IIT-JEE 2009 EXAMINATION PAPER

(QUESTION & SOLUTIONS)

PAPER – I

Part – I (CHEMISTRY)

SECTION – I

Straight Objective Type

12/04/09

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. The term that correct for the attractive forces present in a real gas in the van der Waals equation is -

(A) nb

(B) $\frac{an^2}{V^2}$

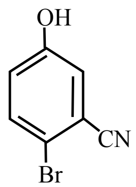
(C) $-\frac{an^2}{V^2}$

(D) – nb

[Ans. B]

Sol. $\left(P + \frac{n^2 a}{V^2}\right)$ (V – nb) = RT

2. The IUPAC name of the following compound is -



(A) 4-Bromo-3-cyanophenol

(B) 2-Bromo-5-hydroxybenzonitrile

(C) 2-Cyano-4- hydroxybromobenzene

(D) 6-Bromo-3-hydroxybenzonitrile

[Ans. B]

Sol. Cyanide group has higher priority than phenol. Hence correct IUPAC name is
2-Bromo-5-hydroxybenzonitrile

3. Given that the abundances of isotopes ^{54}Fe, ^{56}Fe and ^{57}Fe are 5%, 90% and 5 %, respectively, the atomic mass of Fe is -

(A) 55.85 (B) 55.95

(C) 55.75 (D) 56.05

[Ans. B]

Sol. At. mass of Fe = $\frac{54 \times 5 + 56 \times 90 + 57 \times 5}{100}$

$\therefore$ At. mass of Fe = $\frac{270 + 5040 + 285}{100}$

$= \frac{5595}{100} = 55.95$

4. Among cellulose, poly(vinyl chloride), nylon and natural rubber, the polymer in which the intermolecular force of attraction is weakest is -

(A) Nylon (B) Poly(vinyl chloride)

(C) Cellulose (D) Natural Rubber

[Ans. D]

Sol. Natural rubber is an example of elastomers, which has weakest intermolecular forces (dispersion forces).

5. The Henry's law constant for the solubility of N_2 gas in water at 298 K is 1.0×10^5 atm. The mole fraction of N_2 in air is 0.8. The number of moles of N_2 from air dissolved in 10 moles of water at 298 K and 5 atm pressure is -

(A) 4.0×10^{-4} (B) 4.0×10^{-5}

(C) 5.0×10^{-4} (D) 4.0×10^{-6}

[Ans. A]

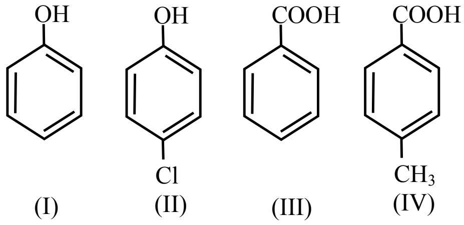
Sol. $p = k_h \times x_2$

$0.8 \times 5 = 10^5 \times \frac{n_{N_2}}{(n_{N_2} + 10)}$

$4 \times 10^{-5} = \frac{n_{N_2}}{n_{N_2} + 10} \simeq \frac{n_{N_2}}{10}$

$n_{N_2} = 4 \times 10^{-4}$

6. The correct acidity order of the following is -



(A) (III) > (IV) > (II) > (I)
(B) (IV) > (III) > (I) > (II)
(C) (III) > (II) > (I) > (IV)
(D) (II) > (III) > (IV) > (I)

[Ans. A]

Sol. Benzoic acid (–COOH) is more acidic than phenol. Electron withdrawing group increase acidic nature & electron donating group decrease acidity.

7. Among the electrolytes Na_2SO_4, $CaCl_2$, $Al_2(SO_4)_3$ and NH_4Cl, the most effective coagulating agent for Sb_2S_3 sol is -

(A) Na_2SO_4
(B) $CaCl_2$
(C) $Al_2(SO_4)_3$
(D) NH_4Cl

[Ans. C]

Sol. Sb_2S_3 is negatively charged sols. Because of the formation higher positive charge ions coagulating power of $Al_2(SO_4)_3$ is the highest

8. The reaction of P_4 with X leads selectively to P_4O_6. The X is -

(A) Dry O_2
(B) A mixture of O_2 and N_2
(C) Moist O_2
(D) O_2 in the presence of aqueous NaOH

[Ans. B]

Sol. $P_4 + O_2$ (mixed with N_2) $\rightarrow P_4O_6$

P_4 is converted to P_4O_6 in the presence of limited supply of oxygen.

SECTION – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

9. The compound(s) formed upon combustion of sodium metal in excess air is(are) -

(A) Na_2O_2
(B) Na_2O
(C) NaO_2
(D) NaOH

[Ans. A, B]

Sol. Sodium on combustion mainly form peroxide and little amount of oxide.

10. The correct statement(s) regarding defects in solids is(are) -

(A) Frenkel defect is usually favoured by a very small difference in the size of cation and anion

(B) Frenkel defect is a dislocation defect

(C) Trapping of an electron in the lattice leads to the formation of F-center

(D) Schottky defects have no effect on the physical properties of solids

[Ans. B, C]

Sol. (B) In Frenkel defect an atom or ion leaves it's normal position & occupies interstitial voids.

(C) e^- trapped in anion vacancy is called F-center.

11. The compound(s) that exhibit(s) geometrical isomerism is(are)

(A) $[Pt(en)Cl_2]$ (B) $[Pt(en)_2]Cl_2$

(C) $[Pt(en)_2Cl_2]Cl_2$ (D) $[Pt(NH_3)_2Cl_2]$

[Ans.C,D]

Sol. Both (C) and (D) have cis- & trans-isomers.

12. The correct statement(s) about the compound $H_3C(HO)HC–CH=CH–CH(OH)CH_3$ (X) is (are)

(A) The total number of stereoisomers possible for X is 6

(B) The total number of diastereomers possible for X is 3

(C) If the stereochemistry about the double bond in X is trans, the number of enantiomers possible for X is 4

(D) If the stereochemistry about the double bond in X is cis, the number of enantiomers possible for X is 2

[Ans. A, D]

Sol. No. of stereoisomers = 6

In cis only 2 isomers are optically active (enantiomers).

SECTION – III
Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 13 to 15

p-Amino-N, N-dimethylaniline is added to a strongly acidic solution of X. The resulting solution is treated with a few drops of aqueous solution of Y to yield blue coloration due to the formation of methylene blue. Treatment of the aqueous solution of Y with the reagent potassium hexacyanoferrate (II) leads to the formation of an intense blue precipitate. The precipitate dissolves on excess addition of the reagent. Similarly, treatment of the solution of Y with the solution of potassium hexacyanoferrate (III) leads to a brown coloration due to the formation of Z.

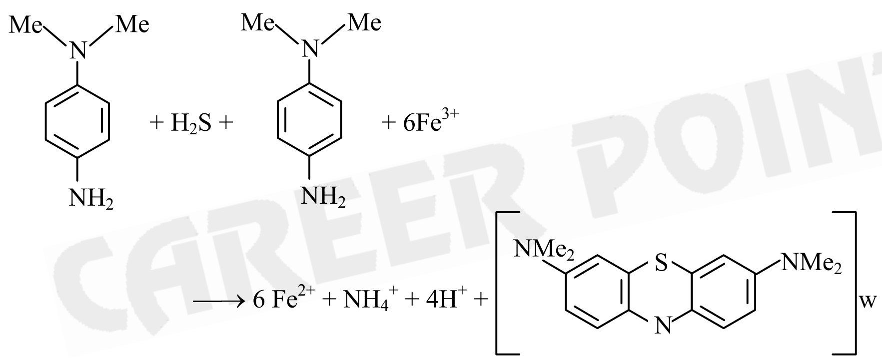
13. The compound X is -

(A) $NaNO_3$ (B) NaCl

(C) Na_2SO_4 (C) Na_2S

[Ans.D]

Sol. $S^{2-} + 2H^+$ (from acid) $\rightarrow H_2S$



Blue Solution

14. The compound Y is –

(A) $MgCl_2$ (B) $FeCl_2$ (C) $FeCl_3$ (D) $ZnCl_2$

[Ans.C]

Sol.

15. The compound Z is –

(A) Mg_2 [$Fe(CN)_6$] (B) $Fe[Fe(CN)_6]$ (C) Fe_4 [$Fe(CN)_6]_3$ (D) $K_2Zn_3[Fe(CN)_6]_2$

[Ans.B]

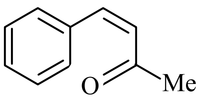
Sol. $Fe^{3+} + [Fe(CN)_6]^{3-} \longrightarrow Fe[Fe(CN)_6]$; Brown Colouration

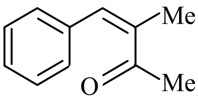
Paragraph for Question Nos. 16 to 18

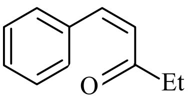
A carbonyl compound P, which gives positive iodoform test, undergoes reaction with MeMgBr followed by dehydration to give an olefin Q. Ozonolysis fo Q leads to a dicarbonyl compound R, Which undergoes intramolecular aldol reaction to give predominantly S.

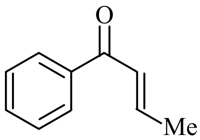
$$P \xrightarrow[\substack{2.\,H^{+},H_2O \\ 3.\,H_2SO_4,\Delta}]{1.\,MeMgBr} Q \xrightarrow[2.\,Zn,\,H_2O]{1.\,O_3} R \xrightarrow[2.\,\Delta]{1.\,OH^{-}} S$$

16. The structure of the carbonyl compound P is –

(A) 

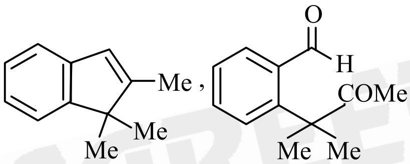
(B) 

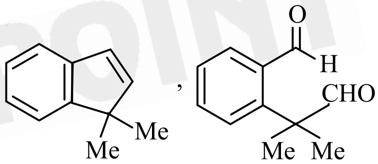
(C) 

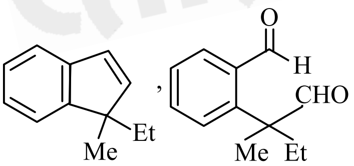
(D) 

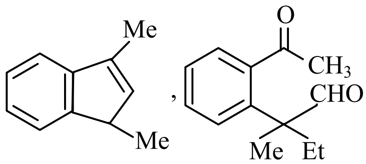
[Ans.B]

17. The structures of the products Q and R, respectively, are –

(A) 

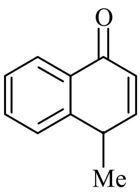
(B) 

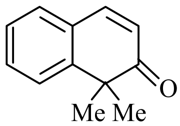
(C) 

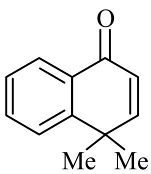
(D) 

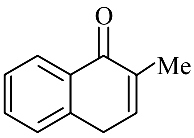
[Ans. A]

18. The structure of the product S is –

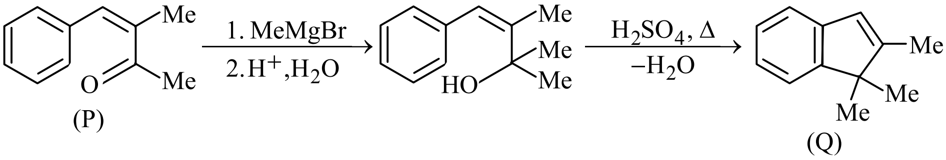
(A) 

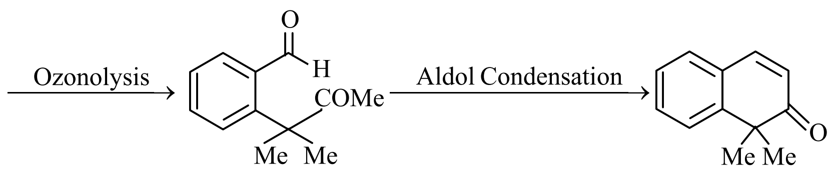
(B) 

(C) 

(D) 

[Ans. B]

Sol. 



(R) (S)

SECTION – IV
Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labeled p, q, r, s and t. any given statement in **Column I** can have correct matching with **ONE OR MORE** statements (s) in **column II**. The appropriate bubbled corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.

	p	**q**	**r**	**s**	**t**
A	●	(q)	(r)	●	●
B	(p)	●	●	(s)	(t)
C	●	●	(r)	(s)	(t)
D	(p)	(q)	(r)	●	●

19. Match each of the diatomic molecules in **Column I** with its property/properties in **Column II**.

Column I	**Column II**
(A) B_2	(p) Paramagnetic
(B) N_2	(q) Undergoes oxidation
(C) O_2^-	(r) Undergoes reduction
(D) O_2	(s) Bond order ≥ 2
	(t) Mixing of 's' and 'p' orbitals

[Ans. (A) → p,q, r,t (B) → q, r, s, t; (C) → p, r ; (D) → p, q, r, s]

Sol. (A) B_2 : Total no. of e^-s = 10

$$\sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2p_y}^1 = \pi_{2p_z}^1$$

Paramagnetic ; BO = $\dfrac{(6-4)}{2} = 1$

(B) N_2 : N = 14

$$\sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2p_y}^2 = \pi_{2p_z}^2\ \sigma_{2p_x}^2$$

Diamagnetic ; BO = $\dfrac{(10-4)}{2} = 3$

(C) O_2^- : N = 1 7

$\sigma_{1s}^2\, \sigma_{1s}^{*2}\, \sigma_{2s}^2\, \sigma_{2s}^{*2}\, \sigma_{2p_x}^2\, \pi_{2p_y}^2 = \pi_{2p_z}^2\;\; \pi_{2p_y}^{*2} = \pi_{2p_z}^{*1}$

$BO = \frac{(10-7)}{2} = \frac{3}{2}$; paramagnetic

O_2^- is an oxidizing agent.

(D) O_2 : N = 16

$\sigma_{1s}^2\, \sigma_{1s}^{*2}\, \sigma_{2s}^2\, \sigma_{2s}^{*2}\, \sigma_{2p_x}^2\, \pi_{2p_y}^2 = \pi_{2p_z}^2\;\; \pi_{2p_y}^{*1} = \pi_{2p_z}^{*1}$

$BO = \frac{(10-6)}{2} = 2$; paramagnetic

20. Match each of the compounds in **Column I** with its characteristic reaction(s) in **Column II**.

Column I	**Column II**
(A) $CH_3CH_2CH_2CN$	(p) Reduction with H_2/Pd-C
(B) $CH_3CH_2OCOCH_3$	(q) Reduction with $SnCl_2$/HCl
(C) $CH_3 - CH = CH - CH_2OH$	(r) Development of foul smell on treatment with chloroform and alcoholic KOH
(D) $CH_3CH_2CH_2CH_2NH_2$	(s) Reduction with disobutylaluminium hydride (DIBAL-H)
	(t) Alkaline hydrolysis

[Ans. (A) → q, s, t; (B) → s, t ; (C) → p ; (D) → r]

Part – II (MATHEMATICS)

SECTION – I

Single Correct Choice Type **12/04/09**

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

21. Tangents drawn from the point P(1, 8) to the circle

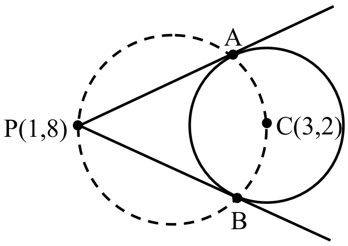
$$x^2 + y^2 - 6x - 4y - 11 = 0$$

touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$

Sol. **[B]**



$\because$ Points PACB are concyclic.

$x^2 + y^2 - 6x - 4y - 11 = 0$ Q

Required equation of circle is given by

$(x - 1)(x - 3) + (y - 8)(y - 2) = 0$

$\Rightarrow x^2 + y^2 - 4x - 10y + 19 = 0$

22. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55 (B) 66 (C) 77 (D) 88

Sol. **[C]**

A + B + C + D + E + F + G = 10

The required solution is

= collect coeff of x^{10} in $(x^1 + x^2 + x^3)^7$

$= {}^9C_3 - 7.\ {}^{7+0-1}C_0$

= 77

Alternate:

Case – (1) : 1111123 $\rightarrow \frac{(7!)}{(5!)} = 42$

Case- (2) : 1111 222 $\rightarrow \frac{(7!)}{(4!)(3!)} = 35$ So total = 77

23. Let P(3, 2, 6) be a point in space and Q be a point on the line

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

Sol. **[A]**

P (3, 2, 6)

$Q \equiv (1 - 3\mu, \mu - 1, 2 + 5\mu)$

$\overline{PQ} = (-3\mu - 2, \mu - 3, 5\mu - 4)$

D.R. of plane is (1, –4, 3)

$(-3\mu - 2) + (-4\mu + 12) + (15\mu - 12) = 0$

$8\mu = 2 \Rightarrow \mu = \frac{1}{4}$

Alternative

[A] $\overrightarrow{PQ} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$

$- (3\hat{i} + 2\hat{j} + 6\hat{k})$

$\vec{n} = \hat{i} - 4\hat{j} + 3\hat{k}$

Dot product = 0 $\Rightarrow 1 + 4 + 6 + \mu(-3 - 4 + 15)$

$- (3 - 8 - 18)$

$\Rightarrow 11 + 8\mu - 13 = 0$

$\Rightarrow 8\mu = 2$

$\mu = \frac{1}{4}$

24. Let z = cos θ + i sin θ. Then the value of

$$\sum_{m=1}^{15} \text{Im}(z^{2m-1})$$

at θ = 2° is

(A) $\frac{1}{\sin 2°}$ (B) $\frac{1}{3\sin 2°}$ (C) $\frac{1}{2\sin 2°}$ (D) $\frac{1}{4\sin 2°}$

Sol. **[D]**

$Z = \cos\theta + i\sin\theta$

$Z^{2m-1} = \cos(2m-1)\theta + i\sin(2m-1)\theta$

$\sum \text{Im}\, Z^{2m-1} = \sin\theta + \sin 3\theta + \ldots + \sin 29\theta$

$= \dfrac{\sin\left(15.\frac{2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} \sin\left(\theta + (15 - 1)\frac{2\theta}{2}\right)$

$= \dfrac{(\sin(15\theta))^2}{\sin\theta} = \dfrac{1}{4\sin 2^o} \quad \forall\ \theta = 2^o$

25. Let z = x + iy be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation

$$z\bar{z}^3 + \bar{z}z^3 = 350$$

is

(A) 48 (B) 32 (C) 40 (D) 80

Sol. **[A]**

Let $Z = x + iy$

$\therefore$ $(x^2 + y^2)(x^2 - y^2 - 2ixy) + (x^2 + y^2)(x^2 - y^2 + 2ixy) = 350$

$\Rightarrow$ $2(x^2 + y^2)(x^2 - y^2) = 350$

$\Rightarrow$ $(x^2 + y^2)(x^2 - y^2) = 175$

$\therefore$ $x^2 + y^2 = 25$

$x^2 - y^2 = 7$

On solving

$x = \pm 4$

$y = \pm 3$

$\therefore$ Area of rectangle = 8 × 6 = 48

26. If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are unit vectors such that

$$(\vec{a}\times\vec{b}).\,(\vec{c}\times\vec{d}) = 1$$

and $\vec{a}\,.\,\vec{c} = \frac{1}{2}$,

then

(A) $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar

(B) $\vec{b}$, $\vec{c}$, $\vec{d}$ are non-coplanar

(C) $\vec{b}$, $\vec{d}$ are non- parallel

(D) $\vec{a}$, $\vec{d}$ are parallel and $\vec{b}$, $\vec{c}$ are parallel

Sol. **[C]**

$(\vec{a}\times\vec{b})\,.\,(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{a}.\vec{c} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix} = 1$

$\Rightarrow$ $\begin{vmatrix} \frac{1}{2} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix} = 1$

$\Rightarrow$ $\frac{1}{2}\,(\vec{b}.\vec{d}) - (\vec{a}.\vec{d})\,(\vec{b}.\vec{c}) = 1$

$(|\vec{a}||\vec{b}|\sin\theta_1\hat{n}_1)\,.\,(|\vec{c}||\vec{d}|\sin\theta_2\hat{n}_2) = 1$

$\Rightarrow$ $\sin\theta_1\sin\theta_2\ \hat{n}_1.\hat{n}_2 = 1$

$\theta_1 = \theta_2 = \frac{\pi}{2};\ \alpha = 0$

$\Rightarrow$ $\vec{a}\perp\vec{b}$ & $\vec{c}$ & $\vec{d}$ & $\vec{n}_1 \,||\, \vec{n}_2$

27. Let f be a non-negative function defined on the interval [0, 1]. If

$$\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt,\ 0 \le x \le 1,$$

and f(0) = 0, then

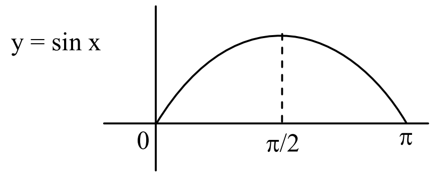
(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Sol. **[C]**



$$\int_0^x \sqrt{1-(f'(t))^2}\ dt = \int_0^x f(t)\ dt$$

On differentiating

$\sqrt{1-(f'(x))^2} = f(x)$

On solving $f'(x) = \sqrt{1-(f(x))^2}$

$f(x) = \sin x$

$\because \sin x < x,\ \forall\ x > 0$

So $f\left(\frac{1}{2}\right) < \frac{1}{2}$

$f\left(\frac{1}{3}\right) < \frac{1}{3}$

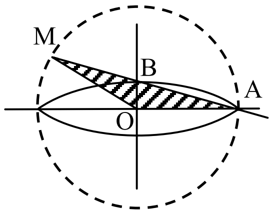
28. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse

$$x^2 + 9y^2 = 9$$

meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$ (C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Sol. **[D]**



equation of AB is x = 3 –3y

equation of auxiliary circle

$x^2 + y^2 = 9$

on solving M = $\left(-\frac{12}{5}, \frac{9}{5}\right)$

Area of Δ OAM = $\frac{1}{2}$(OA) (y_m)

$$= \frac{1}{2}(3)\left(\frac{9}{5}\right) = \frac{27}{10}$$

SECTION – II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONE OR MORE is/are correct.

29. In a triangle ABC with fixed base BC, the vertex A moves such that

$$\cos B + \cos C = 4\sin^2\frac{A}{2}$$

If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, then

(A) b + c = 4a
(B) b + c = 2a
(C) locus of point A is an ellipse
(D) locus of point A is a pair of straight lines

Sol. **[B, C]**

$\cos B + \cos C = 4\sin^2\frac{A}{2}$

$\Rightarrow 2\cos\frac{B+C}{2}\cos\frac{B-C}{2} = 4\sin^2\frac{A}{2}$

$\Rightarrow \cos\frac{B-C}{2} = 2\cos\frac{B+C}{2}$ $\left(\because \sin\frac{A}{2} = \cos\frac{B+C}{2}\right)$

$\Rightarrow \frac{\cos\frac{B-C}{2}}{\cos\frac{B+C}{2}} = 2$

$\Rightarrow$ by componenedo and dividendo

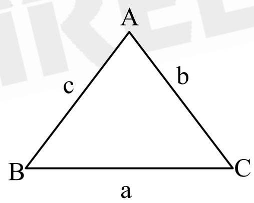
$\frac{\cos\frac{B-C}{2} + \cos\frac{B+C}{2}}{\cos\frac{B-C}{2} - \cos\frac{B+C}{2}} = \frac{3}{1}$

$\Rightarrow \frac{2\cos\frac{B}{2}\cos\frac{C}{2}}{2\sin\frac{B}{2}\sin\frac{C}{2}} = 3$

$\Rightarrow \tan\frac{B}{2}\tan\frac{C}{2} = \frac{1}{3} \Rightarrow \frac{s-a}{s} = \frac{1}{3}$

$\Rightarrow 3s = s + 3a$

$\Rightarrow b + c = 2a$



AB + AC > BC

$\Rightarrow$ Locus of A is ellipse.

30. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

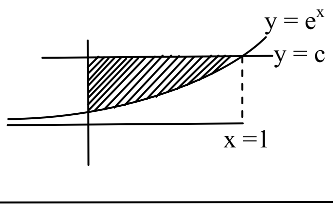
(A) $e - 1$

(B) $\int_1^e \ell n(e+1-y)\,dy$

(C) $e - \int_0^1 e^x dx$

(D) $\int_1^e \ell n\, y\, dy$

Sol. **[B, C, D]**



$y = e^x$

Area $= \int_1^e (\ell ny)\ dy$

$\int_1^e \ell n(1+e-y)\ dy$

$= \left|y(\ell ny - 1)\right|_1^e$

$= 0 - (-1) = 1$

(C) $e - \int_0^1 e^x\ dx$

$= e - |e^x|_0^1 = e - [e-1] \quad = 1$

31. Let

$$L = \lim_{x\to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$$

If L is finite, then

(A) a = 2 (B) a = 1 (C) $L = \frac{1}{64}$ (D) $L = \frac{1}{32}$

Sol. **[A,C]**

$$L = \lim_{x\to 0} \frac{\frac{x^2}{a+\sqrt{a^2-x^2}} - \frac{x^2}{4}}{x^4}$$

$L = \lim_{x\to 0} \frac{4-(a+\sqrt{a^2-x^2})}{4(a+\sqrt{a^2-x^2})x^2}$ $\because$ L is finite so it should be $\frac{0}{0}$ form

So $4 - (2a) = 0 \quad \Rightarrow \ a = 2$

using a = 2

$$L = \lim_{x\to 0} \frac{2-\sqrt{4-x^2}}{4(2+\sqrt{4-x^2})x^2} = \frac{2-\sqrt{4-x^2}}{4(2+\sqrt{4-x^2})x^2} \times \frac{2+\sqrt{4-x^2}}{2+\sqrt{4-x^2}}$$

$$= \lim_{x\to 0} \frac{4-4+x^2}{4(2+\sqrt{4-x^2})x^2.(2+\sqrt{4-x^2})}$$

$$= \frac{1}{16(4)} = \frac{1}{64}$$

32. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$ then

(A) $\tan^2 x = \frac{2}{3}$

(B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(C) $\tan^2 x = \frac{1}{3}$

(D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

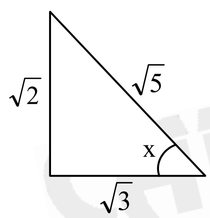
Sol. **[A, B]**

$\frac{(\sin x)^4}{2} + \frac{(\cos x)^4}{3} = \frac{1}{5}$

$3 - 6\cos^2 x + 5(\cos x)^4 = \frac{6}{5}$ Let $\cos x = t$

$25t^4 - 30t^2 + 9 = 0$

$t^2 = \frac{3}{5}$,



$\tan^2 x = \frac{2}{3}$

$(\sin x)^8 = \left(\frac{2}{5}\right)^4 = \frac{16}{625}$

$(\cos x)^8 = \left(\frac{\sqrt{3}}{\sqrt{5}}\right)^4 = \frac{81}{625}$

$\frac{(\sin x)^8}{8} + \frac{(\cos x)^8}{27} = \frac{1}{125}$

SECTION – III

ComprehensionType

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Paragraph for Question Nos. 33 to 35

Let $\mathscr{A}$ be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

33. The number of matrices in $\mathscr{A}$ is

(A) 12 (B) 6 (C) 9 (D) 3

Sol. **[A]**

$$\begin{bmatrix} A & D & E \\ D & B & F \\ E & F & C \end{bmatrix}$$

case (1) : any two of D, E, F are 1

$^3C_2 . 3$

Case (2) : any two of D, E, F are 0

$^3C_2$

So number of matrices $^3C_2 . 3 + {}^3C_2 = 12$

34. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is

(A) less than 4 (B) at least 4 but less than 7

(C) at least 7 but less than 10 (D) at least 10

Sol. **[B]**

$$A = \begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix}$$

$|A| \neq 0$

$|A| = abc - af^2 - cd^2 + 2def - e^2 b \neq 0$

if case (1) $af = 1 \Rightarrow d = 1$ or $e = 1$

case (2) $cd = 1 \Rightarrow e = 1$ or $f = 1$

case (3) $eb = 1 \Rightarrow d = 1$ or $e = 1$

So total six cases occurs.

35. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent, is

(A) 0 (B) More than 2 (C) 2 (D) 1

Sol. **[B]**

All possible cases are

(I) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (II) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (III) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (IV) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (V) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (VI) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

(VII) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (VIII) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (IX) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (X) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (XI) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (XII) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Case V, VII, XI, XII Satisfies for which $|A| = 0$ & at least one of Δ_1 or Δ_2 or Δ_3 are non zero

Paragraph for Question Nos. 36 to 38

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

36. The probability that X = 3 equals

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

Sol. **[A]**

$P(x = 3) \rightarrow$ probability of 3 tosses required

$$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

37. The probability that X ≥ 3 equals

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

Sol. **[B]**

$P(x \geq 3) = 1 - \{P(x = 1) + P(x = 2)\}$

$= 1 - \left(\frac{1}{6} + \frac{5}{6} \times \frac{1}{6}\right)$

$= 1 - \left(\frac{1}{6} + \frac{5}{36}\right) = 1 - \frac{11}{36} = \frac{25}{36}$

38. The conditional probability that X ≥ 6 given X > 3 equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

Sol. **[D]**

$P\left(\frac{x \geq 6}{x > 3}\right) = \frac{P(x \geq 6)}{P(x > 3)}$

$P(x > 3) = 1 - \{P(x = 1) + P(x = 2) + P(x = 3)\}$

$= 1 - \left(\frac{1}{6} + \frac{5}{36} + \frac{25}{216}\right)$

$= 1 - \left\{\frac{36 + 30 + 25}{216}\right\}$

$= 1 - \frac{91}{216} = \frac{125}{216}$

$P(x \geq 6) = \left(\frac{5}{6}\right)^5 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \left(\frac{5}{6}\right)^7 \frac{1}{6} \ldots\ldots\ldots$

$= \left(\frac{5}{6}\right)^5 \frac{1}{6} \left\{1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots\ldots\right\}$

$= \left(\frac{5}{6}\right)^5 \frac{1}{6} \left\{\frac{1}{1 - \frac{5}{6}}\right\} = \frac{5^5}{6^5}$

$P\left(\frac{x \geq 6}{x > 3}\right) = \frac{P(x \geq 6)}{P(x > 3)}$

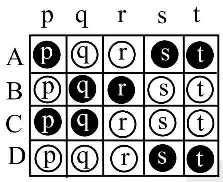
$P = \frac{5^5 / 6^5}{5^3 / 6^3} = \frac{5^2}{6^2} = \frac{25}{36}$

SECTION – IV

Matrix- Match Type

This section contains 2 questions. Each questions contains statements given in two columns, which have to be matched. The statements in Column I are labelled A, B, C and D, while the statements in Column II are labeled p, q, r, s and t. Any given statement in Column I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C –p and q ; and D –s and t; then the correct darkening of bubbles will look like the following.



39. Match the statements/ expressions in Column I with the open intervals in Column- II

Column – I	Column – II
(A) Interval contained in the domain of definition of non- zero solutions of the differential equations $(x-3)^2 y' + y = 0$	(p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	(q) $\left(0, \frac{\pi}{2}\right)$
	(r) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(s) $\left(0, \frac{\pi}{8}\right)$
(D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	(t) $(-\pi, \pi)$

Sol. **A → p, q, s; B → p,t; C → p, q, r, t; D → s;**

$A \rightarrow (x-3)^2 \frac{dy}{dx} + y = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{(x-3)^2}$

or $\frac{dy}{y} = -\frac{1}{(x-3)^2}dx$

$\Rightarrow$ $\ell n\ y = \frac{1}{(x-3)}$ $(x \neq 3)$

$B \to I = \int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)$

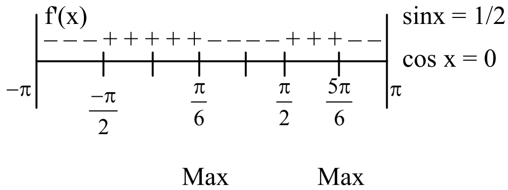
$\Rightarrow$ $I = -I$

$\Rightarrow$ $I = 0$

$C \to f(x) = \cos^2 x + \sin x$

$\Rightarrow$ $f'(x) = -2\cos x \sin x + \cos x$

$= \cos x(-2\sin x + 1)$



$\sin x + \cos x \to (\cos x - \sin x), \cos x > \sin x$

40. Match the conics in Column I with the statements/ expressions in Column- II

Column – I		**Column – II**
(A)	Circle	(p) The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
(B)	Parabola	(q) Points z in the complex plane satisfying $\lvert z + 2\rvert - \lvert z - 2\rvert = \pm 3$
(C)	Ellipse	(r) Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$
(D)	Hyperbola	(s) The eccentricity of the conic lies in the interval $1 \le x < \infty$
		(t) Points z in the complex plane satisfying $\text{Re}(z+1)^2 = \lvert z\rvert^2 + 1$

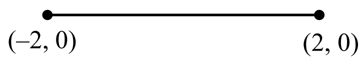
Sol. **A → p; B → s,t; C → r; D → q, s;**

(p) $\frac{1}{\sqrt{h^2 + k^2}} = 2$

$\sqrt{h^2 + k^2} = \frac{1}{2}$

$x^2 + y^2 = \frac{1}{4}$ Circle

(q) $| |z + 2| - |z - 2| = 3$



hyperbola having focus (–2, 0) and (2, 0)

(r) $\frac{x}{\sqrt{3}} = \frac{1 - t^2}{1 + t^2}$, $y = \frac{2t}{1 + t^2}$

$\frac{x^2}{3} + y^2 = 1$ ellipse

(s) Parabola, hyperbola

(t) Let $z = x + iy$

$(x + 1)^2 - y^2 = x^2 + y^2 + 1$

$2x - y^2 = y^2$

$y^2 = x$ parabola

Part – II (PHYSICS)

SECTION – I

Straight Objective Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

41. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0º. Then

(A) at θ = 30º, the block will start sliding down the plane

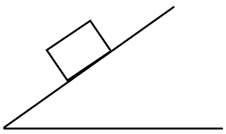
(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at θ = 60º, the block will start sliding down the plane and continue to do so at higher angles

(D) at θ = 60º, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

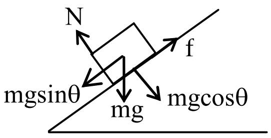
[Ans.B]

Sol.



* At θ = 60º, block on verge of slipping

* At angle θ < 60º



Torque about C.M. due to N

$\tau_N = N \times (0.05)$

$= mg\cos\theta(0.05)$

Torque about C.M. due to f

$\tau_f = mg\sin\theta \times (0.75)$

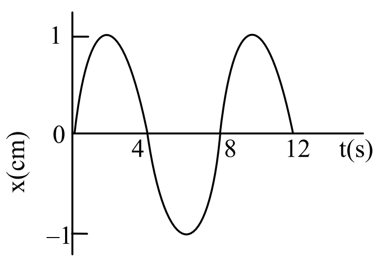
$= mg\sin\theta(0.075)$

If $\tau_f > \tau_N$ then block will topple for line

$mg\cos\theta(0.05) < mg\sin\theta(0.075)$

$\tan\theta > \frac{0.05}{0.075}$; $\tan\theta > \frac{2}{3}$

42. The x-t graph of a particle undergoing simple harmonic is shown below. The acceleration of the particle at t = 4/3 s is-



(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 (B) $\frac{-\pi^2}{32}$ cm/s^2 (C) $\frac{\pi^2}{32}$ cm/s^2 (D) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

[Ans.D]

Sol. A = 1 cm

$x = A\sin\frac{2\pi}{T}t = A\sin\frac{\pi}{4}t$ $(\therefore T = 8 \text{ s})$

$$\frac{d^2x}{dt^2} = -\frac{\pi^2}{16}A\sin\frac{\pi}{4}t$$

$$= -\frac{\pi^2}{16}\times 1\times\sin\frac{\pi}{4}\times\frac{4}{3}$$

$$= -\frac{\pi^2}{16}\times\frac{\sqrt{3}}{2}$$

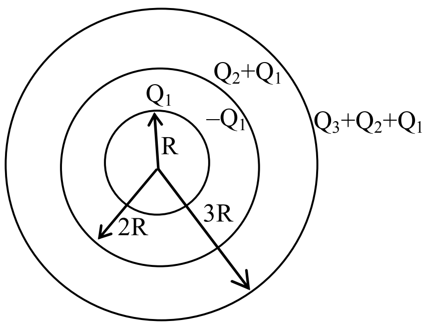
$$= \frac{-\sqrt{3}\,\pi^2}{32}$$

43. Three concentric metallic spherical shells of radii R, 2R, 3R are given charges Q_1, Q_2, Q_3 respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$ is-

(A) 1 : 2 : 3 (B) 1 : 3 : 5 (C) 1 : 4 : 9 (D) 1 : 8 : 18

[Ans.B]

Sol.



$$\frac{Q_3+Q_2+Q_1}{4\pi(3R)^2} = \frac{Q_2+Q_1}{4\pi(2R)^2} = \frac{Q_1}{4\pi R^2}$$

$$\frac{Q_3+Q_2+Q_1}{9} = \frac{Q_2+Q_1}{4} \qquad ......(1)$$

$$\frac{Q_2+Q_1}{4} = \frac{Q_1}{1} \qquad .....(2)$$

$Q_2 + Q_1 = 4Q_1$

$Q_2 = 3Q_1$ ....(3)

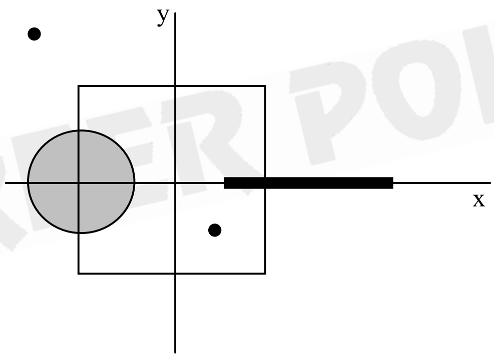
$$\frac{Q_3 + 3Q_1 + Q_1}{9} = \frac{Q_1}{1}$$

$Q_3 + 4Q_1 = 9Q_1$

$Q_3 = 5Q_1$

$Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

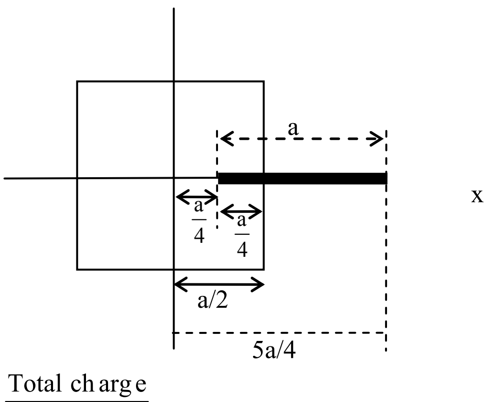
44. A disk of radius a/4 having a uniformly distributed charge 6C is placed in the x-y plane with its centre at (–a/2, 0, 0). A rod of length 'a' carrying a uniformly distributed charge 8C is placed on the x-axis from x = a/4 to x = 5a/4. Two point charges –7C and 3C are placed at (a/4, –a/4, 0) and (–3a/4, 3a/4, 0) respectively. Consider a cubical surface formed by six surfaces x = ± a/2, y = ±a/2, z = ±a/2. The electric flux this cubical surface is-



(A) $\frac{-2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$ (C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$

[Ans.A]

Sol.



$$\text{Flux} = \frac{\text{Total charge}}{\varepsilon_0}$$

Total charge = half of charge of disk + $\frac{1}{4}$th of charge of rod + (–7C)

$= 3 + \frac{1}{4} \times 8 - 7$

$= 3 + 2 - 7$

$= -2C$

Flux = $\frac{-2C}{\varepsilon_0}$

45. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two lines segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are : outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y-coordinate of the centre of mass of the ink in this drawing is-

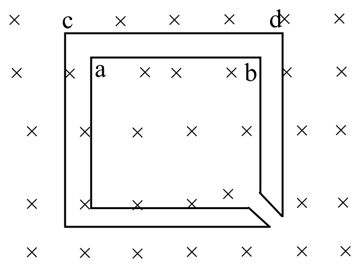
(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

[Ans.A]

Sol. y-coordinate of C.M. of system is given by

$$y_{CM} = \frac{m\times a + m\times a + 6m\times 0 + m\times 0 + m\times(-a)}{10\,m} = \frac{a}{10}$$

46. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then-



(A) $I_1 > I_2$

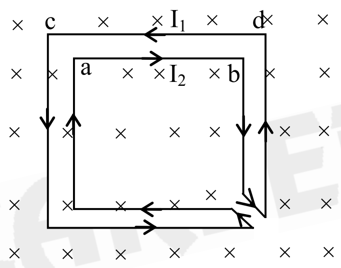
(B) $I_1 < I_2$

(C) I_1 is in the direction **ba** and I_2 is in the direction **cd**

(D) I_1 is in the direction **ab** and I_2 is in the direction **dc**

[Ans.D]

Sol.



There are two rectangles

emf induced = Area × $\frac{dB}{dt}$

Area of outside rectangle is more therefore emf will be more induced in outer rectangle.

∴ current will flow according to outer rectangle i.e. from D to C

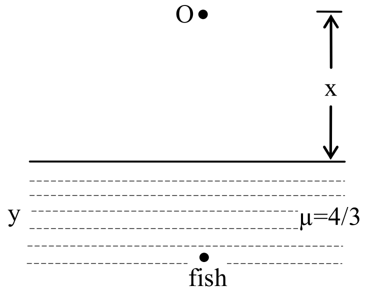
Here $I_1 = I_2$ as same current will flow in the whole rectangle.

47. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as-

(A) 9 m/s (B) 12 m/s (C) 16 m/s (D) 21.33 m/s

[Ans.C]

Sol.



Distance of image of O from fish

$S = \mu x + y$

$\frac{dS}{dt} = \mu \frac{dx}{dt}$

$= \frac{4}{3} \times \frac{dx}{dt}$

Velocity of bird at height 12.8

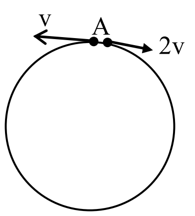
$m \times 10 \times 20 = m \times 10 \times 12.8 + \frac{1}{2} mv^2$ (Energy conservation)

$\sqrt{10 \times \left(\frac{7.2}{10}\right) \times 2} = v$

$v = 12$

$\frac{dx}{dt} = 12$ $\therefore \frac{dS}{dt} = \frac{4}{3} \times 12 = 16$ m/s

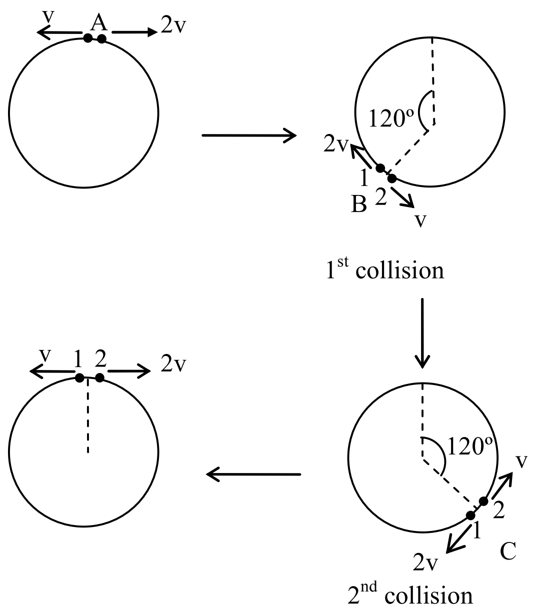
48. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many collisions, other than that at A, these two particles will again reach the point A ?



(A) 4 (B) 3 (C) 2 (D) 1

[Ans.C]

Sol.



First collision will occur at point 'B' as shown in figure. Velocity of balls 1 and 2 will get inter-changed and second collision will occur at point 'C'. Again the velocity of balls 1 and 2 will get inter-changed and the two balls will collide again at 'A'.

SECTION – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

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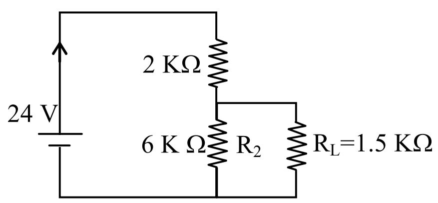
49. For the circuit shown in the figure



(A) the current I through the battery is 7.5 mA

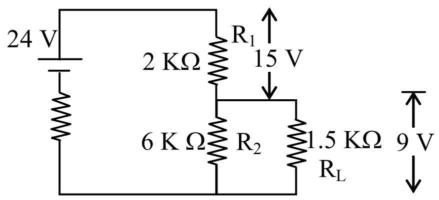
(B) the potential difference across R_L is 18 V

(C) ratio of powers dissipated in R_1 and R_2 is 3

(D) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_1, will decrease by a factor of 9.

[Ans. A, D]

Sol.



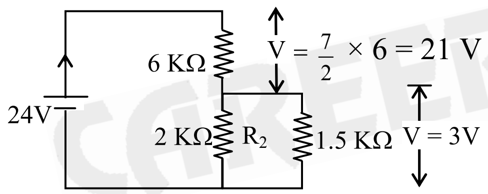
$R_{eq} = \frac{6 \times 1.5}{6 + 1.5} + 2 = \frac{16}{5}$ KΩ

$I = \frac{V}{R_{eq}} = \frac{24 \times 5}{16} = 7.5$ mA

Power dissipated in R_L initially (before interchange)

$P_{R_L} = \frac{V^2}{R_L} = \frac{(9)^2}{1.5} = 54$

When the resistors were interchanged



$R_{eq} = \frac{2 \times 1.5}{2 + 1.5} + 6 = \frac{48}{7}$

$I = \frac{V}{R_{eq}} = \frac{24 \times 7}{48} = \frac{7}{2}$

P_{R_L} (new) $= \frac{V^2}{R_L} = \frac{(3)^2}{1.5} = 6$

The ratio of P_{R_L} / P_{R_L} (new) = 54/6 = 9 times

50. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(A) linear momentum of the system does not change in time

(B) kinetic energy of the system does not change in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does not change in time

[Ans. A]

Sol. $\sum \vec{F}_{ext} = \frac{d\vec{p}}{dt}$

$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$

If net external force in zero, net external torque may of may not be zero

$\therefore \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p}$ doesn't change with time.

Change in potential energy of kinetic energy is related to work done by external as wall as internal forces, hence if net external force is zero P.E. or K.E. may or may not change.

51. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is(are) incorrectly recorded, is (are)

(A) (42, 56) (B) (48, 48) (C) (66, 33) (D) (78, 39)

[Ans. C, D]

Sol. By def :

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad ....(ii)$$

Equation (ii) is satisfied by option A & B only

$\therefore$ option C & D are not acceptable.

52. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

(A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_p . C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

[Ans. B, D]

Sol. **For Diatomic :**

$C_P + C_V = \frac{7}{2}R + \frac{5}{2}R = \frac{12R}{2} = 6R$

$\frac{C_P}{C_V} = \frac{7/2R}{5/2R} = \frac{7}{5}$

$C_P.C_V = \frac{35}{4}R^2$

For Monoatomic :

$C_P + C_V = \frac{5}{2} + \frac{3}{2}R = 4R$

$\frac{C_P}{C_V} = \frac{5/2}{3/2} = \frac{5}{3} = 1.66$

$C_P.C_V = \frac{15}{4}R^2$

SECTION – III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 53 to 55

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}_1^2H$, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D-D reaction is ${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n$ + energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}_1^2H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3.

It may be helpful to use the following. Boltzmann constant $k = 8.6\times10^{-5}$ eV/K; $\frac{e^2}{4\pi\varepsilon_0} = 1.44\times10^{-9}$ eVm.

53. In the core of nuclear fusion reactor, the gas becomes plasma because of –

(A) Strong nuclear force acting between the deuterons

(B) Coulomb force acting between the deuterons

(C) Coulomb force acting between deuteron-electron pairs

(D) the high temperature maintained inside the reactor core

[Ans.D]

Sol. Due to high energy thermal collision, deuteron molecules gets fully ionized. Therefore the temperature should be high.

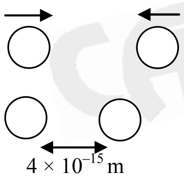
54. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range –

(A) 1.0×10^9 K < T < 2.0×10^9 K (B) 2.0×10^9 K < T < 3.0×10^9 K

(C) 3.0×10^9 K < T < 4.0×10^9 K (D) 4.0×10^9 K < T < 5.0×10^9 K

[Ans.A]

Sol.



: Initial

: Final

Applying energy conservation

$U_i + K_i = U_f + K_f$

$0 + 2 \times 1.5\ kT = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ ($\therefore$ For minimum temperature $K_f = 0$)

$\Rightarrow T = \frac{1.44 \times 10^{-9}}{4 \times 10^{-15}} \times \frac{1}{3 \times 8.6 \times 10^{-5}} = 1.395 \times 10^9$ K

55. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion ?

(A) deuteron density = 2.0×10^{12} cm^{-3}, confinement time = 5.0×10^{-3} s

(B) deuteron density = 8.0×10^{14} cm^{-3}, confinement time = 9.0×10^{-1} s

(C) deuteron density = 4.0×10^{23} cm^{-3}, confinement time = 1.0×10^{-11} s

(D) deuteron density = 1.0×10^{24} cm^{-3}, confinement time = 4.0×10^{-12} s

[Ans.B]

Sol. Lawson number = nt_0 (should be > 5×10^{14} s/cm^3)

In option B : $nt_0 = 7.2 \times 10^{14}$ s/cm^3

Lawson number in option B only is greater than 5×10^{14} s/cm^3

Paragraph for Question Nos. 56 to 58

When a particle is restricted to move along x-axis between x = 0 and x = a, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends x = 0 and x = a. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, ... (n = 1, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line x = 0 to

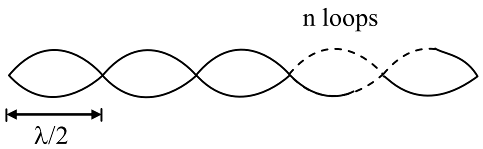
x = a. Take $h = 6.6 \times 10^{-34}$ J-s and $e = 1.6 \times 10^{-19}$ C.

56. The allowed energy for the particle for a particular value of n is proportional to -

(A) a^{-2} (B) $a^{-3/2}$ (C) a^{-1} (D) a^2

[Ans.A]

Sol.



$\frac{n\lambda}{2} = a$

$\lambda = \frac{2a}{n}$

$\lambda = \frac{h}{p}$

$p = \frac{h}{\lambda}$

Energy = $\frac{p^2}{2m} = \frac{h^2}{\lambda^2 \times 2m}$

Energy = $\frac{n^2h^2}{4a^2 \times 2m}$

$\therefore$ Energy $\propto a^{-2}$

57. If the mass of the particle is m = 1.0 × 10^{-30} kg and a = 6.6 nm, the energy of the particle in its ground state is closest to -

(A) 0.8 meV (B) 8 meV (C) 80 meV (D) 800 meV

[Ans.B]

Sol. Energy = $\frac{(6.6)^2 \times 10^{-68}}{4 \times (6.6)^2 \times 10^{-18} \times 2 \times 10^{-30}}$

$= \frac{1}{8} \times \frac{10^{-20}}{1.6 \times 10^{-19}}$ eV

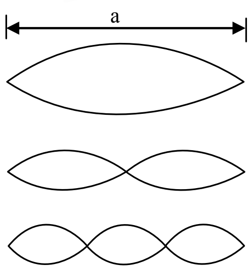
$= \frac{10^{-1}}{8 \times 1.6} = \frac{1}{16 \times 8}$ eV = 8 × 10^{-3} eV = 8 meV

58. The speed of the particle, that can take discrete values, is proportional to -

(A) $n^{-3/2}$ (B) n^{-1} (C) $n^{1/2}$ (D) n

[Ans.D]

Sol.



$\frac{\lambda}{2}$

λ

$\frac{3\lambda}{2}$

For n^{th} state a = $\frac{n\lambda}{2}$

$\lambda = \frac{2a}{n}$

$\frac{h}{\lambda}$ = P (momentum)

$\therefore$ P = $\frac{nh}{2a}$

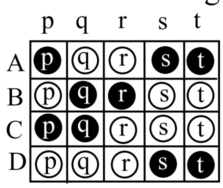
(Momentum) P is proportional to velocity

$\therefore$ Velocity $\propto$ n

SECTION – IV

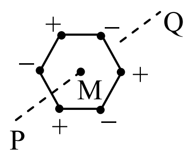
Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement **in Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example.
If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.

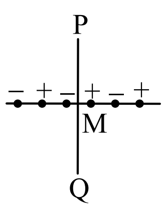


59. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

Column I	**Column II**
(A) $E = 0$	(p) Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.

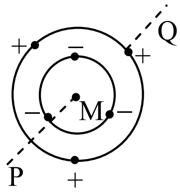


(B) $V \neq 0$	(q) Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.



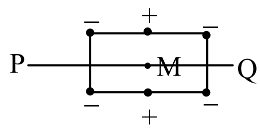
(C) B = 0

(r) Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.

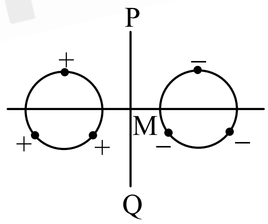


(D) $\mu \neq 0$

(s) Charges are placed at the corners of a rectangle of sides a and 2a and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.



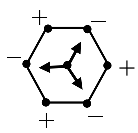
(t) Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.



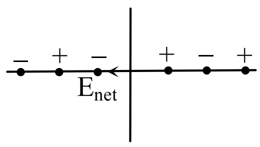
Sol. **A → p, r, s ; B → r, s ; C → p, q, t ; D → r, s**

As option (A) E = 0

(p) E = 0 as

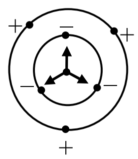


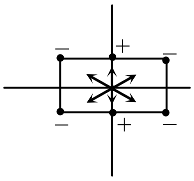
(q) $E_{net} \neq 0$



(r) E due to both + & – charge is zero

∴ E = 0

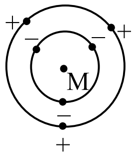


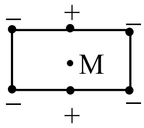
(s) $E = 0$ 

As option (B)

(r) $V_{net} = -ve$

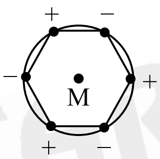
ie $V_{net} \neq 0$



(s) $V_{net} \neq 0$ 

As option (C) & (D)

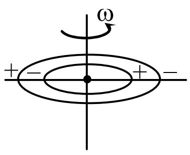
For (p)



during of the system charge particle rotate about PQ in a circle and if total charge is zero then

$B = 0$ as well as $\mu = 0$.

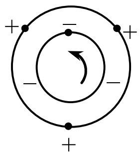
Option (q)



Here various circle are shown because charge are revolving in these circle. Total charge in each circle is zero therefore

$B = 0$ as well as $\mu = 0$

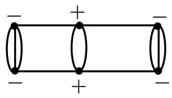
Option (r)



Two circle are shown and B due to both circle charge is in opposite direction.

$\therefore$ However B due to both is not same therefore $B \neq 0$, $\mu \neq 0$

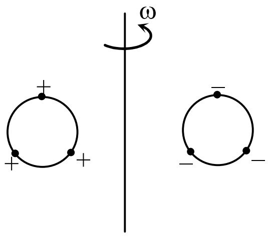
Option (s)



As each moving charge on circle will produce B

$B \neq 0$, $\mu \neq 0$

For option (t)



Net charge on the circle about PQ is zero

$\therefore$ $B = 0$, $\mu = 0$

60. **Column II** shows five systems in which two object are labelled as X and Y. Also in each case a point P is shown. **Column I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II**.

Column I

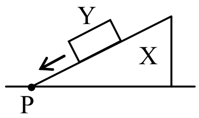
(A) The force exerted by X on Y has a magnitude Mg.

(B) The gravitational potential energy of X continuously increasing.

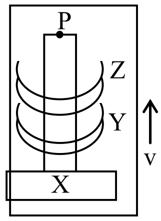
(C) Mechanical energy of the system X + Y

Column II

(p) Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.

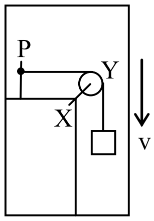


(q) Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.



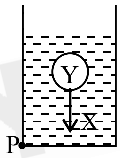
(r) A pulley Y of mass m_0 is fixed to a table is continuously decreasing.

through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.

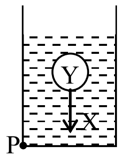


(D) The torque of the weight of Y about point P is zero.

(s) A sphere Y of mass M is put in a X kept in a container at rest. The sphere is released and it moves down in the liquid.



(t) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.



Sol. **A → p,t ; B → q, s, t ; C → p, r, t ; D → ; q**

A → p (As Y moves with constant speed)

t (As Y moves with constant speed)

B → q (As lift is moving up)

s (As liquid is displaced in upward direction)

t (As liquid is displaced in upward direction)

C → p (As Y is moving down and frictional force is acting)

r (Lift is moving down)

t (Y is moving down and viscous force is acting)

D → q (Force passing through point P)