P4/RT-Ex-07-336

Con. 5529–07. [REVISED COURSE] CD–5662

(3 Hours) [Total Marks : 100

N.B. : Answer any **five** questions.

1. (a) State the three axioms of probability. 5
   (b) Explain the concept of Joint and conditional probability with one example each. 5
   (c) Let $B_1, B_2, \ldots B_n$ be partitions of an event space B_i, $i = 1, 2 \ldots n$, for the event B that has occured. Suppose now that an event A occurs. Find expression for p(B/A) in terms of B_i. 10

2. (a) Define and give one example each :– 6
   (i) Probability distribution function of continuous and discrete random variable
   (ii) Probability density function of continuous and discrete random variable.
   (b) Let X be a continuous random variable with p.d.f. $f(x) = k \cdot x \cdot (1 - x)$; $0 \le x \le 1$. Find K and determine a number b such that $P(x \le b) = P(x \ge b)$. Also obtain distribution function. 8
   (c) (i) State important properties of characteristic function. 2
   (ii) Find the probability density function $f_x(x)$ whose characteristic function is given below :– 4

$$\phi_x(w) = 1 - |w| \quad ; \quad |w| \le 1$$
$$= 0 \quad ; \quad |w| \ge 1$$

3. (a) If the probability density function of x is 4

$$f_x(x) = e^{-x} \quad , x > 0$$

   Find the probability density function of $y = x^3$.
   (b) If x is a continuous random variable with uniform probability density function in $(0, 2\pi)$. Find the probability density function and distribution function of y = cos x. 6
   (c) The joint probability density function of two random variables is given by – 10

$$f_{xy}(x, y) = 15 \; e^{-3x - 5y} \quad ; \quad x > 0, \quad y > 0$$
$$= 0 \quad ; \quad \text{else}$$

   (i) Find the probability that –

$$1 < x < 2 \quad \text{and} \quad 0{\cdot}2 < y < 0{\cdot}3$$

   (ii) Find the probability that –

$$x < 2 \quad \text{and} \quad y > 0{\cdot}2$$

   (iii) Find the marginal probability distributions of x and y.

4\. (a) If x, y are two independent random variables and if z = x/y, then prove that probability density function of z is given by – 8

$$f_z(z) = \int_{-\infty}^{\infty} |y| \cdot f_x(yz) \cdot f_y(y)\,dy$$

(b) If x and y are two independent random variables with probability density function – 6

$f_x(x) = e^{-x}$; $x > 0$ and

$f_y(y) = 3 \cdot e^{-3y}$; $y > 0$

Find : fz (z) if z = x / y

(c) Find the moment generating function (M·G·F) of Poisson distribution and hence find mean and variance. 6

5\. (a) Suppose x and y are two random variables. Define covariance and correlation coefficient of x and y. When do we say that x and y are :– 10

(i) Orthogonal

(ii) Independent and

(iii) Uncorrelated ? Are uncorrelated variables independent ?

(b) If a random process is given by – 10

x (t) = A cos wt

Where w is constant and A is random variable with uniform distribution over (0, 1). Find the mean $M_x(t)$, autocorrelation $R_{xx}(t_1, t_2)$ and auto covariance $C_{xx}(t_1, t_2)$ of x(t).

6\. (a) What is Random Process ? State four classes of random processes giving one example each. 6

(b) A Random Process is defined by, $x(t) = \sin(w_0 t + \theta)$ where θ is uniformly distributed in (0, 2π) and w_o is constant. 8

Verify where x(t) is a wide sense stationary process.

(c) Prove that if the input to a linear time invariant system is w.s.s then the output is also wss. 6

7\. (a) Write brief notes on :– 12

(i) Ergodic Process

(ii) M / M / 1 Queue

(iii) Poisson Process.

(b) (i) State important properties of power spectral density. 2

(ii) Find the power spectral density of WSS random process whose autocorrelation function is given by – 6

$$R(\tau) = \frac{a^2}{2} \cos b\tau$$