T.E. (EXTC) Sem V (R)

Con. 5914–09. (REVISED COURSE) SP–8606

(3 Hours) [Total Marks : 100

Signals & Systems

21/12/09

2.30 to 5.30

N.B. : (1) Question No. 1 is **compulsory**,
(2) Answer **five** question in **all**.
(3) **All** parts of the same questions must be written in **continuation**.
(4) Assume **suitable** data, if **required** and state them **clearly**.

1. Answer any **four** :– 20

(a) A discrete time periodic sequence is given by $x_p(n) = \cos\left[\frac{n\pi}{2}\right]$

(i) Determine the period of the sequence

(ii) Sketch the sequence x(n) for the variable n for one period.

(b) Show that $\int_{-D}^{D} |x(t)|\, dt < \alpha$ is a sufficient condition for the existence of the Fourier transform of x(t) .

(c) The impulse response of a linear time invariant system is

$h(n) = \{ 1, 2, 1, -1 \}$
$\uparrow$

Determine the response of the system to the input signal

$x(n) = \{ 1, 2, 3, 1 \}$
$\uparrow$

(d) Derive the initial and final value theorem of z transform.

(e) Determine the direct form - I realization for the following transfer function

$H(z) = 1 - 0.5z^{-1} + 0.2z^{-2}$

2. (a) Determine the range of values of the parameter for which the following linear time-invariant systems with impulse response. 10

(i) $h(n) = a^n u(n)$

(ii) $h(n) = \begin{cases} a^n & n \geq 0 \\ b^n & n < 0 \end{cases}$ is stable.

(b) Consider the analog signal $x_a(t) = 3 \cos 100 \pi t$ 10

(i) Determine the minimum required sampling rate to avoid aliasing.

(ii) Suppose that the signal is sampled at the rate Fs = 200 Hz. What is the discrete time signal obtained after sampling ?

(iii) Suppose that the signal is sampled at the rate Fs = 75 Hz. What is the discrete time signal obtained after sampling.

3. (a) A causal system has a difference equation as given below $y(n) = 0{\cdot}5\ y(n-1) - 0{\cdot}25y\ (n-2) + x(n)$ what is the ROC of the transfer function of the system. **10**

(b) (i) Find the condition for an LTI DTS described as **10**

$$y(k+1) - \alpha\ y(k) = \alpha\ r(k)$$

to be BIBO stable.

(ii) Check the BIBO stability of T.F $H(z) = \dfrac{z-2}{z(z-0{\cdot}8)}$

4. (a) Find the Fourier transform of the unit impulse train function $\delta_T(t)$, where $\delta_T(t)$ is defined by **10**

$$\delta_T(t) = \ldots\ldots + \delta(t + 2T) + \delta(t + T) + \delta(t - T) + \delta(t - 2T) + \ldots\ldots \} = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

(b) Evaluate the following convolutions – **10**

(i) $\delta(t) * \delta(t)$

(ii) $x(t) * \delta(t - t_o)$

5. (a) Find the z transform of – **10**

(i) $x(n) = (-1)^n \cos\frac{\pi}{3}n \quad u(n)$

(ii) $x(n) = (n + 1)a^n$.

(b) Determine a cascade realization for the following transfer function **10**

$$H(z) = \frac{0{\cdot}7\left(z^2 - 0{\cdot}36\right)}{z^2 + 0{\cdot}1z - 0{\cdot}72}$$

6. (a) Find the Fourier series for the function x(t) defined by **10**

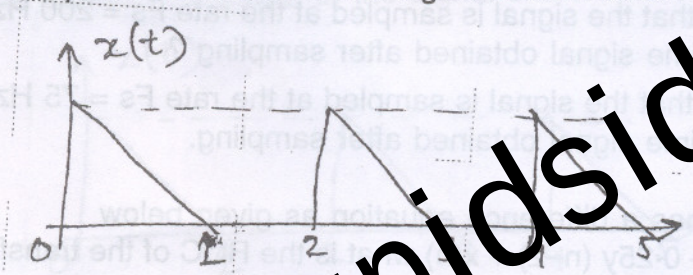
$$x(t) = \begin{cases} 0 & \frac{-T}{2} < t < 0 \\ A \operatorname{Sin} w_0 t & 0 < t < \frac{T}{2} \end{cases}$$

and $x(t + T) = x(t)$, $w_0 = \frac{2\pi}{T}$

(b) A discrete time system is described by the difference equation, **10**
$y(k - 3) + 2y(k - 2) + 3y(k - 1) + y(k) = 2u(k)$. Draw the block diagram representing the system. Generate the state variable description in terms of appropriate A, B, C & D matrices.

7. (a) Show that if $x_e(n)$ is an even signal and $x_0(n)$ is an odd signal, then $x_e(n) \cdot x_0(n)$ is an odd signal. **20**

(b) Find the Laplace transform of the signal shown below



(c) Find the state transition matrix for

$$A = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}$$

(d) Obtain the inverse z transform of a rational function

$$f(z) = \frac{z^2}{z^3 - 1{\cdot}7z^2 + 0{\cdot}8z + 0{\cdot}1}$$