

# FIITJEE Solutions to IITJEE-2004 Mains Paper

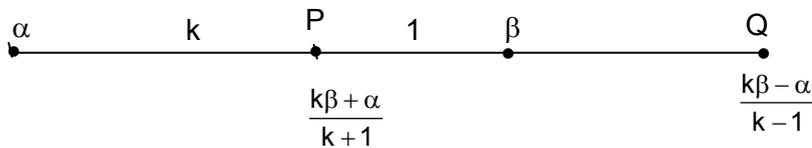
## Mathematics

Time: 2 hours

Note: Question number 1 to 10 carries 2 marks each and 11 to 20 carries 4 marks each.

1. Find the centre and radius of the circle formed by all the points represented by  $z = x + iy$  satisfying the relation  $\frac{|z - \alpha|}{|z - \beta|} = k$  ( $k \neq 1$ ) where  $\alpha$  and  $\beta$  are constant complex numbers given by  $\alpha = \alpha_1 + i\alpha_2$ ,  $\beta = \beta_1 + i\beta_2$ .

Sol.



Centre is the mid-point of points dividing the join of  $\alpha$  and  $\beta$  in the ratio  $k : 1$  internally and externally.

$$\text{i.e. } z = \frac{1}{2} \left( \frac{k\beta + \alpha}{k+1} + \frac{k\beta - \alpha}{k-1} \right) = \frac{\alpha - k^2\beta}{1 - k^2}$$

$$\text{radius} = \left| \frac{\alpha - k^2\beta}{1 - k^2} - \frac{k\beta + \alpha}{1 + k} \right| = \left| \frac{k(\alpha - \beta)}{1 - k^2} \right|.$$

Alternative:

$$\text{We have } \frac{|z - \alpha|}{|z - \beta|} = k$$

$$\text{so that } (z - \alpha)(\bar{z} - \bar{\alpha}) = k^2(z - \beta)(\bar{z} - \bar{\beta})$$

$$\text{or } z\bar{z} - \alpha\bar{z} - \bar{\alpha}z + \alpha\bar{\alpha} = k^2(z\bar{z} - \beta\bar{z} - \bar{\beta}z + \beta\bar{\beta})$$

$$\text{or } z\bar{z}(1 - k^2) - (\alpha - k^2\beta)\bar{z} - (\bar{\alpha} - k^2\bar{\beta})z + \alpha\bar{\alpha} - k^2\beta\bar{\beta} = 0$$

$$\text{or } z\bar{z} - \frac{(\alpha - k^2\beta)}{1 - k^2}\bar{z} - \frac{(\bar{\alpha} - k^2\bar{\beta})}{1 - k^2}z + \frac{\alpha\bar{\alpha} - k^2\beta\bar{\beta}}{1 - k^2} = 0$$

$$\text{which represents a circle with centre } \frac{\alpha - k^2\beta}{1 - k^2} \text{ and radius } \sqrt{\frac{(\alpha - k^2\beta)(\bar{\alpha} - k^2\bar{\beta})}{(1 - k^2)^2} - \frac{\alpha\bar{\alpha} - k^2\beta\bar{\beta}}{(1 - k^2)}} = \left| \frac{k(\alpha - \beta)}{1 - k^2} \right|.$$

2.  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$ ,  $\vec{d}$  are four distinct vectors satisfying the conditions  $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$  and  $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ , then prove that  $\vec{a} \cdot \vec{b} + \vec{c} \cdot \vec{d} \neq \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{d}$ .

Sol. Given that  $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$  and  $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$

$$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = (\vec{c} - \vec{b}) \times \vec{d} = \vec{d} \times (\vec{b} - \vec{c}) \Rightarrow \vec{a} - \vec{d} \parallel \vec{b} - \vec{c}$$

$$\Rightarrow (\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) \neq 0 \Rightarrow \vec{a} \cdot \vec{b} + \vec{d} \cdot \vec{c} \neq \vec{d} \cdot \vec{b} + \vec{a} \cdot \vec{c}.$$

3. Using permutation or otherwise prove that  $\frac{n^2!}{(n!)^n}$  is an integer, where n is a positive integer.

**Sol.** Let there be  $n^2$  objects distributed in n groups, each group containing n identical objects. So number of arrangement of these  $n^2$  objects are  $\frac{n^2!}{(n!)^n}$  and number of arrangements has to be an integer.

Hence  $\frac{n^2!}{(n!)^n}$  is an integer.

4. If M is a  $3 \times 3$  matrix, where  $M^T M = I$  and  $\det(M) = 1$ , then prove that  $\det(M - I) = 0$ .

**Sol.**  $(M - I)^T = M^T - I = M^T - M^T M = M^T (I - M)$   
 $\Rightarrow |(M - I)^T| = |M - I| = |M^T| |I - M| = |I - M| \Rightarrow |M - I| = 0$ .

Alternate:  $\det(M - I) = \det(M - I) \det(M^T) = \det(MM^T - M^T)$   
 $= \det(I - M^T) = -\det(M^T - I) = -\det(M - I)^T = -\det(M - I) \Rightarrow \det(M - I) = 0$ .

5. If  $y(x) = \int_{\pi^2/16}^{x^2} \frac{\cos x \cdot \cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}} d\theta$  then find  $\frac{dy}{dx}$  at  $x = \pi$ .

**Sol.**  $y = \int_{\pi^2/16}^{x^2} \frac{\cos x \cdot \cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}} d\theta = \cos x \int_{\pi^2/16}^{x^2} \frac{\cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}} d\theta$

so that  $\frac{dy}{dx} = -\sin x \int_{\pi^2/16}^{x^2} \frac{\cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}} d\theta + \frac{2x \cos x \cdot \cos x}{1 + \sin^2 x}$

Hence, at  $x = \pi$ ,  $\frac{dy}{dx} = 0 + \frac{2\pi(-1)(-1)}{1 + 0} = 2\pi$ .

6. T is a parallelopiped in which A, B, C and D are vertices of one face. And the face just above it has corresponding vertices A', B', C', D'. T is now compressed to S with face ABCD remaining same and A', B', C', D' shifted to A'', B'', C'', D'' in S. The volume of parallelopiped S is reduced to 90% of T. Prove that locus of A'' is a plane.

**Sol.** Let the equation of the plane ABCD be  $ax + by + cz + d = 0$ , the point A' be  $(\alpha, \beta, \gamma)$  and the height of the parallelopiped ABCD be h.

$$\Rightarrow \frac{|a\alpha + b\beta + c\gamma + d|}{\sqrt{a^2 + b^2 + c^2}} = 0.9h \Rightarrow a\alpha + b\beta + c\gamma + d = \pm 0.9h\sqrt{a^2 + b^2 + c^2}$$

$\Rightarrow$  the locus of A'' is a plane parallel to the plane ABCD.

7. If  $f : [-1, 1] \rightarrow \mathbb{R}$  and  $f'(0) = \lim_{n \rightarrow \infty} n f\left(\frac{1}{n}\right)$  and  $f(0) = 0$ . Find the value of  $\lim_{n \rightarrow \infty} \frac{2}{\pi} (n+1) \cos^{-1}\left(\frac{1}{n}\right) - n$ .

Given that  $0 < \left| \lim_{n \rightarrow \infty} \cos^{-1}\left(\frac{1}{n}\right) \right| < \frac{\pi}{2}$ .

**Sol.**  $\lim_{n \rightarrow \infty} \frac{2}{\pi} (n+1) \cos^{-1} \frac{1}{n} - n = \lim_{n \rightarrow \infty} n \left[ \frac{2}{\pi} \left(1 + \frac{1}{n}\right) \cos^{-1} \frac{1}{n} - 1 \right]$

$= \lim_{n \rightarrow \infty} n f\left(\frac{1}{n}\right) = f'(0)$  where  $f(x) = \frac{2}{\pi} (1+x) \cos^{-1} x - 1$ .

Clearly,  $f(0) = 0$ .

Now,  $f'(x) = \frac{2}{\pi} \left[ (1+x) \frac{-1}{\sqrt{1-x^2}} + \cos^{-1} x \right]$   
 $\Rightarrow f'(0) = \frac{2}{\pi} \left[ -1 + \frac{\pi}{2} \right] = \frac{2}{\pi} \left[ \frac{\pi-2}{2} \right] = 1 - \frac{2}{\pi}$ .

8. If  $p(x) = 51x^{101} - 2323x^{100} - 45x + 1035$ , using Rolle's Theorem, prove that atleast one root lies between  $(45^{1/100}, 46)$ .

**Sol.** Let  $g(x) = \int p(x) dx = \frac{51x^{102}}{102} - \frac{2323x^{101}}{101} - \frac{45x^2}{2} + 1035x + c$   
 $= \frac{1}{2}x^{102} - 23x^{101} - \frac{45}{2}x^2 + 1035x + c$ .

Now  $g(45^{1/100}) = \frac{1}{2}(45)^{102} - 23(45)^{101} - \frac{45}{2}(45)^2 + 1035(45)^{1/100} + c = c$

$g(46) = \frac{(46)^{102}}{2} - 23(46)^{101} - \frac{45}{2}(46)^2 + 1035(46) + c = c$ .

So  $g'(x) = p(x)$  will have atleast one root in given interval.

9. A plane is parallel to two lines whose direction ratios are  $(1, 0, -1)$  and  $(-1, 1, 0)$  and it contains the point  $(1, 1, 1)$ . If it cuts coordinate axis at A, B, C, then find the volume of the tetrahedron OABC.

**Sol.** Let  $(l, m, n)$  be the direction ratios of the normal to the required plane so that  $l - n = 0$  and  $-l + m = 0$   
 $\Rightarrow l = m = n$  and hence the equation of the plane containing  $(1, 1, 1)$  is  $\frac{x}{3} + \frac{y}{3} + \frac{z}{3} = 1$ .

Its intercepts with the coordinate axes are A  $(3, 0, 0)$ ; B  $(0, 3, 0)$ ; C  $(0, 0, 3)$ . Hence the volume of OABC

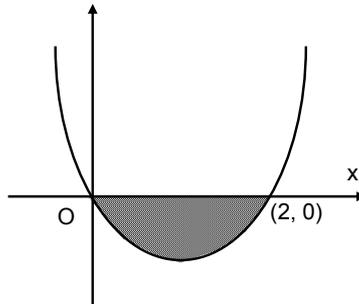
$= \frac{1}{6} \begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \frac{27}{6} = \frac{9}{2}$  cubic units.

10. If A and B are two independent events, prove that  $P(A \cup B) \cdot P(A' \cap B') \leq P(C)$ , where C is an event defined that exactly one of A and B occurs.

**Sol.**  $P(A \cup B) \cdot P(A') P(B') \leq (P(A) + P(B)) P(A') P(B')$   
 $= P(A) \cdot P(A') P(B') + P(B) P(A') P(B')$   
 $= P(A) P(B') (1 - P(A)) + P(B) P(A') (1 - P(B))$   
 $\leq P(A) P(B') + P(B) P(A') = P(C)$ .

11. A curve passes through  $(2, 0)$  and the slope of tangent at point P  $(x, y)$  equals  $\frac{(x+1)^2 + y - 3}{(x+1)}$ . Find the equation of the curve and area enclosed by the curve and the x-axis in the fourth quadrant.

**Sol.**  $\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1}$   
 or,  $\frac{dy}{dx} = (x+1) + \frac{y-3}{x+1}$   
 Putting  $x+1 = X, y-3 = Y$   
 $\frac{dY}{dX} = X + \frac{Y}{X}$   
 $\frac{dY}{dX} - \frac{Y}{X} = X$



$$\text{I.F} = \frac{1}{X} \Rightarrow \frac{1}{X} \cdot Y = X + c$$

$$\frac{y-3}{x+1} = (x+1) + c.$$

It passes through (2, 0)  $\Rightarrow c = -4$ .

$$\text{So, } y - 3 = (x + 1)^2 - 4(x + 1)$$

$$\Rightarrow y = x^2 - 2x.$$

$$\Rightarrow \text{Required area} = \left| \int_0^2 (x^2 - 2x) dx \right| = \left| \left[ \frac{x^3}{3} - x^2 \right]_0^2 \right| = \frac{4}{3} \text{ sq. units.}$$

12. A circle touches the line  $2x + 3y + 1 = 0$  at the point (1, -1) and is orthogonal to the circle which has the line segment having end points (0, -1) and (-2, 3) as the diameter.

**Sol.** Let the circle with tangent  $2x + 3y + 1 = 0$  at (1, -1) be

$$(x - 1)^2 + (y + 1)^2 + \lambda (2x + 3y + 1) = 0$$

$$\text{or } x^2 + y^2 + x(2\lambda - 2) + y(3\lambda + 2) + 2 + \lambda = 0.$$

It is orthogonal to  $x(x + 2) + (y + 1)(y - 3) = 0$

$$\text{Or } x^2 + y^2 + 2x - 2y - 3 = 0$$

$$\text{so that } \frac{2(2\lambda - 2)}{2} \cdot \left(\frac{2}{2}\right) + \frac{2(3\lambda + 2)}{2} \cdot \left(\frac{-2}{2}\right) = 2 + \lambda - 3 \Rightarrow \lambda = -\frac{3}{2}.$$

Hence the required circle is  $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ .

13. At any point P on the parabola  $y^2 - 2y - 4x + 5 = 0$ , a tangent is drawn which meets the directrix at Q. Find the locus of point R which divides QP externally in the ratio  $\frac{1}{2} : 1$ .

**Sol.** Any point on the parabola is P  $(1 + t^2, 1 + 2t)$ . The equation of the tangent at P is  $t(y - 1) = x - 1 + t^2$  which meets the directrix  $x = 0$  at Q  $\left(0, 1 + t - \frac{1}{t}\right)$ . Let R be (h, k).

Since it divides QP externally in the ratio  $\frac{1}{2} : 1$ , Q is the mid point of RP

$$\Rightarrow 0 = \frac{h + 1 + t^2}{2} \text{ or } t^2 = -(h + 1)$$

$$\text{and } 1 + t - \frac{1}{t} = \frac{k + 1 + 2t}{2} \text{ or } t = \frac{2}{1 - k}$$

$$\text{So that } \frac{4}{(1 - k)^2} + (h + 1) = 0 \text{ Or } (k - 1)^2 (h + 1) + 4 = 0.$$

Hence locus is  $(y - 1)^2 (x + 1) + 4 = 0$ .

14. Evaluate  $\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} dx$ .

**Sol.** 
$$I = \int_{-\pi/3}^{\pi/3} \frac{(\pi + 4x^3) dx}{2 - \cos\left(|x| + \frac{\pi}{3}\right)}$$

$$2I = \int_{-\pi/3}^{\pi/3} \frac{2\pi dx}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} = \int_0^{\pi/3} \frac{2\pi dx}{2 - \cos\left(x + \frac{\pi}{3}\right)}$$

$$I = \int_{\pi/3}^{2\pi/3} \frac{2\pi dt}{2 - \cos t} \Rightarrow I = 2\pi \int_{\pi/3}^{2\pi/3} \frac{\sec^2 \frac{t}{2} dt}{1 + 3 \tan^2 \frac{t}{2}} = 2\pi \int_{1/\sqrt{3}}^{\sqrt{3}} \frac{2 dt}{1 + 3t^2} = \frac{4\pi}{3} \int_{1/\sqrt{3}}^{\sqrt{3}} \frac{dt}{\left(\frac{1}{\sqrt{3}}\right)^2 + t^2}$$

$$I = \frac{4\pi}{3} \sqrt{3} \left[ \tan^{-1} \sqrt{3}t \right]_{1/\sqrt{3}}^{\sqrt{3}} = \frac{4\pi}{\sqrt{3}} \left[ \tan^{-1} 3 - \frac{\pi}{4} \right] = \frac{4\pi}{\sqrt{3}} \tan^{-1} \left( \frac{1}{2} \right).$$

15. If  $a, b, c$  are positive real numbers, then prove that  $[(1+a)(1+b)(1+c)]^7 > 7^7 a^4 b^4 c^4$ .

**Sol.**  $(1+a)(1+b)(1+c) = 1 + ab + a + b + c + abc + ac + bc$

$$\Rightarrow \frac{(1+a)(1+b)(1+c) - 1}{7} \geq (ab \cdot a \cdot b \cdot c \cdot abc \cdot ac \cdot bc)^{1/7} \quad (\text{using AM} \geq \text{GM})$$

$$\Rightarrow (1+a)(1+b)(1+c) - 1 > 7(a^4 \cdot b^4 \cdot c^4)^{1/7}$$

$$\Rightarrow (1+a)(1+b)(1+c) > 7(a^4 \cdot b^4 \cdot c^4)^{1/7}$$

$$\Rightarrow (1+a)^7 (1+b)^7 (1+c)^7 > 7^7 (a^4 \cdot b^4 \cdot c^4).$$

16. 
$$f(x) = \begin{cases} b \sin^{-1} \left( \frac{x+c}{2} \right), & -\frac{1}{2} < x < 0 \\ \frac{1}{2}, & x = 0 \\ \frac{e^{\frac{a}{2}x} - 1}{x}, & 0 < x < \frac{1}{2} \end{cases}$$

If  $f(x)$  is differentiable at  $x = 0$  and  $|c| < \frac{1}{2}$  then find the value of 'a' and prove that  $64b^2 = (4 - c^2)$ .

**Sol.**  $f(0^+) = f(0^-) = f(0)$

$$\text{Here } f(0^+) = \lim_{x \rightarrow 0^+} \frac{e^{\frac{ax}{2}} - 1}{x} = \lim_{x \rightarrow 0^+} \frac{e^{\frac{ax}{2}} - 1}{\frac{ax}{2}} \cdot \frac{a}{2} = \frac{a}{2}.$$

$$\Rightarrow b \sin^{-1} \frac{c}{2} = \frac{a}{2} = \frac{1}{2} \Rightarrow a = 1.$$

$$L f'(0_-) = \lim_{h \rightarrow 0^-} \frac{b \sin^{-1} \frac{(h+c)}{2} - \frac{1}{2}}{h} = \frac{b/2}{\sqrt{1 - \frac{c^2}{4}}}$$

$$R f'(0_+) = \lim_{h \rightarrow 0^+} \frac{\frac{e^{h/2} - 1}{h} - \frac{1}{2}}{h} = \frac{1}{8}$$

$$\text{Now } L f'(0_-) = R f'(0_+) \Rightarrow \frac{\frac{b}{2}}{\sqrt{1 - \frac{c^2}{4}}} = \frac{1}{8}$$

$$4b = \sqrt{1 - \frac{c^2}{4}} \Rightarrow 16b^2 = \frac{4 - c^2}{4} \Rightarrow 64b^2 = 4 - c^2.$$

17. Prove that  $\sin x + 2x \geq \frac{3x \cdot (x+1)}{\pi} \quad \forall x \in \left[ 0, \frac{\pi}{2} \right]$ . (Justify the inequality, if any used).

**Sol.** Let  $f(x) = 3x^2 + (3 - 2\pi)x - \pi \sin x$

$$f(0) = 0, f\left(\frac{\pi}{2}\right) = -ve$$

$$f'(x) = 6x + 3 - 2\pi - \pi \cos x$$

$$f'(x) = 6 + \pi \sin x > 0$$

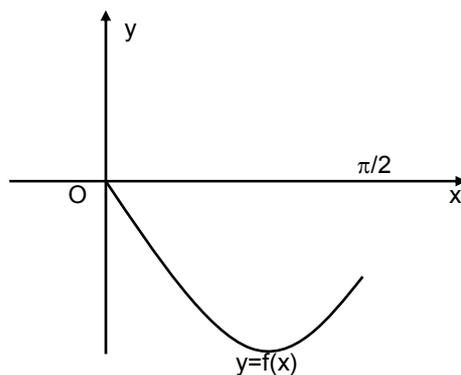
$\Rightarrow f(x)$  is increasing function in  $\left[0, \frac{\pi}{2}\right]$

$\Rightarrow$  there is no local maxima of  $f(x)$  in  $\left[0, \frac{\pi}{2}\right]$

$\Rightarrow$  graph of  $f(x)$  always lies below the x-axis

in  $\left[0, \frac{\pi}{2}\right]$ .

$$\Rightarrow f(x) \leq 0 \text{ in } x \in \left[0, \frac{\pi}{2}\right].$$



$$3x^2 + 3x \leq 2\pi x + \pi \sin x \Rightarrow \sin x + 2x \geq \frac{3x(x+1)}{\pi}$$

18.  $A = \begin{bmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{bmatrix}$ ,  $B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$ ,  $U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$ ,  $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$ . If there is vector matrix X, such that  $AX = U$  has

infinitely many solutions, then prove that  $BX = V$  cannot have a unique solution. If  $afd \neq 0$  then prove that  $BX = V$  has no solution.

**Sol.**  $AX = U$  has infinite solutions  $\Rightarrow |A| = 0$

$$\begin{vmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{vmatrix} = 0 \Rightarrow ab = 1 \text{ or } c = d$$

$$\text{and } |A_1| = \begin{vmatrix} a & 0 & f \\ 1 & c & g \\ 1 & d & h \end{vmatrix} = 0 \Rightarrow g = h; \quad |A_2| = \begin{vmatrix} a & f & 1 \\ 1 & g & b \\ 1 & h & b \end{vmatrix} = 0 \Rightarrow g = h$$

$$|A_3| = \begin{vmatrix} f & 0 & 1 \\ g & c & b \\ h & d & b \end{vmatrix} = 0 \Rightarrow g = h, c = d \Rightarrow c = d \text{ and } g = h$$

$BX = V$

$$|B| = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix} = 0 \quad (\text{since } C_2 \text{ and } C_3 \text{ are equal}) \quad \Rightarrow BX = V \text{ has no unique solution.}$$

$$\text{and } |B_1| = \begin{vmatrix} a^2 & 1 & 1 \\ 0 & d & c \\ 0 & g & h \end{vmatrix} = 0 \quad (\text{since } c = d, g = h)$$

$$|B_2| = \begin{vmatrix} a & a^2 & 1 \\ 0 & 0 & c \\ f & 0 & h \end{vmatrix} = a^2cf = a^2df \quad (\text{since } c = d)$$

$$|B_3| = \begin{vmatrix} a & 1 & a^2 \\ 0 & d & 0 \\ f & g & 0 \end{vmatrix} = a^2 df$$

since if  $adf \neq 0$  then  $|B_2| = |B_3| \neq 0$ . Hence no solution exist.

19. A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast 4 balls are white. Find the probability that in the next two draws exactly one white ball is drawn. (leave the answer in terms of  ${}^n C_r$ ).

**Sol.** Let  $P(A)$  be the probability that atleast 4 white balls have been drawn.  
 $P(A_1)$  be the probability that exactly 4 white balls have been drawn.  
 $P(A_2)$  be the probability that exactly 5 white balls have been drawn.  
 $P(A_3)$  be the probability that exactly 6 white balls have been drawn.  
 $P(B)$  be the probability that exactly 1 white ball is drawn from two draws.

$$P(B/A) = \frac{\sum_{i=1}^3 P(A_i) P(B/A_i)}{\sum_{i=1}^3 P(A_i)} = \frac{\frac{{}^{12}C_2 {}^6C_4}{{}^{18}C_6} \cdot \frac{{}^{10}C_1 {}^2C_1}{{}^{12}C_2} + \frac{{}^{12}C_1 {}^6C_5}{{}^{18}C_6} \cdot \frac{{}^{11}C_1 {}^1C_1}{{}^{12}C_2}}{\frac{{}^{12}C_2 {}^6C_4}{{}^{18}C_6} + \frac{{}^{12}C_1 {}^6C_5}{{}^{18}C_6} + \frac{{}^{12}C_0 {}^6C_6}{{}^{18}C_6}}$$

$$= \frac{{}^{12}C_2 {}^6C_4 {}^{10}C_1 {}^2C_1 + {}^{12}C_1 {}^6C_5 {}^{11}C_1 {}^1C_1}{{}^{12}C_2 ({}^{12}C_2 {}^6C_4 + {}^{12}C_1 {}^6C_5 + {}^{12}C_0 {}^6C_6)}$$

20. Two planes  $P_1$  and  $P_2$  pass through origin. Two lines  $L_1$  and  $L_2$  also passing through origin are such that  $L_1$  lies on  $P_1$  but not on  $P_2$ ,  $L_2$  lies on  $P_2$  but not on  $P_1$ .  $A, B, C$  are three points other than origin, then prove that the permutation  $[A', B', C']$  of  $[A, B, C]$  exists such that  
 (i).  $A$  lies on  $L_1$ ,  $B$  lies on  $P_1$  not on  $L_1$ ,  $C$  does not lie on  $P_1$ .  
 (ii).  $A'$  lies on  $L_2$ ,  $B'$  lies on  $P_2$  not on  $L_2$ ,  $C'$  does not lie on  $P_2$ .

**Sol.**  $A$  corresponds to one of  $A', B', C'$  and  
 $B$  corresponds to one of the remaining of  $A', B', C'$  and  
 $C$  corresponds to third of  $A', B', C'$ .  
 Hence six such permutations are possible  
 eg One of the permutations may  $A \equiv A'; B \equiv B', C \equiv C'$   
 From the given conditions:  
 $A$  lies on  $L_1$ .  
 $B$  lies on the line of intersection of  $P_1$  and  $P_2$   
 and ' $C$ ' lies on the line  $L_2$  on the plane  $P_2$ .  
 Now,  $A'$  lies on  $L_2 \equiv C$ .  
 $B'$  lies on the line of intersection of  $P_1$  and  $P_2 \equiv B$   
 $C'$  lie on  $L_1$  on plane  $P_1 \equiv A$ .  
 Hence there exist a particular set  $[A', B', C']$  which is the permutation of  $[A, B, C]$  such that both (i) and (ii) is satisfied. Here  $[A', B', C'] \equiv [CBA]$ .

**Note:** **FIITJEE** solutions to IIT-JEE, 2004 Main Papers created using memory retention of select **FIITJEE** students appeared in this test and hence may not exactly be the same as the original paper. However, every effort has been made to reproduce the original paper in the interest of the aspiring students.