

Con. 3496-08.

[REVISED COURSE]

CO-2944

(3 Hours)

[Total Marks : 100

N.B. : Answer any five questions.

1. (a) Give the following definitions of probability with the short comings if any :- 8
- (i) A – priori or classical definition
 - (ii) A – posteriori or relative frequency definition.
 - (iii) Axiomatic definition.

- (b) State and prove Bayes' theorem. 4

- (c) In a factory, four machines A_1, A_2, A_3 and A_4 produce 10%, 20%, 30% and 40% of the items respectively. The percentage of defective items produced by them is 5%, 4%, 3% and 2% respectively. An item selected at random is found to be defective. What is the probability that it was produced by the machine A_2 ? 8

2. (a) Define discrete and continuous random variables. Give one example of each type. 10
Define Expectation of discrete random variable and continuous random variable.

- (b) The joint density function of two continuous random variables is given by 10

$$f(x, y) = \begin{cases} xy/8 & 0 < x < 2, \quad 1 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

- Find : (a) $E(X)$
(b) $E(Y)$
(c) $E(2X + 3Y)$.

3. (a) Suppose X and Y are two random variables. Define covariance and correlation coefficient of X and Y. When do we say that X and Y are - 10

- (i) Orthogonal
- (ii) Independent and
- (iii) Uncorrelated

Are uncorrelated random variables independent ?

- (b) Prove - $|C_{xy}| \leq \sigma_x \cdot \sigma_y$. 10

If x, y are two random variates with standard deviations σ_x and σ_y and if C_{xy} is the covariance between them.

4. (a) Define Entropy of a Discrete Random Variable. 2+8

- (b) A random sample X has the following probability mass function.

X = x	1	2	3	4	5	6
P(X = x)	3/8	3/8	1/8	1/16	1/32	1/32

Find the Entropy.

(c) The joint probability function of two random variables x and y is given by :-

$$f(x, y) = \begin{cases} c(x^2 + 2y) & x = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases} \quad y = 1, 2, 3, 4$$

- Find (a) the value of c ,
 (b) $p(x = 2, y = 3)$
 (c) $p(x \leq 1, y > 2)$ and
 (d) marginal probability functions of x and y .

5. (a) Let $f_{xy}(x, y) = 1, \quad 0 < |y| < x < 1$ 10
 $= 0$ otherwise.
 Determine $E(X/Y)$ and $E(Y/X)$

(b) Define a random process giving an example. 10
 Define (i) mean
 (ii) autocorrelation and
 (iii) autocovariance of a random process.

6. (a) If $X(t)$ is an ergodic process, show that $S_{xx}(w) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-jw\tau} d\tau$ where 10
 $\tau = t_2 - t_1$, t_1 and t_2 being two instants of time.

(b) Explain power spectral density function. State its important properties and prove any one property. 10

7. (a) Explain in brief :- 10
 (i) WSS process
 (ii) Poisson process
 (iii) Queueing system.

(b) If $x = \cos \theta$ and $y = \sin \theta$ where θ is uniformly distributed over $(0, 2\pi)$. 10
 Prove that -
 (i) x and y are uncorrelated
 (ii) x and y are not independent.