

# SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act,1956)

Course & Branch :B.E/B.Tech – Common to ALL Branches (Except Bio groups)

Title of the Paper :Engineering Mathematics – II Max. Marks :80

Sub. Code :6C0016

Time : 3 Hours

Date :10/05/2010

Session :FN

## PART - A

(10 x 2 = 20)

Answer ALL the Questions

1. If  $\tan(x/2) = \tanh(y/2)$  prove that  $\cos x \cosh y = 1$ .
2. If  $x + \frac{1}{x} = 2 \cos \alpha, y + \frac{1}{y} = 2 \cos \beta, z + \frac{1}{z} = 2 \cos \gamma$ , prove that  $xyz + \frac{1}{xyz} = \cos(\alpha + \beta + \gamma)$ .
3. Find the equation of the line passing through (2,3,4) and perpendicular to the plane  $y+3x+2z=6$ .
4. Find the coordinates of centre and radius of the sphere  $2x^2 + 2y^2 + 2z^2 - 4x + 8y + 10z + 9/2 = 0$ .
5. Evaluate  $I = \int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta$
6. Evaluate  $\int_0^{\infty} x^6 5^{-x} dx$ .
7. Find the directional derivative of  $f(x,y,z) = 2x^2 + 3y^2 + z^2$  at the point P(2,1,3) in the direction of  $\mathbf{a} = \mathbf{i} - 2\mathbf{k}$ .

8. Evaluate  $\iint_S (x^3 dydz + x^2 ydzdx + x^2 zdx dy)$ , where S is the surface of the cube  $x = 0, y = 0, z = 0, x=1, y=1, z=1$ .
9. Prove that  $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
10. Evaluate  $\int_0^a \int_0^b (x+y)dx dy$ .

PART – B

(5 x 12 = 60)

Answer ALL the Questions

11. (a) Solve approximately  $\cos\left(\frac{\pi}{3} + \theta\right) = 0.49$ .  
 (b) Separate into real and imaginary parts of  $\tan^{-1}(x+iy)$ .  
 (or)
12. Expand  $\cos^5\theta \sin^7\theta$  in a series of sines of multiples of  $\theta$ .
13. (a) The plane  $x - y - z = 2$  is rotated through  $90^\circ$  about its line of intersection with the plane  $x + 2y + z = 2$ . Find its equation in the new position.  
 (b) Find the equation of the spheres passing through the circle  $x^2 + y^2 + z^2 - 6x - 2z + 5 = 0, y = 0$  and touching the plane  $3x + 4z + 5 = 0$ .  
 (or)
14. Find the shortest distance and its equation between the lines.  
 $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$  and  $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ .

15. Prove  $\beta(n, \frac{1}{2}) = 2^{2n-1} \beta(n, n)$  and hence deduce the duplication formula.

(or)

16. (a) Evaluate  $\int_0^1 \frac{dx}{\sqrt{\log(1/x)}}$ .

(b) Prove that  $\beta(m, n) = \Gamma(m) \Gamma(n) / \Gamma(m+n)$ .

17. Verify divergence theorem for  $F = (x^2 - yz)\mathbf{i} + (y^2 - zx)\mathbf{j} + (z^2 - xy)\mathbf{k}$  taken over rectangular parallelepiped  $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$ .

(or)

18. Determine  $f(r)$  so that the vector  $f(r) r$  is both solenoidal and irrotational.

19. (a) Evaluate by changing the order of integration  $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{xdydx}{\sqrt{x^2 + y^2}}$ .

(b) Derive the reduction formula for  $\int_0^{\pi/2} \sin^m x \cos^n x dx$ .

(or)

20. (a) Evaluate  $\int_0^{\log a} \int_0^x \int_0^{x+y} e^{(x+y+z)} dz dy dx$ .

(b) Prove that  $\int_0^{\pi/2} \log \sin x dx = -\frac{\pi}{2} \log 2$ .