(\because \theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)\right)$$

$$\Rightarrow f(\theta) = \tan \theta$$

$$\Rightarrow \frac{d(f(\theta))}{d(\tan \theta)} = 1$$

67. If  $z$  is any complex number satisfying  $|z - 3 - 2i| \leq 2$ , then the minimum value of  $|2z - 6 + 5i|$  is

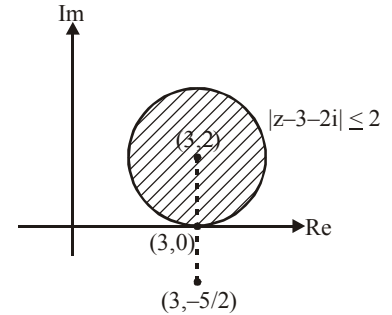
**Sol. Ans. 5**

We have to find minimum value of  $2 \left| z - \left( 3 - \frac{5}{2}i \right) \right|$

$$= 2 \times (\text{minimum distance between } z \text{ and point } \left( 3, -\frac{5}{2} \right))$$

$$= 2 \times (\text{distance between } (3,0) \text{ and } \left( 3, -\frac{5}{2} \right))$$

$$= 2 \times \frac{5}{2} = 5 \text{ units.}$$



**68.** The minimum value of the sum of real numbers  $a^{-5}$ ,  $a^{-4}$ ,  $3a^{-3}$ ,  $1$ ,  $a^8$  and  $a^{10}$  with  $a > 0$  is

**Sol. Ans. 8**

As  $a > 0$

and all the given terms are positive

hence considering A.M.  $\geq$  G.M. for given numbers :

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^8 + a^{10}}{7} \geq \left( a^{-5} \cdot a^{-4} \cdot a^{-3} \cdot a^{-3} \cdot a^{-3} \cdot a^8 \cdot a^{10} \right)^{\frac{1}{7}}$$

$$\Rightarrow \frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^8 + a^{10}}{7} \geq 1 \quad \Rightarrow \quad (a^{-5} + a^{-4} + 3a^{-3} + a^8 + a^{10})_{\min} = 7$$

where  $a^{-5} = a^{-4} = a^{-3} = a^8 = a^{10}$  i.e.  $a = 1$

$$\Rightarrow (a^{-5} + a^{-4} + 3a^{-3} + a^8 + a^{10} + 1)_{\min} = 8 \quad \text{when } a = 1$$

**69.** Let  $f : [1, \infty) \rightarrow [2, \infty)$  be a differentiable function such that  $f(1) = 2$ . If  $6 \int_1^x f(t) dt = 3x f(x) - x^3$

for all  $x \geq 1$ , then the value of  $f(2)$  is

**Sol. Ans. 6**

*(Comment : The given relation does not hold for  $x = 1$ , therefore it is not an identity. Hence there is an error in given question. The correct identity must be-)*

$$6 \int_1^x f(t) dt = 3x f(x) - x^3 - 5, \quad \forall x \geq 1$$

Now applying Newton Leibnitz theorem

$$6f(x) = 3x f'(x) - 3x^2 + 3f(x)$$

$$\Rightarrow 3f(x) = 3x f'(x) - 3x^2$$

Let  $y = f(x)$

$$\Rightarrow x \frac{dy}{dx} - y = x^2 \quad \Rightarrow \quad \frac{xdy - ydx}{x^2} = dx \quad \Rightarrow \quad \int d\left(\frac{y}{x}\right) = \int dx$$

$$\Rightarrow \frac{y}{x} = x + C \quad (\text{where } C \text{ is constant})$$

$$\Rightarrow y = x^2 + Cx$$

$$\therefore f(x) = x^2 + Cx$$

$$\text{Given } f(1) = 2 \quad \Rightarrow \quad C = 1$$

$$\therefore f(2) = 2^2 + 2 = 6$$