SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.TECH - Common to ALL Branches (Except Bio Groups)
Title of the paper: Engineering Mathematics - I
Semester: I — Max. Marks: 80
Sub.Code: ET102/3ET102A/4ET102A/5ET102A — Time: 3 Hours
Date: 08-12-2008 — Session: FN

PART – A (10 x 2 = 20)

Answer All the Questions

1. Write down the expansion for tan nθ interms of power of tanθ.

2. Prove that $\tan h^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \log x$ for x>0.

3. Find the point where the line
$\frac{x}{-1} = \frac{y-1}{2} = \frac{z+1}{2}$ Meets the plane
$3x - 4y + 7z - 1 = 0$

4. Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 + 6x + 4y - 7z + 1 = 0$, $x + y - 2z + 5 = 0$ and passes through the point (1, -2, 0)

5. Find the rank of $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 2 \\ 3 & -1 & 3 \end{bmatrix}$

6. Find the Eigen values of A^2 given $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

7. Evaluate $\iiint dv$, where v is the region of replace bounded by $x^2+y^2+2^2 = 1$

8. Prove that β (m, n) = β (n, m)

9. Write down the Reduction formula for $\int sin^2 x\, dx$

10. Find the value of $\int_0^{\frac{\pi}{2}} sin^3 x\, cos^{5/2} x\, dx$

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Expand sin 7θ as a polynomial in sin θ, Hence show that Sin π/7 sin 2π/7 sin 3π/7 sin 4π/7 sin 5π/7 sin6π/7 = -7/64

(b) If tan x/2 = tan h y/2, prove that sin hy = tanx and y = log tan $\left(\frac{\pi}{4} + \frac{x}{2}\right)$

(or)

12. (a) Show that

$cos^5\theta sin^4\theta = \frac{1}{2^8}$ [Cos 9θ + cos 7θ - 4cos 5θ - 4cos 3θ + 6cosθ]

(b) Separate $\tan^{-1}(x + iy)$ into real and imaginary parts.

13. (a) The plane 4x +4y -8z + 1 = 0 is rotated through a right angle about the line of intersection with the plane 3x+y-5=0. Find the education of this plane in its new portion.

(b) Find the shortest distance between the lines

$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{2-7}{1}$ And

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ Also find the equation to the line of shortest distance.

(or)

14. (a) Show that the line x+y+z-3=0=2x+3y+4z-5 and 4x-y+5z-7=0=2x-5y-z-3 are coplanar. Find the equation to the plane containing them.

(b) Find the equation to the sphere pacing through the points (0, -2, 3), (1, 5, -1), (2, 0, 1) and (4, -1, 2).

15. (a) Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

(b) Find λ and μ such that

$x+2y+\lambda 2=1$, $x+2\lambda y+z=\mu$,

$\lambda x+2y+z=1$ has (i) no solution, (ii) unique solution (iii) many solution.

(or)

16. (a) Find the Eigen values and Eigen vectors of $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

(b) Reduce the quadratic form $2x^2+5y^2+3z^2+4xy$ to canonical form by an orthogonal reduction and hence find its rank, index, signature and the nature of the quadric form.

17. (a) Prove that $\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta = \frac{\pi}{8}\log 2$

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dy\, dx$ by changing the order of integration.

(or)

18. (a) If $I_n = \int_0^{\frac{\pi}{2}} x^x \cos^s x\, dx$ prove that

$$I_n = \frac{n-1}{n} I_{m-2} - \frac{1}{x^2}$$

(b) Evaluate I = $\iiint xyz\, dx\, dz\, dy\, dz$ where D is the region bounded by the +ve octane of the sphere $x^2 + y^2 + z^2 = a^2$

19. Show that $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx = \frac{\sqrt{(x+1)}}{(x+1)^{x+1}}$ and hence deduce that

$$\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$$

(or)

20. (a) Establish the relationship between Beta and Gamma functions.

(b) Show that

$\int_{-a}^{a} (a+x)^{m-1}(a-x)^{n-1}\, dx = (2a)^{m+n-1}\beta$ (m, n) if m, n>o.