Total No. of Questions : 12] **[Total No. of Printed Pages : 7**

[3761]-106

F. E. (Semester - II) Examination - 2010

ENGINEERING MATHEMATICS - II

(June 2008 Pattern)

Time : 3 Hours] **[Max. Marks : 100**

Instructions :

(1) In section I, attempt Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6. In section II, attempt Q. No. 7 or 8, Q. No. 9 or 10, Q. No. 11 or 12.

*(2) Answers to the **two sections** should be written in **separate answer-books.***

(3) Figures to the right indicate full marks.

(4) Neat diagram must be drawn wherever necessary.

(5) Use of non-programmable electronic pocket calculator is allowed.

(6) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Form a Differential Equation whose general solution is

$xy = ae^{x} + be^{-x} + x^{3}$ **[05]**

(B) Solve the following : **(Any Three)** **[12]**

(1) $(x^{2}y - 2xy^{2})\, dx = (x^{3} - 3x^{2}y)\, dy$

(2) $\tan y \dfrac{dy}{dx} + \tan x = \cos y \cos^{2}x$

(3) $x\,dy - y\,dx = (x^{2} + y^{2})(x\,dx + y\,dy)$

(4) $(x^{2}y + y^{4})\, dx + (2x^{3} + 4xy^{3})\, dy = 0$

OR

Q.2) (A) Form a Differential Equation whose general solution is $y = \log \cos(x - a) + b$ **[05]**

(B) Solve the following : **(Any Three)** **[12]**

(1) $x\dfrac{dy}{dx} + 3y = x^4 e^{1/x^2} y^3$

(2) $\left[\log(x^2 + y^2) + \dfrac{2x^2}{x^2 + y^2}\right] dx + \dfrac{2xy}{x^2 + y^2} dy = 0$

(3) $\left(\dfrac{y}{x} \sec y - \tan y\right) dx = (x - \sec y \log x)\, dy$

(4) $\dfrac{dy}{dx} = \dfrac{2x - 3y + 1}{3x + 4y - 5}$

Q.3) Solve **any three** :

(a) A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of air being 40°C. What will be the temperature of the body after 40 minutes from the original ? **[05]**

(b) In a circuit containing inductance L, resistance R and voltage E, the current I is given by $E = RI + L\dfrac{dI}{dt}$. Given L = 640H, R = 250 ohms, E = 500 volts, I being zero when t = 0, find the time that elapses, before I reaches 90% of its maximum value. **[06]**

(c) A particle of mass m is projected upward with velocity V. Assuming the air resistance is k times its velocity, write the equation of motion and show it will reach maximum height in time $\dfrac{m}{k}\log\left(1 + \dfrac{kV}{gm}\right)$. Find also the distance travelled at any time t. **[06]**

(d) A particle executes S.H.M. When it is 2 cm from the mid path, its velocity is 10cm/sec. and when it is 6 cm., from centre its velocity is 2 cm/sec. Find its period and greatest acceleration. **[05]**

OR

Q.4) Solve **any three** of the following :

(a) A steam pipe 20 cm in diameter is protected with a covering 6cm thick for which the coefficient of thermal conductivity is k = 0.0003. Find the heat lost per hour through a meter length of the pipe, if the inner surface of the pipe is at 200°C and the outer surface of the covering is at 30°C. Also, find temperature at a distance 12 cm from the centre of the pipe. **[06]**

(b) The charge Q on a plate of condenser of capacity C is charged through a resistance R, by steady voltage V. If Q = 0 at t = 0, find charge as a function of t. **[05]**

(c) A particle is moving in a straight line with an acceleration $k\left[x + \frac{a^4}{x^3}\right]$, directed towards origin. If it starts from rest at a distance 'a' from the origin, prove that it will arrive at origin at the end of time $\frac{\pi}{4\sqrt{k}}$. **[06]**

(d) Find the orthogonal trajectories of $r = a(1 - \cos\theta)$. **[05]**

Q.5) (A) Expand $f(x) = x\sin x$ as a Fourier Series in the interval $0 \le x \le 2\pi$. **[08]**

(B) Show that $B(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1 + x)^{m+n}}\,dx$. **[04]**

(C) If $I_n = \int_0^{\pi/2} \cos^n x \cos nx\,dx$ prove that $I_n = \frac{1}{2} I_{n-1} = \frac{\pi}{2^{n+1}}$. **[05]**

OR

Q.6) (A) Obtain the constant term and the coefficient of first sine and cosine terms in the Fourier Expansion of y as given in following table : **[07]**

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

(B) If $I_n = \int_{\pi/4}^{\pi/2} \cot^n\theta \, d\theta$, prove that $I_n = \frac{1}{n-1} - I_{n-2}$.

Hence evaluate $\int_{\pi/4}^{\pi/2} \cot^6\theta \, d\theta$ **[06]**

(C) Evaluate $\int_0^\infty x^7 e^{-2x^2} dx$ **[04]**

SECTION - II

Q.7) (A) Trace the following curves : **(Any Two)** **[08]**

(1) $y^2 (4 - x) = x (x - 2)^2$

(2) $r = a \cos 2\theta$

(3) $a^2y^2 = x^2 (a^2 - x^2)$

(B) Prove that $\int_0^\infty \frac{1}{x^2} \log (1 + ax^2) \, dx = \pi\sqrt{a} \quad (a>0)$

Deduce that $\int_0^\infty \frac{1}{x^2} \log \left(1 + x^2\right) dx = \pi$ **[04]**

(C) Find the length of the arc of the cardioide $r = a (1 - \cos\theta)$, which lies outside the circle $r = a \cos\theta$ **[05]**

OR

Q.8) (A) Trace the following curves : **(Any Two)** **[08]**

(1) $yx^2 = a^2 (a - y)$

(2) $x = a (t + \sin t)$

$y = a (1 + \cos t)$

(3) $r = a (1 + 2\cos\theta)$

(B) Show that $\frac{d}{dt}\left(\operatorname{erf}\left(\sqrt{t}\right)\right) = \frac{e^{-t}}{\sqrt{\pi t}}$.

Hence evaluate $\int_0^\infty e^{-t} \operatorname{erf}\left(\sqrt{t}\right) dt$ **[05]**

(C) Find the length of arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ in the positive quadrant. **[04]**

Q.9) (A) Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0, 3x - 4y + 5z - 15 = 0$ and intersecting the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogonally **[05]**

(B) Obtain the equation of the right circular cone, which passes through (1, 3, 4) with vertex (2, 2, 1) and axis parallel to the line . **[05]**

(C) Find the equation of the right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 = 9, x - y + z = 3$. **[06]**

OR

Q.10) (A) Find the equation of the sphere which is tangential to the plane $2x - 2y - z + 16 = 0$ at $(-3, 4, 2)$ and passing through the point $(-2, 0, 3)$. **[06]**

(B) Obtain the equation of the right circular cylinder of radius 5 and axis $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{1}$. **[05]**

(C) The axis of a right circular cone whose vertex is origin 'O' makes equal angles with the co-ordinate axes, and the cone passes through the line drawn from O with direction cosines proportional to 1, –2, 2. Find the equation of the cone. **[05]**

Q.11) (A) Evaluate $\int_0^1 \int_{y^2}^{y} \frac{y\,dxdy}{(1-x)\sqrt{x-y^2}}$ **[06]**

(B) Find the area common to the circles $x^2 + y^2 = 9$ and $x^2 + y^2 = 6x$. **[05]**

(C) Find the C.G. (Centre of Gravity) of the area enclosed by the curves $y^2 = 4ax$, $y = 2x$. **[06]**

OR

Q.12) (A) Evaluate $\iint_R \frac{\sqrt{x^2+y^2}}{x^2}\,dxdy$, where R is the region enclosed by the curves $x^2 + y^2 = 2x$, $y = x$ and $y = 0$, in the first quadrant. **[05]**

(B) Find the volume bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the paraboloid $x^2 + y^2 = 3z$. **[06]**

(C) Show that the Moment of Inertia (M.I.) of a loop of the curve $r^2 = a^2\cos 2\theta$, about a line through the pole perpendicular to its plane, is $\frac{Ma^2\pi}{8}$, where M is the mass of the loop. **[06]**
