

PART II - Physics

SECTION - I (Total Marks : 21)

(Single Correct Answer Type)

This Section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

24. The wavelength of the first spectral line in the Balmer series of Hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is:
(A) 1215 Å (B) 1640 Å (C) 2430 Å (D) 4687 Å

24. (A) For first spectral line in the Balmer series

$$\frac{1}{\lambda} = Rz^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \quad z = 1$$

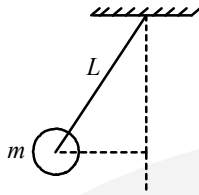
$$\frac{1}{\lambda} = R \left(\frac{1}{4} - \frac{1}{9} \right) = R (5/36) \quad \dots (1)$$

For second spectral line in the Balmer series of singly - ionized helium

$$\begin{aligned} \frac{1}{\lambda} &= R(4) \left(\frac{1}{2^2} - \frac{1}{4^2} \right) \\ &= R \cdot \frac{4}{4} \left(1 - \frac{1}{4} \right) = R (3/4) \quad \dots (2) \end{aligned}$$

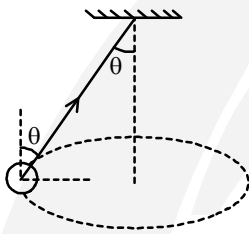
$$\begin{aligned} \Rightarrow (\lambda^1 / \lambda) &= [(5 \times 4) / (36 \times 3)] \\ \lambda^1 &= [5\lambda / (9 \times 3)] = [(5 \times 6561) / (9 \times 3)] = 1215 \text{ Å} \end{aligned}$$

25. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is:



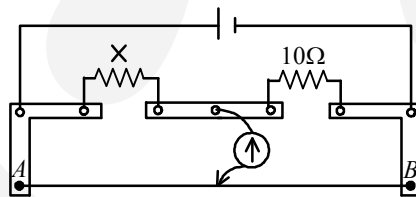
- (A) 9 (B) 18 (C) 27 (D) 36

25. (D) $T \sin \theta = m \omega^2 r = m \omega^2 L \sin \theta$
 $T = m \omega^2 L$



$$\omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0.5 \times 0.5}} = (18 / 0.5) = 36$$

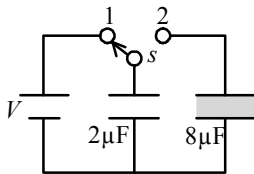
26. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm. mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is



- (A) 10.2 ohm (B) 10.6 ohm (C) 10.8 ohm (D) 11.1 ohm

26. (B) $(X / 10) = [(52 + 1) / (48 + 2)] \Rightarrow x = 10.6 \Omega$

27. A $2\ \mu\text{F}$ capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



- (A) 0% (B) 20% (C) 75% (D) 80%

27. (D) Initial energy stored in capacitor $C_1 = (Q^2 / 2C_1) = (Q^2 / 4)$

Where $Q = C_1 V$

let $C_1 = 2\ \mu\text{F}$ $C_2 = 8\ \mu\text{F}$

Finally charge on $C_1 = (Q / 5)$

Energy on $C_1 = [Q^2 / 50 C_1] = (Q^2 / 100)$

Energy on $C_2 = (16 Q^2 / 25) = (16 Q^2 / 25)$

$$\text{Energy dissipated, } \Delta E = \frac{Q^2}{4} - \left(\frac{Q^2}{100} + \frac{Q^2}{25} \right) = [(Q^2 / 4) - (Q^2 / 20)] = (Q^2 / 5)$$

$$\therefore \text{ \% of its stored energy dissipated} = \left(\frac{Q^2 \times 4}{5 \times Q^2} \times 100 \right) = 80\%$$

28. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km / hr towards a tall building which reflects the sound waves. The speed of sound in air is $320\ \text{ms}^{-1}$. The frequency of the siren heard by the car driver is:

- (A) 8.50 kHz (B) 8.25 kHz (C) 7.75 kHz (D) 7.50 kHz

28. (A) Velocity of sound, $C = 320\ \text{ms}^{-1}$

frequency of the siren heard by the car driver

$$f = f_0 [(C + V) / (C - V)] = 8[(320 + 10) / (320 - 10)] = 8.50\ \text{kHz}$$

29. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1 , the work done in the process is

- (A) $(9 / 8) RT$ (B) $(3 / 2) RT$ (C) $(15 / 8) RT$ (D) $(9 / 2) RT$

29. (A) No. of moles, $n = (5.6 / 22.4) = (1 / 4)$

Initial Temperature = T_1

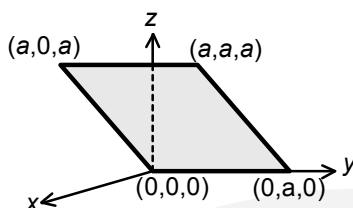
Let final temperature = T_2

$$\Rightarrow T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 (V_1 / V_2)^{\gamma-1} = T_1 (5.6 / 0.7)^{5/3-1} = 4 T_1$$

$$\text{Workdone} = [(nR\Delta T) / (\gamma - 1)] = (9 / 8) RT_1$$

30. Consider an electric field $\vec{E} = E_0 \hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is:



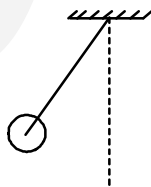
- (A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$ (C) E_0a^2 (D) $(E_0a^2 / \sqrt{2})$
30. (C) $\phi = (E_0\hat{i}) \cdot (a\hat{j} \times (a\hat{k} - a\hat{i})) = E_0a^2$

SECTION - II (Total Marks : 16)

(Multiple Correct Choice Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

31. A metal rod of length ' L ' and mass ' m ' is pivoted at one end. A thin disk of mass ' M ' and radius ' R ' ($< L$) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A) The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true ?

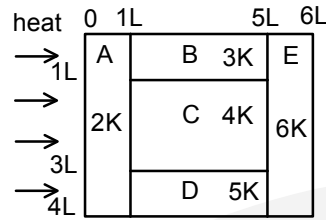


- (A) Restoring torque in case A = Restoring torque in case B
 (B) Restoring torque in case A < Restoring torque in case B
 (C) Angular frequency for case A > Angular frequency for case B
 (D) Angular frequency for case A < Angular frequency for case B
31. (A,D) Torque due to weight is same in both case \Rightarrow (A)
 $T \propto \sqrt{I_{sys}} \Rightarrow \omega \propto [1 / \sqrt{I_{sys}}]$
 Case A : When disc is not rotating then M.I of system about point of suspension = I_A
 Case B : When disc is rotating then M.I of system about point of suspension = I_B

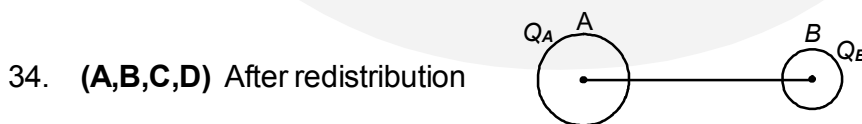
$$\therefore I_B = I_{rod}^{Pivot} + I_{disc}^{Pivot} \quad I_A = I_{rod} + \left(I_{disc}^{Pivot} + \frac{MR^2}{2} \right)$$

$$\therefore I_B < I_A \Rightarrow \omega_A < \omega_B \Rightarrow (D)$$

32. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat ' Q ' flows only from left to right through the blocks. Then in steady state



- (A) Heat flow through A and E slabs are same
 (B) Heat flow through slab E is maximum
 (C) Temperature difference across slab E is smallest
 (D) Heat flow through $C =$ heat flow through $B +$ heat flow through D
32. **(ABCD)** $A, (BCD), E$ in series. Thus $H_A = H_E = H_B + H_C + H_D \Rightarrow (A), (B)$.
 For parallel slabs $B, C, D; H \propto KA \therefore H_B : H_C : H_D = 3KL : 8KL : 5KL \Rightarrow (D)$.
 For series, $K \Delta\theta$ is same. Since K_E is largest, $\Delta\theta_E$ is least. $\Rightarrow (C)$.
33. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is / are true ?
 (A) They will never come out of the magnetic field region
 (B) They will come out travelling along parallel paths
 (C) They will come out at the same time
 (D) They will come out at different times
33. **(B,D)** $R_p = (m_p V / q_p B)$ $R_e = m_e V / q_e B$ $\therefore m_p > m_e \Rightarrow R_p > R_e$
 $T_p = \pi m_p / qB$ $T_e = \pi m_e / qB$ $\Rightarrow T_p > T_e \Rightarrow (D)$
 Since they come out parallelly $\Rightarrow (B)$
34. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B (< R_A)$ are kept for apart and each is given charge '+ Q '. Now they are connected by a thin metal wire. Then
 (A) $E_A^{inside} = 0$ (B) $Q_A > Q_B$
 (C) $(\sigma_A / \sigma_B) = (R_B / R_A)$ (D) $E_A^{on\ surface} < E_B^{on\ surface}$



Since potential of both sphere will be same $\frac{Q_A}{4\pi\epsilon_0 R_A} = \frac{Q_B}{4\pi\epsilon_0 R_B} \Rightarrow Q_A > Q_B \Rightarrow (B)$

$E_A^{inside} = 0$ (from Gauss law) $\Rightarrow (A)$

$\sigma_A = \frac{Q_A}{4\pi R_A^2}, \sigma_B = \frac{Q_B}{4\pi R_B^2} \Rightarrow \frac{\sigma_A}{\sigma_B} = \frac{Q_A}{Q_B} \cdot \left(\frac{R_B}{R_A}\right)^2 = \frac{R_B}{R_A} \Rightarrow (C)$

$E_A^{on\ surface} \propto \sigma_A$ & $E_B^{on\ surface} \propto \sigma_B$
 $\Rightarrow E_A^{on\ surface} < E_B^{on\ surface} \Rightarrow (D)$

SECTION - III (Total Marks : 15)

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the first paragraph **2 multiple choice questions** and based on the paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for questions 35 to 36

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let ' N ' be the number density of free electrons, each of mass ' m '. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ' ω_p ', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω , where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p , all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

35. Taking the electronic charge as ' e ' and the permittivity as ' ϵ_0 ', use dimensional analysis to determine the correct expression for ω_p .

(A) $\sqrt{\frac{Ne}{m\epsilon_0}}$ (B) $\sqrt{\frac{m\epsilon_0}{Ne}}$ (C) $\sqrt{\frac{Ne^2}{m\epsilon_0}}$ (D) $\sqrt{\frac{m\epsilon_0}{Ne^2}}$

35. (C) Let $\omega_p \propto (Ne^2)^a (m\epsilon_0)^b$
 $\Rightarrow T^{-1} = (Ne^2)^a (m\epsilon_0)^b$
 $= [L^{-3} Q^2]^a [MM^{-1} L^{-3} T^2 Q^2]^b$
 $= L^{-3a} Q^{2a} L^{-3b} T^{2b} Q^{2b}$
 $= L^{-3(a+b)} Q^{2(a+b)} T^{2b}$
 $\Rightarrow a + b = 0$ and $2b = -1$
 $\Rightarrow b = -(1/2) \Rightarrow a = -b = (1/2)$
 $\Rightarrow \omega_p \propto (Ne^2)^{1/2} (m\epsilon_0)^{-1/2}$
 $\Rightarrow \omega_p \propto \sqrt{\frac{Ne^2}{m\epsilon_0}}$

36. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27} \text{ m}^{-3}$. Take $\epsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(A) 800 nm (B) 600 nm (C) 300 nm (D) 200 nm

36. (B) $f = (\omega / 2\pi)$

$$\omega = \sqrt{\frac{4 \times 10^{27} \times (1.6 \times 10^{-19})^2}{10^{-30} \times 10^{-11}}}$$

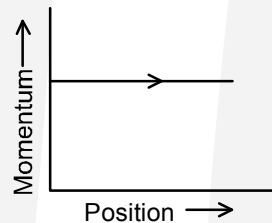
$$\omega = \sqrt{(1.6 \times 10^{-19})^2 \times 4 \times 10^{68}} = 2 \times 1.6 \times 10^{15} = 3.2 \times 10^{15}$$

$$\Rightarrow \lambda = (c / f) = \frac{3 \times 10^8}{\frac{3.2 \times 10^{15}}{2\pi}} = \frac{6\pi}{3.2} \times 10^{-7} m$$

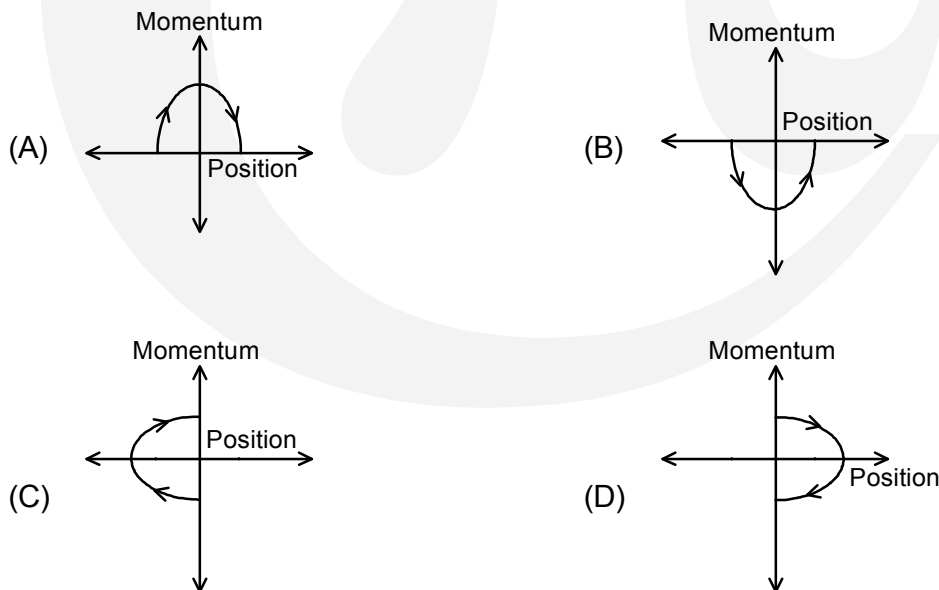
$$\Rightarrow \lambda = (600\pi / 3.2) \text{ nm} \approx 600 \text{ nm}$$

Paragraph for questions 37 to 39

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs. $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We used the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

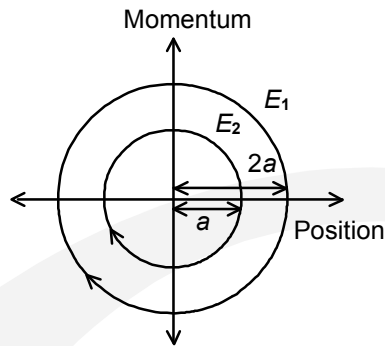


37. The phase space diagram for a ball thrown vertically up from ground is

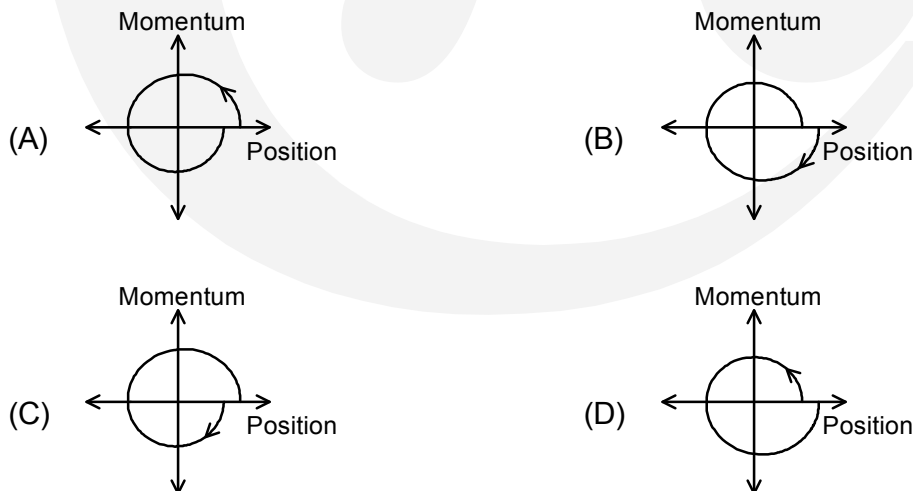
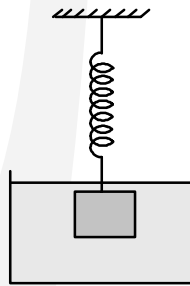


37. (D) For ball thrown up, initially momentum has maximum positive value and position is zero. Then as ball moves up, the momentum decreases & position increases. At highest position, momentum is zero, position is maximum. Then ball returns during which position decreases and momentum increases in negative.

38. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



- (A) $E_1 = \sqrt{2} E_2$ (B) $E_1 = 2 E_2$ (C) $E_1 = 4 E_2$ (D) $E_1 = 16 E_2$
38. **(C)** $(E_1 / E_2) = (2a / a)^2 = 4$
 $\Rightarrow E_1 = 4E_2$
39. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is:



39. **(B)** The mass has zero momentum at highest position. From here, it moves down, during which position decreases but momentum increases in negative value. After one cycle, its position is slightly less than initial value (damped oscillation).

SECTION - IV (Total Marks : 28)

(Integer Answer Type)

This section contains **7 questions**. The answer to each question is a **single digit integer** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

40. Steel wire of length ' L ' at 40°C is suspended from the ceiling and then a mass ' m ' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length ' L '. The coefficient of linear thermal expansion of the steel is $10^{-5} / ^\circ\text{C}$, Young's modulus of steel is 10^{11} Nm^{-2} and radius of the wire is 1 mm. Assume that $L \gg$ diameter of the wire. Then the value of ' m ' in kg is nearly.

40. (3) $(\Delta L / L) : \text{Same} \Rightarrow (mg / YA) = \alpha \Delta\theta$
 $\Rightarrow m = [(\alpha\Delta\theta \cdot YA) / g] = \pi \text{ kg} \approx \mathbf{3 \text{ kg}}$

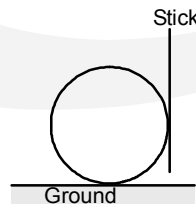
41. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is.

41. (1) $(1 / \lambda) = 10^9 \text{ s}, A_0 = 10^{10}$
 $\Rightarrow \lambda N_0 = 10^{10}$
 $\Rightarrow 10^{10} = [\lambda \cdot (m / M)] = [m / (10^9 \times 10^{-25})]$
 $\Rightarrow m = 10^{-6} \text{ kg} = \mathbf{1 \text{ mg}}$

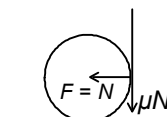
42. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ . The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10 \mu$, then N is.

42. (5) $F_u = mg (\sin \theta + \mu \cos \theta)$
 $F_d = mg (\sin \theta - \mu \cos \theta) \ \& \ F_u = 3F_d$
 $\Rightarrow \mu = (1 / 2) \tan \theta = (1 / 2)$
 $\Rightarrow N = 10 \mu = \mathbf{5}$

43. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 ms^{-2} . The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is $(P / 10)$. The value of P is.



43. (4) $NR - \mu NR = 2mR^2\alpha$



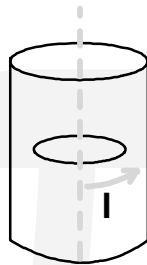
$\Rightarrow N(1 - \mu) = 2ma \quad \Rightarrow 2(1 - (P / 10)) = 2 \times 0.3 \times 2$
 $\Rightarrow \mathbf{P = 4}$

44. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corner of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m², then N is.

$$44. \quad (9) \quad I = 2 \left[\left\{ \frac{2}{5} mr^2 + m \left(\frac{a}{\sqrt{2}} \right)^2 \right\} + \left\{ \frac{2}{5} mr^2 \right\} \right]$$

$$= 9 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

45. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N \mu_0 I_0 \sin(300t)$, then ' N ' is.



$$45. \quad (6) \quad I = iN \quad \Rightarrow \quad i = (I/N)$$

[N : No. of turns per unit length assuming the tube as solenoid]

$$B = \mu_0 ni = \mu(N/L) i = (\mu_0 I / L)$$

$$\therefore \phi = BA = [(\mu_0 I / L) \cdot A]$$

$$\Rightarrow e = -(d\phi / dt) = [-(\mu_0 A / L) \cdot (dI / dt)]$$

$$\therefore \mu = i_1 A = [(\mu_0 A^2 / LR) (dI / dt)], \text{ where } A = \pi r^2$$

$$\Rightarrow \mu = [(A^2 / LR) \times 300] \mu_0 I_0 \sin(300t)$$

$$\Rightarrow N = 0.6 \pi^2 = 6$$

46. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side ' a '. The surface tension of the soap film is γ . The system of charges and planar film are

in equilibrium, and $a = k \left[\frac{q^2}{\gamma} \right]^{1/N}$, where ' k ' is a constant. Then N is.

46. (3) Consider the upper half of the setup.

For equilibrium of the system

$$F_{\text{s.T.}} = F_{\text{elec.}}$$

$$\Rightarrow 2 \cdot \gamma a = 2 \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2} + \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{(\sqrt{2}a)^2} \cdot \frac{1}{\sqrt{2}} \right]$$

$$\Rightarrow \gamma a^3 = q^2 \left[(1 / 4\pi\epsilon_0) \cdot (1 + (1 / 2\sqrt{2})) \right]$$

$$\therefore a = \text{constant } (q^2 / \gamma)^{1/3} \quad \therefore N = 3.$$

Alternately,

$$F_{\text{elec}} \propto a^{-2} \quad \text{while } F_{\text{ST}} \propto \gamma a. \quad \therefore (q^2 / \gamma) \propto a^3.$$