T.E. / Sem VI / Rev. / E&TC / PRP / K-7 / 6 cons

[REVISED COURSE]

CON/4739-06. YM-7156

(3 Hours) [Total Marks : 100

N.B. : Answer any five question.

1. (a) Give the following definitions of probability with the shortcomings if any:
   (i) *A-priori* or classical definition
   (ii) *A-posteriori* or relative frequency definition
   (iii) Axiomatic definition. [8]

(b) State Total probability theorem and Bayes' theorem.
Suppose box I contains 5 white balls and 6 black balls and box II contains 6 white balls and 4 black balls. A box is selected at random and then a ball is chosen at random from the selected box.
   (i) What is the probability that the ball chosen will be a white ball?
   (ii) Given that the ball chosen is white, what is the probability that it came from box I? [4+8]

2 (a) Define discrete and continuous random variables by giving examples. Define the distribution function and probability mass function/probability density function of a random variable. A random variable has the following exponential probability density function: $f_x(x) = Ke^{-|x|}$. Determine the value of K and the corresponding distribution function $F_x(x)$. [14]

(b) Obtain the distribution function of $Y = aX + b$, where X is uniformly distributed in (c,d). [6]

3 (a) Define the characteristic function $\phi_X(w)$ of a random variable X.

Show that $\phi_X(w)$ can be expressed as: $\phi_X(w) = \sum_{n=0}^{\infty} m_n \frac{j^n w^n}{n!}$ where

$m_n = \frac{1}{j^n}[\frac{d^n}{dw^n}\phi_X(w)]_{w=0}$ is the $n\,th$ order moment of the r.v X.

[12]

(b) Find the characteristic function of the geometric distribution given by

$P(X = r) = q^r p,\ r = 0,1,2,...$

$p + q = 1$

Hence find the mean and variance. [8]

4. (a) Suppose X and Y are two random variables. Define covariance and correlation coefficient of X and Y. When do we say that X and Y are (i) orthogonal (ii) independent and (iii) uncorrelated ? Are uncorrelated random variables independent? [10]

(b) Suppose that X and Y are continuous random variables with joint probability density function :

$$f_{XY}(x, y) = \frac{1}{2} x e^{-y}, 0 < x < 2, y > 0$$
$$= 0 \quad elsewhere ,$$

find :

i) the joint distribution function of X, Y and

ii) the marginal probability density functions of X and Y. [10]

5 (a) Find the probability density function of $Z = X + Y$ where X and Y are (i) any two random variables (ii) independent. If X and Y are independent, Binomial random variables with parameters (m,p) and (n,p) respectively, obtain the distribution of $X+Y$. [12]

(b) Given :

$$f(x, y) = k, \quad 0 < x < y < 1$$
$$= 0 \quad otherwise$$

Determine k and the conditional densities $f_{x|y}(x|y)$ and $f_{y|x}(y|x)$. [8]

6 (a) Define a random process giving an example. Define (i) mean (ii) autocorrelation and (iii) auto covariance of a random process. [8]

(b) Write brief notes on :

(i) Ergodic process

(ii) Poisson process

(iii) Renewal process

(iv) M/M/1 queue. [12]

7 (a) Explain what is meant by a wide sense stationary process. A random process is given by $X(t) = a\cos[\omega_0 t + \phi]$ where a and ω_0 are constants and ϕ is a random variable uniform in $[-\pi, \pi]$. Show that it is a wide sense stationary process. [10]

(b) Obtain the power spectrum of the above process $X(t) = a\cos[\omega_0 t + \phi]$. [10]