IIT-JEE 2003 Mains Questions & Solutions - Physics
(The questions are based on memory)

Break-up of questions:

Mechanics	Sound	Heat	Electromagnetism	Optics	Modern Physics
6	2	2	5	2	3

1. N divisions on the main scale of a vernier callipers coincide with $N + 1$ divisions on the vernier scale. If each division on the main scale is of a units, determine the least count of the instrument.

[2]

Sol. Least count of vernier callipers = value of one division of main scale - value of one division of vernier scale

Now $N \times a = (N + 1)a'$ {a' = value of one division of vernier scale)

$$a' = \frac{N}{N+1}a$$

$\therefore$ Least count = $a - a' = \dfrac{a}{N+1}$

2. Characteristic X-rays of frequency 4.2×10^{18} Hz are produced when transitions from L shell to K shell take place in a certain target material. Use Mosley's law to determine the atomic number of the target material. Given Rydberg constant $R = 1.1 \times 10^{7}\ \text{m}^{-1}$.

[2]

Sol. According to Bohr's model

$$\Delta E = \lambda v = Rhc(z-b)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \quad \text{for } L \to K$$

$$\sqrt{v} = \sqrt{\frac{3Rc}{4}}(z-1) \qquad \text{(b = 1)}$$

$$\sqrt{4.2\times10^{18}} = \sqrt{\frac{3\times1.1\times10^{7}\times3\times10^{8}}{4}}\ (z-1)$$

Solving $(z-1) = 41.194$

$z \approx 42$

3. In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The air column in pipe resonates with a tuning fork of frequency 480 Hz when the minimum length of the air column is 16 cm. Find the speed of sound in air at room temperature.

[2]

Sol. $$\frac{V}{4[L+0.6r]} = 480$$

$\therefore$ $V = 480 \times 4 \times [0.16 + 0.6 \times 0.025] = 336$ m/s

4. An insulated box containing a monatomic gas of molar mass M moving with a speed v_o is suddenly stopped. Find the increment in gas temperature as a result of stopping the box.

[2]

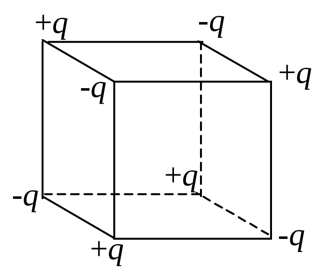
Sol. Decrease in kinetic energy = Increase in internal energy.

$$\frac{1}{2}mv^2 = \frac{m}{M}\frac{R}{\gamma-1}\Delta T$$

$$\frac{1}{2}v^2 = \frac{1}{M}\frac{3}{2}R\Delta T$$

$$\Delta T = \frac{Mv^2}{3R}$$

5. Eight point charges are placed at the corners of a cube of edge a as shown in the figure. Find the work done in disassembling this system of charges.



[2]

Sol. For potential energy total number of charge pairs = 28.

Let U_1 = Potential energy of charge pairs with separation a (12 pairs)

U_2 = Potential energy of charge pairs with separation $\sqrt{2}\,a$ (12 pairs)

U_3 = Potential energy of charge pairs with separation $\sqrt{3}\,a$ (4 pairs)

$$U_1 = \frac{-12Kq^2}{a} \qquad \left[K = \frac{1}{4\pi\varepsilon_o}\right]$$

$$U_2 = \frac{12Kq^2}{\sqrt{2}\,a}$$

$$U_3 = \frac{-4Kq^2}{\sqrt{3}\,a}$$

U = total potential energy of system = $U_1 + U_2 + U_3$

$$= \frac{Kq^2}{a}\left[-12 + \frac{12}{\sqrt{2}} - \frac{4}{\sqrt{3}}\right] = -5.824\frac{Kq^2}{a}$$

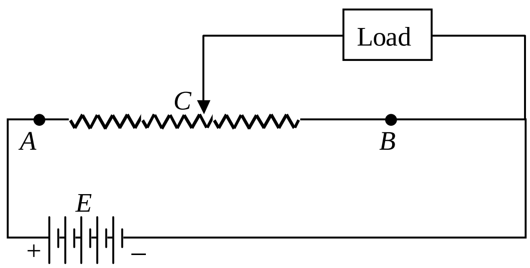
When the charges are separated to infinity, potential energy $U_\infty = 0$

Change in energy = 0 - (-5.824 $\frac{Kq^2}{a}$) = 5.824

So work done by external force = change in potential energy = 5.824 $\frac{Kq^2}{a}$

6. Show by diagram, how can we use a rheostat as the potential divider. **[2]**

Sol.



7. A radioactive element decays by β emission. A detector records n beta particles in 2 seconds and in next 2 seconds it records 0.75n beta particles. Find mean life correct to nearest whole number. Given ln |2| = 0.6931, ln |3| = 1.0986. **[2]**

Sol. $N = N_o e^{-\lambda t}$

$N_2 = N_o e^{-2\lambda}$

and $N_4 = N_o e^{-4\lambda}$

$\therefore$ $n = N_o - N_2 = N_o\left(1 - e^{-2\lambda}\right)$

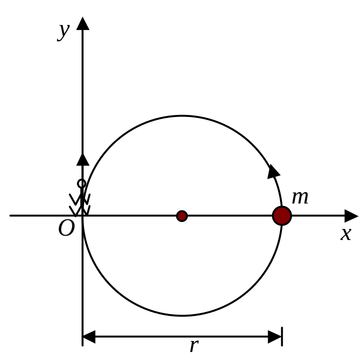
and $0.75n = N_o\left(e^{-2\lambda} - e^{-4\lambda}\right)$

Solving we get,

$\lambda = 0.145\ s^{-1}$

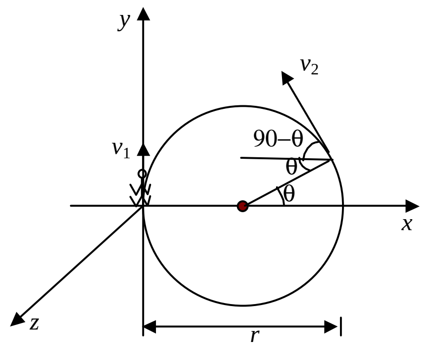
Average life = $\frac{1}{\lambda}$ = 6.896 second.

8. A man and a mass m are initially situated on the diametrically opposite ends as shown in the figure. At some instant they start moving with constant speeds v_1 and v_2. If the man moves in $\hat{j}$ direction and mass moves in a circle of radius r as shown in the figure.



Find the linear momentum of mass with respect to man as a function of time. **[2]**

Sol. Let at any time t mass is situated as shown.



$$\vec{v}_2 = -v_2 \sin\theta\hat{i} + v_2\cos\theta\hat{j}$$

$$= -v_2 \sin\left(\frac{2v_2}{r}t\right)\hat{i} + v_2\cos\left(\frac{2v_2}{r}t\right)\hat{j}$$

$$\because \quad v_2 = \frac{r}{2}\omega$$

Relative velocity of the mass with respect to the person is

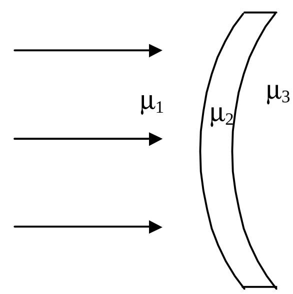
$$= \vec{v}_2 - \vec{v}_1$$

$$= -v_2 \sin\left(\frac{2v_2}{r}.t\right)\hat{i} + \left(v_2\cos\left(\frac{2v_2}{r}.t\right) - v_1\right)\hat{j}$$

Relative momentum of mass with respect to the person will be

$$m_2\left(-v_2 \sin\left(\frac{2v_2}{r}.t\right)\hat{i} + \left(v_2\cos\left(\frac{2v_2}{r}.t\right) - v_1\right)\hat{j}\right)$$

9. In the figure, light is incident on the thin lens as shown. The radius of curvature for both the surfaces is R. Determine the focal length of this system.



[2]

Sol. For refraction at 1'st surface,

$$\frac{\mu_2}{\mu_1 v_1} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\frac{1}{R}$$

$$u = \infty$$

$$\therefore \quad \frac{1}{v_1} = \frac{(\mu_2 - \mu_1)}{\mu_2.R} \qquad (1)$$

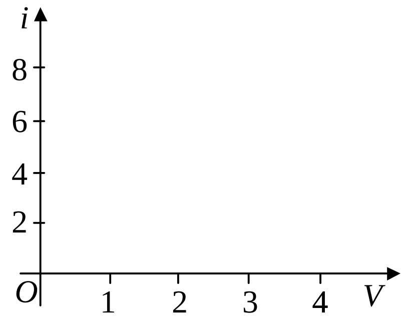
For refraction at 2'nd surface,

$$\frac{\mu_3}{\mu_2.v} - \frac{1}{v_1} = \left(\frac{\mu_3 - \mu_2}{\mu_2.R}\right)$$

Now $v = f$, by putting the value of v_1 from (1)

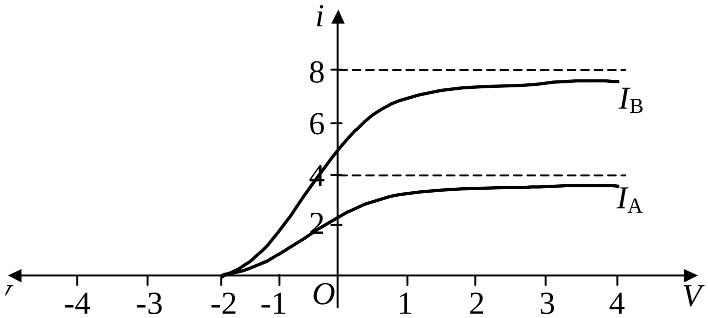
$$\therefore \quad f = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

10. In a photoelectric effect experiment, photons with kinetic energy = 5 eV are incident on a metal surface having work function 3 eV. For intensity of incident photons $I_A = 10^5$ W/m^2 saturation current of 4 μA is obtained. Sketch the graph between i and anode voltage for I_A and $I_B = 2I_A$.

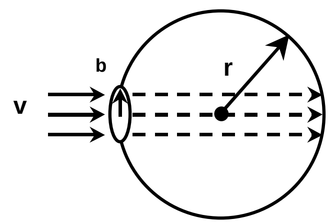


[2]

Sol. Since doubling the intensity doubles the number of photoelectrons, therefore the saturation current will be doubled in the later case.



11. A soap bubble is being blown at the end of a very narrow tube of radius b. Air (density ρ) moves with a velocity v inside the tube and comes to rest inside the bubble. The surface tension of the soap solution is T. After some time the bubble, having grown to a radius r, separates from the tube. Find the value of r. Assume that $r >> b$ so that your can consider the air to be falling normally on the bubble's surface.

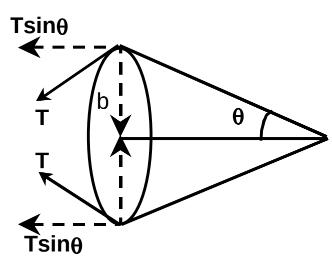


[4]

Sol. Surface Tension force = $2\pi b \times 2T \sin\theta$

Mass of the air per second entering the bubble = $\rho A v$

Momentum of air per second = Force due to air = $\rho A v^2$



The bubble will separate from the tube when force due to moving air becomes equal to the surface tension force inside the bubble.

$2\pi b \times 2T \sin\theta = \rho A v^2$

putting $\sin\theta = \frac{b}{r}, A = \pi b^2$ and solving we get

$$r = \frac{4T}{\rho v^2}$$

12. There is a crater of depth $\frac{R}{100}$ on the surface of the moon (radius R). A projectile is fired vertically upward from the crater with a velocity, which is equal to the escape velocity v from the surface of the moon. Find the maximum height attained by the projectile.

[4]

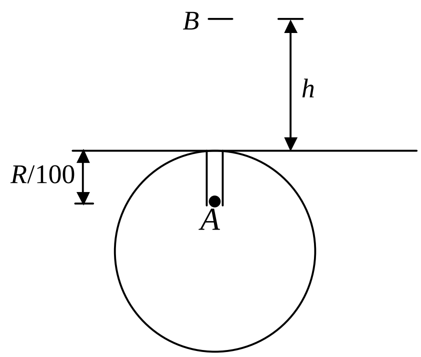
Sol. Let particle be projected from A and it reaches to point B at a height h from surface.

It is projected with escape velocity

$$= \sqrt{\frac{2GM}{R}}$$

Energy at A = energy at B.

K. E. + P. E. (at A) = P. E. at B



$$\frac{1}{2}m\frac{2GM}{R} - \frac{GmM}{2R^3}\left[3R^2 - \left(\frac{99}{100}\right)^2 R^2\right] = -\frac{GMm}{(R+h)}$$

$$\frac{1}{2}m\frac{2GM}{R} - \frac{2.0199GmM}{2R} = -\frac{GMm}{(R+h)}$$

$$-\frac{0.0199GMm}{2R} = -\frac{GMm}{(R+h)}$$

$$R + h = \frac{2R}{0.0199} = 100.5R$$

$$h = 99.5R$$

13. A positive point charge q is fixed at origin. A dipole with a dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along positive x-axis. Find

(a) the kinetic energy of the dipole when it reaches a distance d from the origin, and

(b) the force experienced by the charge q at this moment.

[4]

Sol.(a) $$U = \frac{-1}{4\pi\varepsilon_o}\frac{q}{x^2}p$$

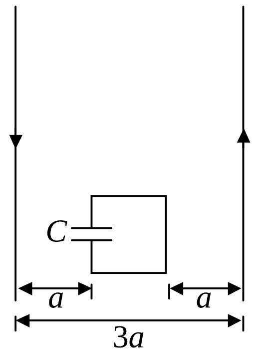
$\therefore$ Kinetic energy at a distance $x = d$ will be $= \frac{1}{4\pi\varepsilon_o}\frac{qp}{d^2}$

(b)

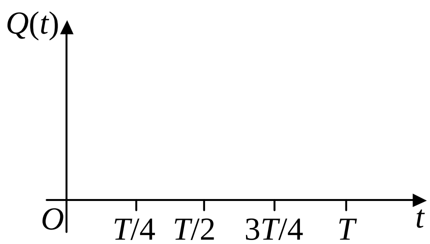
$$F = -\frac{dU}{dx}$$

$$F = \frac{-2qp}{4\pi\varepsilon_o d^3}$$ (-ve sign implies an attractive force)

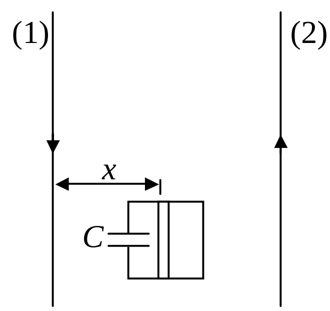
14. Two infinitely long parallel wires carrying currents $I = I_o \sin \omega t$ in opposite directions are placed a distance $3a$ apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown. Find the maximum current in the square loop. Also sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anticlockwise direction for the current in the loop as positive.



[4]



Sol. In the square loop magnetic field due to both the wires is out of paper.



For a elemental strip of thickness dx at a distance x from wire 1, magnetic field due to wire (1) and (2) will be

$$B = \frac{\mu_o i}{2\pi x} + \frac{\mu_o i}{2\pi(3a - x)}$$

Flux in the strip

$$d\Phi = B.dA = \frac{\mu_o i}{2\pi}\left[\frac{1}{x} + \frac{1}{3a - x}\right].adx$$

$$\therefore \quad \Phi = \frac{\mu_o ia}{2\pi}\left[\ln|x| - \ln(3a - x)\right]_a^{2a}$$

$$\Phi = \frac{\mu_o ai}{\pi}\sin\omega t.\ln(2)$$

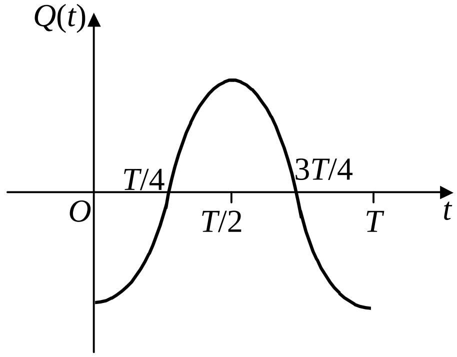
$$\mathcal{E}_{ind} = \left|\frac{d\Phi}{dt}\right| = \frac{\omega\mu_o ai\ln(2)\cos\omega t}{\pi}$$

Now $Q = C\mathcal{E}_{ind}$ and $I = \frac{dQ}{dt}$

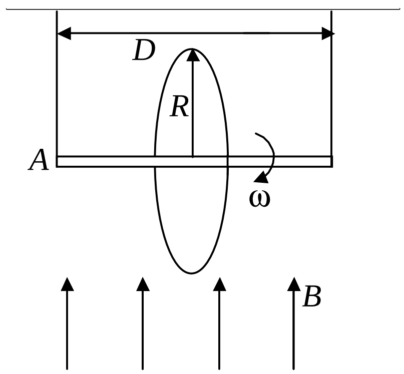
$$\therefore \quad i = \frac{C\omega^2\mu_o ai\ln(2)\sin\omega t}{\pi}$$

For $i_{\max}$, $\sin \omega t = 1$

$$i_{\max} = \frac{C\omega^2\mu_o a i \ln(2)}{\pi}$$



15. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_o. Now a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $3T_o/2$.



[4]

Sol. Let for ω string breaks; $2T_o = mg$, $T_o = mg/2$

current = $\frac{\omega}{2\pi} \times Q$

Magnetic moment = IA

$$\tau = M \times B = IAB$$

$$(T_1 - T_2)\frac{D}{2} = IAB$$

$$T_1 - T_2 = \frac{2IAB}{D}$$

$$T_1 + T_2 = mg$$

$$2T_1 = \frac{2IAB}{D} + mg$$

$$\frac{2 \times 3T_o}{2} = \frac{2\omega Q \times \pi R^2 \times B}{2\pi D} + mg$$

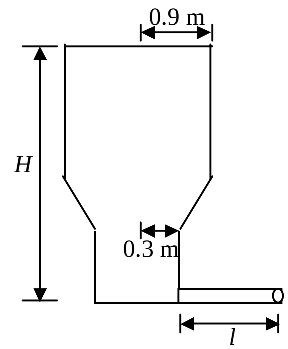
$$\frac{3mg}{2} = \frac{2\omega Q\pi R^2 \times B}{2\pi D} + mg$$

$$\frac{mg}{2} = \frac{\omega QR^2 B}{D}$$

putting, $mg/2 = T_o$

$$\omega = \frac{DT_o}{QR^2 B}$$

16. A liquid of density 900 kg/m^3 is filled in a cylindrical tank of upper radius 0.9 m and lower radius 0.3 m. A capillary tube of length l is attached at the bottom of the tank as shown in the figure. The capillary has outer radius 0.002 m and inner radius a. When pressure P is applied at the top of the tank volume flow rate of the liquid is 8×10^{-6} m^3/s and if capillary tube is detached, the liquid comes out from the tank with a velocity 10 m/s. Determine the coefficient of viscosity of the liquid.
[Given: $\pi a^2 = 10^{-6}$ m^2 and $a^2/l = 2 \times 10^{-6}$ m]



[4]

Sol. When capillary is not connected

$$P + P_o + \rho g H + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2 + P_o$$

$$p + \rho g H = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2 v_2}{A_1}$$

$$P + \rho g H = \frac{1}{2}\rho\left(v_2^2 - \left(\frac{A_2}{A_1}v_2^2\right)\right) = \frac{1}{2}\rho v_2^2\left(1 - \left(\frac{3}{a}\right)^2\right) = \frac{1}{2}\rho 100\left(1 - \frac{1}{a}\right) = \frac{800\rho}{18}$$

Excess pressure is $P + \rho_g W$

$$\Delta P = \frac{800\rho}{18}$$

$$\frac{dv}{dt} = \frac{\pi r^4 \Delta P}{8\eta l}$$

$$8 \times 10^{-6} = \frac{2 \times 10^{-6} \times 10^{-6} \times 800\rho}{8 \times 18 \times \eta}$$

$\eta = 1.25 \times 10^{-3}$ Ns/m^2

17. A string of mass per unit length μ is clamped at both ends such that one end of the string is at $x = 0$ and the other is at $x = l$. When string vibrates in fundamental mode, amplitude of the mid point of the string is a, and tension in the string is T. Find the total oscillation energy stored in the string.

[4]

Sol. The amplitude at a distance x from the origin is given by

$$A = a\sin kx$$

Considering an element of length dx of the string at a distance x from the origin.
The total energy of this element = its maximum kinetic energy

$$= \frac{1}{2}dm\omega^2 A^2 = \frac{1}{2}\mu dx 4\pi^2 f^2 a^2 \sin^2 kx$$

$$= 2\pi^2 \mu f^2 a^2 \sin^2 kxdx$$

Total energy of the string = $\int_0^L 2\pi^2 \mu f^2 a^2 \sin^2 kxdx$

$$= \pi^2 \mu f^2 a^2 \int_0^L (1-\cos 2kx)dx = \pi^2 \mu f^2 a^2 \left[x - \frac{\sin 2kx}{2}\right]_0^L$$

$$= \pi^2 \mu f^2 a^2 \left[L - \frac{\sin 2kL}{2}\right] = \pi^2 \mu f^2 a^2 L$$

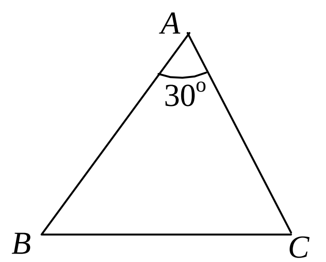
Since $\sin 2kL = \sin \frac{4\pi}{\lambda}\frac{\lambda}{2} = \sin 2\pi = 0$

$$= \pi^2 \mu \frac{V^2}{\lambda^2} a^2 L = \frac{\pi^2 L}{4L^2} a^2 L = \frac{\pi^2 a^2 T}{4L}$$

18. A prism of refracting angle 30^o is coated with a thin film of transparent material of refractive index 2.2 on face AC of the prism. A light of wavelength 6600 Å is incident on face AB such that angle of incidence is 60^o, find

(a) the angle of emergence, and
[Given refractive index of the material of the prism is $\sqrt{3}$]

(b) the minimum value of thickness of the coated film on the face AC for which the light emerging from the face has maximum intensity.



[4]

Sol.(a) $1.\sin 60^o = \sqrt{3} \,.\, \sin r$

$\therefore \quad \sin r = \frac{1}{2} \quad \Rightarrow \quad r = 30^o$

In ΔADE

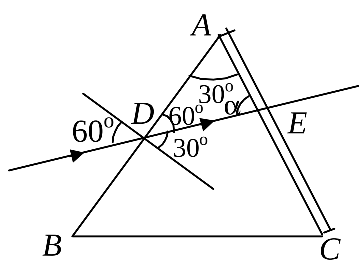
$$30 + (90 - 30) + \alpha = 180$$

$$\alpha = 90^o$$

$\therefore$ Ray refracted on the first surface of the prism is incident normally on the face AC.

Hence it will emerge undeviated.

$\therefore$ angle of emergence is zero.



(b) Intensity will be maximum if constructive interference takes place in the transmitted system.

$$2\mu t = n\lambda$$

$$t = \frac{\lambda}{2\mu} \qquad \text{(n = 1 for minimum thickness)}$$

$$t_{\min} = \frac{6000}{2\times 2.2} = 1500\ \text{Å}$$

19. Two point masses m_1 and m_2 are connected by a spring of natural length l_o. The spring is compressed such that the two point masses touch each other and then they are fastened by a string. Then the system is moved with a velocity v_o along positive x-axis. When the system reaches the origin the strings breaks (t = 0). The position of the point mass m_1 is given by

$$x_1 = v_o t - A(1-\cos\omega t) \quad \text{where } A \text{ and } \omega \text{ are constants.}$$

Find the position of the second block as a function of time. Also find the relation between A and l_o.

[4]

Sol. Since there is no external force, momentum of the system is conserved.

$$\Rightarrow \quad m_1\frac{dx_1}{dt} + m_2\frac{dx_2}{dt} = (m_1+m_2)v_o$$

$$\Rightarrow \quad m_2\frac{dx_2}{dt} = (m_1+m_2)v_o - m_1\frac{dx_1}{dt}$$

$$= (m_1+m_2)v_o - m_1(v_o - A\omega\sin\omega t)$$

$$= m_2 v_o + m_1 a\omega\sin\omega t$$

$$\Rightarrow \quad \frac{dx_2}{dt} = v_o + \frac{m_1}{m_2}A\omega\sin\omega t$$

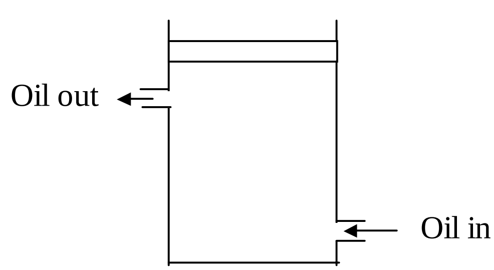
$$x_2 = \int_0^t v_o dt + \frac{m_1 A\omega}{m_2}\int_0^t \sin\omega t dt = v_o t + \frac{m_1}{m_2}(1-\cos\omega t)$$

$$x_2 - x_1 = \frac{m_1 A}{m_2}(1-\cos\omega t) + A(1-\cos\omega t)$$

Maximum value of x_2 - x_1 = $2l_o$

$$\Rightarrow \quad 2l_o = \frac{2m_1 A}{m_2} + 2A = \frac{2(m_1+m_2)A}{m_2} \qquad \Rightarrow \quad l_o = \frac{(m_1+m_2)A}{m_2}$$

20. The top of an insulated cylindrical container is covered by a disc having emissivity 0.6 and conductivity 0.167 W/K/m and thickness 1 cm. The temperature is maintained by circulating oil as shown.



[4]

(a) Find the radiation loss to the surroundings in $J/m^2/s$ if temperature of the upper surface of disc is 127 ^oC, and temperature of surroundings is 27^oC.

(b) Also find the temperature of the circulating oil. Neglect the heat loss due to convection.

[Given $\sigma = \frac{17}{3} \times 10^{-8}$ $Wm^{-2}K^{-4}$].

20. (a) Rate of heat loss per unit area due to radiation

$$= e\sigma\left(T^4 - T_o^4\right) = 0.6 \times \frac{17}{3} \times 10^{-8}\left[(400)^4 - (300)^4\right]$$

$= 595$ J/m^2. s

(b) Suppose temperature of oil is θ then rate of heat flow through conduction = rate of heat loss due to radiation.

$$\therefore \quad \frac{0.167 \times A(\theta - 127)}{1 \times 10^{-2}} = 595\ A \qquad [A = \text{area of the disc}]$$

After solving we get,

$\theta = 162.6$ ^oC.