T.E | VI | Rev | EXTC | Probability & Random process |
Nov | '05 | I.T. EXAM.

PH 98
Con. 4408-05. PR-4853

(REVISED COURSE)

(3 Hours) [Total Marks : 100

N.B. Answer any **five** questions.

1. (a) Let $B_1, B_2 \ldots . B_n$ be partitions of an event space B_i, i = 1, 2, ... n, for the event B that has occurred. Suppose now an event A occurs. Find expression for P(B|A) in terms of B_i. **10**
   (b) Two balanced dices are being rolled simultaneously. If sum of the numbers shown at a time by the two faces is 7. What is the probability that the number shown by one of the face to the dice in this case is 1. **10**

2. (a) Explain with sketches how the probabilistic behaviour of a random variable X is defined by its probability density function and its relation with cumulative distribution function, expected value, and variance. **14**
   (b) Given $f(x) = ce^{-\alpha |x|}$ and $P[\,|x| < \upsilon\,]$. Find the value of normalization constant C and $F[\,|x| < \upsilon]$. **6**

3. (a) (i) Explain what is a moment generating function of a random variable. **4**
   (ii) If X is a random variable and f(x) is given by $f(x) = \frac{1}{b} e^{-(x-a)/b}$, find the first and second moments of X. **8**
   (b) If X and Y are independent random variables and z = x + y, find f(z) by the transform method. **8**

4. (a) X and Y are random variables. Show that the conditional probability density function of Y, given X = x is given by $f_y(y/x) = \dfrac{f_{xy}(x,y)}{f_x(x)}$ X and Y are random variables. **10**

   (b) Given $f_{xy}(x, y) = \begin{cases} \frac{8}{9} xy , & 1 < x < y < 2 \\ 0, & \text{Otherwise} \end{cases}$ **10**

   Find the marginal density function of X and Y and conditional densities of Y given X = x, and of X given Y = y.

5. (a) X and Y are random variables. Show that their joint central moment is given by– **8**
   $C_{xy}(x, y) = R_{xy}(x, y) - E(X)\,E(Y)$

   (b) $f_{xy}(x, y) = \begin{bmatrix} 2e^{-x}e^{-y} & , 0 \le y \le x < \infty \\ 0, & \text{elsewhere} \end{bmatrix}$. **12**

   Find the correlation coefficient of X and Y. Are X and Y are independent.

6. (a) If X(t) is an ergodic process show that – **10**

   $S_{xx}(w) = \int_{-\infty}^{\infty} R_{xx}(\tau)\, e^{-jwt}\, d\tau$ where $\tau = t_2 - t_1$ t_1 and t_2 being two instants of time.

   (b) A random process is given by $X(t) = A\cos(\omega_0 t + \theta)$, where A and ω_0 is constant and θ is a random variable uniformly distributed in the interval $(-\pi, -\pi,)$. **10**
   Determine the power spectrum density of X(t).

7. (a) (i) Explain how a random process can be described by a set of indexed random variables and hence derive expressions for its mean, autocorrelation and autocovariance functions. What will be properties of these functions, if the random process is wide-sense stationary ? **8**
   (ii) Write down the expression of the probability density function if the process is gaussian. Hence explain how a wide-sense stationary process, if gaussian, is stationary in the strict sense also. **4**
   (b) A stationary process is given by– **8**
   $X(t) = 10\cos[\,100\,t + \theta\,]$
   where θ is a random variable with uniform probability distribution in the interval $[-\pi, \pi]$. Show that it is a wide sense stationary process.