SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E / B.Tech - (Common to ALL Branches)
(Except to Bio-Groups)

Title of the paper: Engineering Mathematics - I

Semester: I — Max. Marks: 80

Sub.Code: 6C0002 — Time: 3 Hours

Date: 05-05-2007 — Session: AN

PART – A (10 x 2 = 20)

Answer All the Questions

1. Find the sum and product of the eigen values of the matrix

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

2. Prove the matrix $M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is orthogonal.

3. Prove that $\dfrac{e+1}{e-1} = \dfrac{\frac{1}{1!}+\frac{1}{3!}+....}{\frac{1}{2!}+\frac{1}{4!}+....}$

4. Find the coefficient of x^n in the expansion of log $(2x + 3)$.

5. Define curvature of a curve.

6. Find the envelope of the family of lines $\dfrac{x}{t} + yt = 2c$, where t being the parameter.

7. If x = r cos θ and y = r sin θ prove that $\frac{\partial(r,\theta)}{\partial(x,y)} = \frac{1}{r}$.

8. If u = f(x – y, y – z, z – x) then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

9. Find the particular integral of $(D-1)\,y = e^x$.

10. Solve $xy'' + y' + \frac{y}{x} = 0$.

PART – B (5 x 12 = 60)

Answer ALL the Questions

11. Reduce the quadratic form $8x_1^2+7x_2^2+3x_3^2-12x_1x_2+4x_1x_3-8x_2x_3$ in to its canonical form by using orthogonal reduction.

(or)

12. Verify Cayley – Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$ Also find A^{-1} and A^4.

13. (a) Prove that $\frac{5}{1.2.3}+\frac{7}{3.4.5}+\frac{9}{5.6.7}+\ldots = 3\log 2 - 1$.

(b) Prove that $\left(1+\frac{1}{2!}+\frac{1}{4!}+\ldots\right)^2 = 1+\left(1+\frac{1}{3!}+\frac{1}{5!}+\ldots\right)^2$.

(or)

14. (a) If x is small prove that $(1+x)^{\frac{1}{1-x}} = 1+x+x^2+\frac{3x^3}{2}$ approximately.

(b) Find the coefficient of x^n in the expansion of $\frac{1+2x-3x^2}{e^x}$.

15. Find the evolute of $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ as the envelope of normals.

(or)

16. Find the circle of curvature of $\sqrt{x} + \sqrt{y} = \sqrt{a} dt \left(\frac{a}{4}, \frac{a}{4} \right)$.

17. A rectangular open box, open at the top is to have a volume of 32 c. ft. Find the dimensions of it, requiring least material for its construction.

(or)

18. Evaluate $\int_0^x \frac{\log(1+xy)}{1+y^2} dy$. Hence find the value of $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$.

19. Solve $\frac{dx}{dy} + y = S\mathrm{int}\, x + \frac{dy}{dt} = \cos t$ where x(0)=2 and y(0)=0.

(or)

20. Solve by method of variation of parameters.

$$\frac{d^2y}{dx^2} + 4y = \sec 2x.$$