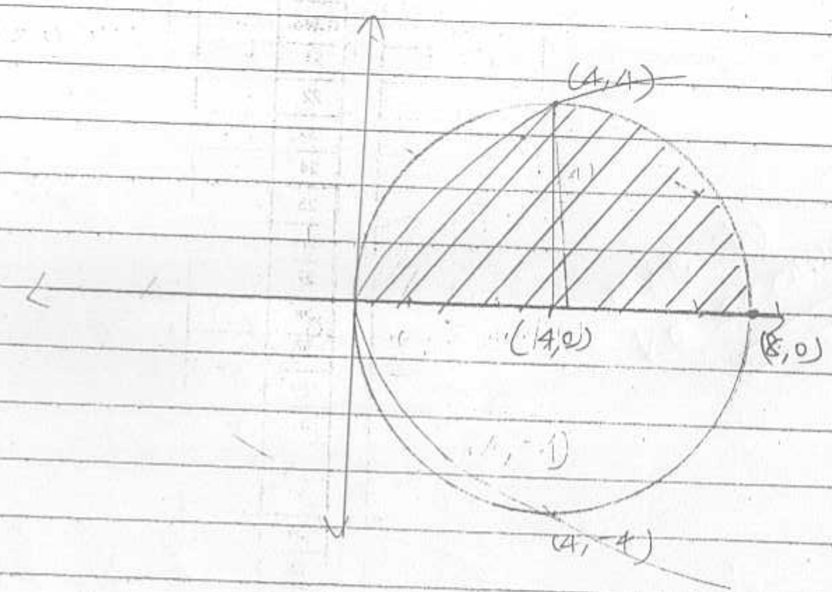
28)



$$x^2 + y^2 = 8x$$

$$x^2 - 8x + y^2 = 0$$

$$x^2 - 8x + 16 + y^2 = 16$$

$$(x-4)^2 + y^2 = 4^2$$

$$y^2 = 4x$$

$$x^2 + y^2 = 8x$$

$$x^2 + 4x = 8x$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0 \text{ or } x = 4$$

$$y^2 = 4x$$

$$y^2 = 16$$

$$y = \pm 4$$

Area of shaded region $= \int_0^4 \text{parabola} + \int_4^8 \text{circle}$

$$= \int_0^4 \sqrt{4x}\, dx + \int_4^8 \sqrt{8x - x^2}\, dx$$

$$= \int_0^4 2\sqrt{x}\, dx + \int_4^8 \sqrt{16 - (x-4)^2}\, dx$$

$$= \frac{2 \times 2}{3}\left[x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2}\right]_4^8 + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)^8$$

$$= \frac{32}{3} + \frac{\overset{4}{\cancel{8}}\pi}{\cancel{2}}$$

$$= \frac{32}{3} + 4\pi$$

$$= 4\left(\frac{8}{3} + \pi\right) \text{ sq units}$$

$$\boxed{\text{area} = 4\left(\frac{8}{3} + \pi\right) \text{ sq units}}$$

27) $I = \int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx$

$\frac{x}{a+x} = \sin^2\theta$

$x = a\sin^2\theta + x\sin^2\theta$

$x(\cos^2\theta) = a\sin$

$x = a\tan^2\theta$

$x = a\tan^2\theta$

$dx = a\sec^2\theta \times 2\tan\theta\, d\theta$

$\theta = 0 \quad x = 0$

$\theta = \frac{\pi}{4} \quad x = a$

$$I = \int_0^{\pi/4} \sin^{-1}\sqrt{\frac{a\tan^2\theta}{a(1+\tan^2\theta)}} \times a\sec^2\theta\; 2\tan\theta\, d\theta$$

$$I = \int_0^{\pi/4} \sin^{-1}\sqrt{\frac{\tan^2\theta}{\sec^2\theta}} \times 2a \sec^2\theta \tan\theta \, d\theta$$

$$= \int_0^{\pi/4} \sin^{-1}\sqrt{\sin^2\theta} \times 2a \sec^2\theta \tan\theta \, d\theta$$

$$= \int_0^{\pi/4} \sin^{-1}(\sin\theta) \times 2a \sec^2\theta \tan\theta \, d\theta$$

$$= 2a \int_0^{\pi/4} \underbrace{\theta}_{u} \; \underbrace{\tan\theta \sec^2\theta \, d\theta}_{dV}$$

$$dv \quad \int = \int \tan\theta \sec^2\theta \, d\theta$$

$$\int dv = \int \tan\theta \sec^2\theta \, d\theta$$

$$= \int \tan\theta \, d(\tan\theta)$$

$$= \frac{\tan^2\theta}{2} + c.$$

$$I = 2a \int_0^{\pi/2} \left[\theta \times \cancel{\frac{\tan^2\theta}{2}} \right]_0^{\pi/4} - 2a \int_0^{\pi/4} \frac{\tan^2\theta}{2} \times 1 \times d\theta.$$

$$= 2a\left[\frac{\pi}{4}\times\frac{1}{2} - 0\right] - a \int_0^{\pi/4} \tan^2\theta \, d\theta$$

$$= 2a\left[\frac{\pi}{8}\right] - a \int_0^{\pi/4} \sec^2\theta - 1 \, d\theta$$

$$= \frac{a\pi}{4} - a \int_0^{\pi/4} \sec^2\theta d\theta + a\int_0^{\pi/4} d\theta$$

$$= \frac{a\pi}{4} - a\left[\tan\theta\right]_0^{\pi/4} + \frac{a\pi}{4}$$

$$= \frac{a\pi}{2} - a\times 1$$

$$= \frac{a\pi}{2} - a.$$

$$\boxed{\text{ans} = \frac{a\pi}{2} - a.}$$

2(i) ~~P(A) ⇒~~

Let A – Event that bulb is manufactured by A

B – " " " " " B

C – " " " " " C

$P(A) = 0.6 \quad P(B) = 0.3 \quad P(C) = 0.1$

Let E_1 – Event that bulb from A is defective

E_2 – " " " " B " "

E_3 " " " " C " "

$$P(A \mid \text{defective bulb}) = \frac{P(\text{defective bulb} \mid A) \times P(A)}{P(\text{def bulb}/B) \cdot P(B) + P(\text{def bulb}/C) \times P(C) + P(\text{def bulb}/A) \times P(A)}$$

$$= \frac{\cancel{0.01 \times 0.6}}{}$$

$$\frac{0.006}{0.006 + 0.006 + 0.003}$$

$$\frac{0.006}{0.015}$$

$$\frac{6}{15}$$

$$= \frac{0.006}{0.006 + 0.006 + 0.003}$$

$$= \frac{0.006}{0.015}$$

$$= \frac{6}{15}$$

$$= \frac{2}{5}$$

Probability that defective bulb is from A = $\frac{2}{5}$.

24) $x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1} B$$

To find A^{-1} C-cofactor

$C_{11} = -6 \quad C_{12} = +14 \quad C_{13} = -15$

$C_{21} = +17 \quad C_{22} = 5 \quad C_{23} = 9$

$C_{31} = 13 \quad C_{32} = -8 \quad C_{33} = -1$

$$\text{adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$|A| = 1(-12+6) - 2(-8-6) - 3(-6-9)$$
$$= -6 - 2(-14) - 3(-15)$$
$$= -6 + 28 + 45$$
$$= 22 + 45$$
$$= 67$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$X = A^{-1} B$$

$$= \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$X = \frac{1}{67}\begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ +60 + 18 - 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

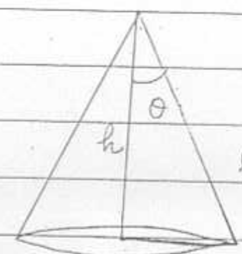
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$x = +3$
$y = -2$
$z = +1$

$x = 3$
$y = -2$
$z = 1$

$3 \times 3 = 9$, 6×6

23)



Let r = radius of cone

h = ht of cone

Given l = slant ht of cone

Let θ be semivertical $\angle^{le}$.

$$l^2 = r^2 + h^2$$

$$r^2 = l^2 - h^2$$

Volume = $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi (l^2 - h^2) h$$

For max. vol

$$\frac{dv}{dh} = 0 \qquad \frac{d^2v}{dh^2} < 0$$

$$\frac{dv}{dh} = \frac{1}{3}\pi(l^2 - 3h^2) = 0$$

$$\Rightarrow l^2 - 3h^2 = 0$$

$$3h^2 = l^2$$

$$h = \frac{l}{\sqrt{3}}$$

$$\frac{d^2v}{dh^2} = +\frac{1}{3}\pi \times -6h$$

$$= -2\pi h < 0 \text{ (for all } h \text{ as } h>0)$$

$h = \frac{l}{\sqrt{3}}$ corresponds to max volume

$$\therefore r^2 = l^2 - h^2$$

$$r = \frac{\sqrt{2}}{\sqrt{3}}\, l$$

$$\tan\theta = \frac{r}{h}$$

$$= \frac{\frac{\sqrt{2}\,\cancel{l}}{\cancel{\sqrt{3}}}}{\frac{\cancel{l}}{\cancel{\sqrt{3}}}}$$

$$= \sqrt{2}$$

$$\Rightarrow \theta = \tan^{-1}\sqrt{2}$$

$$\boxed{\Rightarrow \text{semivertical } \angle^{le} = \tan^{-1}\sqrt{2}}$$

22) $\frac{dy}{dx} + 2y\tan x = \sin x$.

$\frac{dy}{dx} + y \times 2\tan x = \sin x$

$y\sec^2 x =$

$2y\sec^2 x\tan$

$+\sec^2 x\frac{dy}{dx}$

$2y\sec x\tan x$

$\frac{dy}{dx}$

$$\text{I.F.} = \text{Integ factor} = e^{\int 2\tan x\,dx}$$
$$= e^{2\log|\sec x|}$$
$$= e^{\log \sec^2 x}$$
$$= \sec^2 x$$

Multiplying Both sides of diff eqn by $\sec^2 x$

$$\sec^2 x \frac{dy}{dx} + \sec^2 x \times y \times 2\tan x = \sin x \sec^2 x$$

$$\frac{d}{dx}(y \times \sec^2 x) = \sin x \sec^2 x.$$

$$y \times \sec^2 x = \int \sin x \sec^2 x\,dx$$

$$= \underbrace{\int \frac{\tan x}{\sqrt{\tan^2 x+1}} \sec^2 x\,dx}_{I}$$

$$I = \int \frac{\tan x}{} \sec^2 x\,dx$$

~~$\sin x =$~~

$$\sin x = \frac{1}{\operatorname{cosec} x}$$
$$= \frac{1}{\sqrt{\cot^2 x+1}}$$
$$= \frac{\tan x}{\sqrt{\tan^2 x+1}}$$

$$I = \frac{1}{2}\int \frac{2\tan x \sec^2 x}{\sqrt{\sec x}}\, dx$$

$$\sec^2 x = t$$

$$\Rightarrow 2\tan x \sec^2 x\, dx = dt$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{\cancel{2}} \times \left[\frac{\sqrt{t}}{\frac{1}{\cancel{2}}}\right] + c$$

$$= \sqrt{t} + c$$

$$= \sec x + c$$

$$y \times \sec^2 x = \sec x + c$$

$$y = \cos x + c\cos^2 x$$

21) $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4} = \lambda$

Any pt on the line can be written as $x = 2\lambda - 1$
$y = 3\lambda - 2$
$z = 4\lambda - 3$

To ~~find pt of~~ Let $P(2\lambda-1, 3\lambda-2, 4\lambda-3)$ be pt of intersection of the line with $x + y + 4z = 6$

$\Rightarrow$ P lies on the plane

$\Rightarrow \quad 2\lambda - 1 + 3\lambda - 2 + 16\lambda - 12 = 6$

$21\lambda - 15 = 6$

$21\lambda = 21$

$\lambda = 1$

$\therefore \quad P(2\times1-1,\ 3\times1-2,\ 4\times1-3)$

$P\cancel{(0\,1, 2, 3)}$ $P(1,1,1)$

20) $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$

$= \dfrac{1}{abc} \begin{vmatrix} a^3+a & a^2b & a^2c \\ ab^2 & b^3+b & b^2c \\ c^2a & c^2b & c^3+c \end{vmatrix}$ (Multiplying ~~each~~ row wise by a, b, c respy. & dividing by abc)

$= \dfrac{abc}{abc} \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

$= \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

$R_1 \leftrightarrow R_1 + R_2 + R_3$

$= \begin{vmatrix} a^2+b^2+c^2+1 & a^2+b^2+c^2+1 & a^2+b^2+c^2+1 \\ & & b^2 \end{vmatrix}$

19) $y = 2x^3 - 15x^2 + 36x - 21$

$\frac{dy}{dx} = 6x^2 - 30x + 36 = 0$ $\left(\because \frac{dy}{dx} = 0\right.$ for line $\parallel^{l}$ to x axis$)$

$\Rightarrow 6(x^2 - 5x + 6) = 0$

$(x-2)(x-3) = 0$

$\Rightarrow x = 2$ (or) $x = 3$

At $x = 2$

$y = 2\times8 - 15\times4 + 36\times2 - 21$

$= 16 - 60 + 72 - 21$

$= 7$

$x = 3$

$y = 2\times27 - 15\times9 + 36\times3 - 21$

$= 54 - 135 + 108 - 21$

$= 6$

Pts are $(2,7)$ and $(3,6)$

~~$y-7$~~ At $(2,7)$

$\frac{y-7}{x-2} = 0$

$\Rightarrow y = 7$

At $(3,6)$

$$\frac{y-6}{x-3} = 0$$

$y = 6$

$\therefore$ Pts are $(2,7)$ $(3,6)$

eqn of tgts at $(2,7)$: $y = 7$

$(3,6)$ $y = 6$

Rough

$\frac{\cos\theta}{\frac{\cos^2\theta}{\sin\theta}}$ $\frac{\sin\theta}{\cos\theta}$

$a\left(-\sin\theta + \frac{1}{\tan\theta/2}\sec^2\theta/2\right)$

$\left(-\sin\theta + \frac{1}{2\cos^2\theta/2\cos\theta/2} \frac{\sin\theta/2}{\cos\theta/2}\right.$

$\frac{\cos^2\theta}{\sin\theta}$

$-\sin\theta + \frac{1}{\sin\theta}$

$16 - 60 + 72 - 21$

$-44 + 51$

7

$54 - 135 + 108 - 21$

$33 - 2\cancel{5}7$

6

Linear prog.

25) Let the no. of units of A = x

no. of units of B = y

$$200x + 100y \geq 4000$$

$$x + 2y \geq 50$$

$$40x + 40y \geq 1400$$

$$2x + y \geq 40$$

$$x + 2y \geq 50$$

$$x + y \geq 35$$

$$x \geq 0$$

$$y \geq 0$$

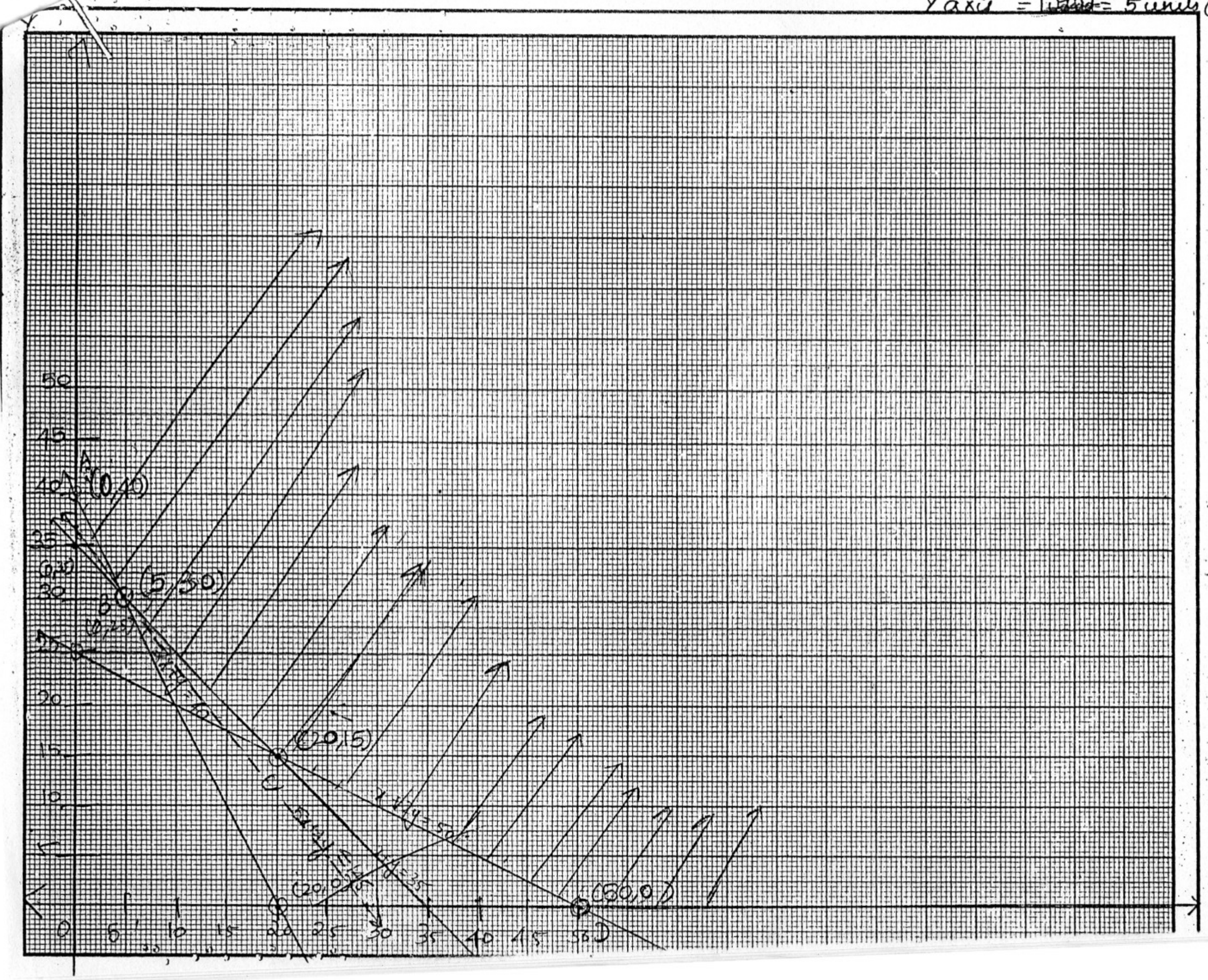
Cost = $Z = 5x + 4y$

A (0,40) Z_A = 160 Rs

B(5,30) $Z_B = 25 + 120$ = Rs 145

C (20,15) $Z_C = 100 + 60$ = Rs 160

X axis = 1 ~~[illegible]~~ cm = 5 units (A)
Y axis = 1 ~~[illegible]~~ cm = 5 units (B)



Here Z_B is minimum.

i.e. At (5, 30).

$5x + 4y < 145$ has no region common ~~when~~ with the unbounded region

$\therefore$ Rs145 is minimum

$\therefore$ 5 units of A

30 unit of B

Min cost = Rs145

8) $\cancel{y}$ $x = a(\cos\theta + \log\tan\frac{\theta}{2})$

$y = a\sin\theta$

$\frac{dy}{d\theta} = a\cos\theta$

$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\theta/2} \times \sec^2\theta/2 \times \frac{1}{2}\right]$

$= a\left(-\sin\theta + \frac{1}{2} \times \frac{\frac{1}{\cos^2\theta/2}}{\frac{\sin\theta/2}{\cos\theta/2}}\right)$

$= a\left(-\sin\theta + \frac{1}{2 \times \sin\theta/2 \cos\theta/2}\right)$

$= a\left(-\sin\theta + \frac{1}{\sin\theta}\right)$

$= a\left(\frac{-\sin^2\theta + 1}{\sin\theta}\right)$

$= a\cos^2\theta$

$\pi \times 16$ $x^2 + y^2 = 16$

$\int_0^4 \sqrt{16 - x^2}\, dx$

$\left[\frac{x}{2}\sqrt{16-x^2}\right]_0^4 + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)_0^4$

$+ 8 \times \frac{\pi}{2}$

$$\frac{dy}{dx} = \frac{\cancel{a}\cos\theta}{\frac{\cancel{a}\cos^2\theta}{\sin\theta}}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

at $\theta = \frac{\pi}{4}$ $\quad \frac{dy}{dx} = \tan\left(\frac{\pi}{4}\right) = 1$

$$\boxed{\frac{dy}{dx} = 1 \quad \text{at } \theta = \frac{\pi}{4}}$$

17) $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

$= \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$$= \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right] + \tan^{-1}\left[\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right]$$

$$= \tan^{-1}\left[\frac{11}{23}\right] + \tan^{-1}\left[\frac{12}{34}\right]$$

$$= \tan^{-1}\left[\frac{\frac{11}{23}+\frac{12}{34}}{1-\frac{11}{23}\times\frac{12}{34}}\right]$$

$$\begin{array}{r} 374 \\ +276 \\ \hline 650 \end{array}$$

$$\begin{array}{r} 782 \\ -132 \\ \hline 650 \end{array}$$

$$= \tan^{-1}\left[\frac{34\times 11 + 12\times 23}{23\times 34 - 132}\right]$$

$$= \tan^{-1}\left[\frac{374+276}{782-132}\right]$$

$$= \tan^{-1}\left[\frac{650}{650}\right]$$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

$= RHS$

$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

16) $(a,b)\ R\ (c,d) \Rightarrow a+d = b+c$ $\quad N \times N$.

$a+a = a+a$

$\Rightarrow (a,a)\ R\ (a,a)$

$\cancel{\Rightarrow a+b}$

$a+b = b+a$

$\Rightarrow (a,b)\ R\ (a,b)$

$\Rightarrow$ R is reflexive

$(a,b)\ R\ (c,d)$

$\Rightarrow a+d = b+c$

$\Rightarrow \cancel{d+a = c+b}$

$\Rightarrow \cancel{c}$

$\Rightarrow b+c = a+d$

$\Rightarrow c+b = d+a.$

$\Rightarrow (c,d)\ R\ (a,b)$

$\Rightarrow$ $(a,b)\ R\ (c,d)$ implies that $(c,d)\ R\ (a,b)$

$\therefore$ R is symmetric

Let $(a,b)\ R\ (c,d)$

$(c,d)\ R\ (e,f)$

$a+d = b+c$

$c+f = d+e$

Adding; $a+f = b+e$

$\Rightarrow (a,b)\ R\ (e,f)$

$(a,b)\ R\ (c,d)$ $(c,d)\ R\ (e,f) \Rightarrow (a,b)\ R\ (e,f)$

∴ R is reflexive, symmetric, transitive

R is an equivalence relation

15) $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & ; x < 0 \\ c & ; x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & ; x > 0 \end{cases}$

∵ f is continuous at 0.

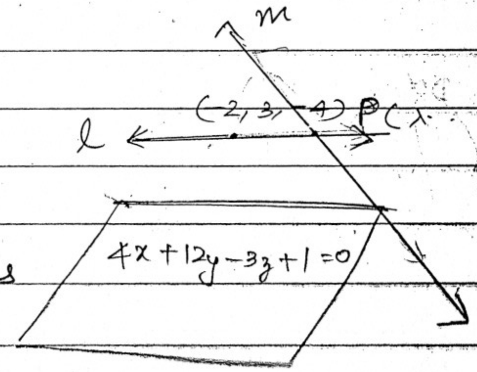
$\lim\limits_{x \to 0+} f(x) = \lim\limits_{x \to 0-} f(x) = f(0)$

$\text{LHL} = \lim\limits_{x \to 0} \dfrac{\sin(a+1)x + \sin x}{x}$

$= \lim\limits_{x \to 0} \dfrac{\sin(a+1)x}{x(a+1)} (a+1) + \lim\limits_{x \to 0} \dfrac{\sin x}{x}$

line m is:

29) $\dfrac{x+2}{3} = \dfrac{y+\frac{3}{2}}{2} = \dfrac{z+\frac{4}{3}}{\frac{5}{3}} = \lambda$



(P.)

Any Pt. on the line can be written as

$x = 3\lambda - 2$

$y = 2\lambda - \frac{3}{2}$

$z = \frac{5\lambda - 4}{3}$

~~Let~~ Let P be pt of Intersection of $\overleftrightarrow{l}$ & $\overleftrightarrow{m}$

$(-2,3,-4)$ lies on l.

DR's of l are $(3\lambda - 2 + 2)$, $(2\lambda - \frac{3}{2} - 3)$, $(\frac{5\lambda}{3} - \frac{4}{3} + 4)$

3λ, $(2\lambda - \frac{9}{2})$, $(\frac{5\lambda}{3} + \frac{8}{3})$

l is $\parallel^{l}$ to the plane

$\Rightarrow$ It is $\perp^{r}$ to normal to plane

$\therefore$ DRs of normal to the plane $4, 12, -3$

$\therefore \quad 4 \times 3\lambda + 12 \times \left(2\lambda - \frac{9}{2}\right) - 3\left(\frac{5\lambda}{3} + \frac{8}{3}\right) = 0$

$12\lambda + 24\lambda - 54 - \overset{5\lambda}{\cancel{}} - 8 = 0$

$36\lambda - 5\lambda - 62 = 0$

$31\lambda = 62$

$\lambda = 2.$

$\therefore \quad P\left(3 \times 2 - 2,\ 2 \times 2 - \frac{3}{2},\ \frac{5 \times 2 - 4}{3}\right)$

$\therefore \quad P\left(4, \frac{5}{2}, 2\right)$

$\therefore$ ~~The dist bet P &~~

Contd.

Fictitious Roll No.
(To be entered by Board)

अपना अनुक्रमाँक इस उत्तर–पुस्तिका पर न लिखें
Please do not write your Roll Number on this Answer-Book

अतिरिक्त उत्तर–पुस्तिकाओं की संख्या...............
Supplementary Answer-Book (S) NO. 1

$\therefore$ dist bet P & $(-2, 3, -4)$

$$= \sqrt{(4+2)^2 + (\tfrac{5}{2}-3)^2 + (2+4)^2}$$

$$= \sqrt{36 + \tfrac{1}{4} + 36}$$

$$\sqrt{\frac{144+1+144}{2}}$$

$$= \sqrt{\frac{288+1}{2}}$$

$$= \frac{17}{2} \text{ units}$$

$\boxed{\text{distance} = 8.5 \text{ units.}}$

$= (a+1) + 1$

LHL $= a+2.$

$$RHL = \lim_{x\to 0} \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}$$

$$= \lim_{x\to 0} \frac{x + bx^2 - x}{bx^{3/2}\left(\sqrt{x+bx^2} + \sqrt{x}\right)}$$

{Multiplying nr. & dr. by $\left(\sqrt{x+bx^2} + \sqrt{x}\right)$ }

$$= \lim_{x\to 0} \frac{\cancel{b}\cancel{x^2}\, x^{1/2}}{\cancel{b}\cancel{x^{3/2}}\left(\sqrt{x+bx^2} + \sqrt{x}\right)}$$

$$= \lim_{x\to 0} \frac{\cancel{x^{1/2}}}{\cancel{x^{1/2}}\left(\sqrt{1+bx} + 1\right)}$$

$$RHL = \frac{1}{\left(\sqrt{1+b0} + 1\right)}$$

$$= \frac{1}{2}$$

$LHL = RHL = f(0) = c$

$\Rightarrow a+2 = \frac{1}{2} = c$

$c = \frac{1}{2}$

$a = -\frac{3}{2}$

b can be any real value.

14) $I = \int_0^{\pi/2} \log \sin x \, dx$

$I = \int_0^{\pi/2} \log \sin(\frac{\pi}{2} - x) \, dx$

$= \int_0^{\pi/2} \log \cos x \, dx$

$$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x) \; dx$$

$$= \int_0^{\pi/2} \log \frac{\sin x \cos x \times 2}{2} \, dx$$

$$= \int_0^{\pi/2} \log \frac{\sin 2x}{2} \, dx$$

$$= \underbrace{\int_0^{\pi/2} \log \sin 2x \, dx}_{I_1} - \int_0^{\pi/2} \log 2 \, dx$$

$$I_1 = \int_0^{\pi/2} \log \sin 2x \, dx$$

$t = 2x$ $\qquad x = 0 \quad t = 0$

$dt = 2\,dx$ $\qquad x = \pi/2 \quad t = \pi$

$$J_1 = \frac{1}{2} \int^{\pi} \log \sin t \, dt$$

$$I_1 = \frac{1}{2}\int_0^{\pi} \log \sin t \, dt$$

$$\sin(\pi - t) = \sin t$$

$$\Rightarrow I_1 = 2 \times \frac{1}{2}\int_0^{\pi/2} \log \sin t \, dt$$

$$= \int_0^{\pi/2} \log \sin t \, dt$$

$$I_1 = I$$

$$\therefore 2I = I - \frac{\pi}{2}\log 2$$

$$\Rightarrow I = -\frac{\pi}{2}\log 2$$

$$\Rightarrow \int_0^{\pi/2} \log \sin x \, dx = -\pi/2 \log 2$$

13) $x^2\frac{dy}{dx} = y^2+2xy$

$\frac{dy}{dx} = \frac{y^2+2xy}{x^2}$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\frac{v^2x^2+2xvx}{x^2} = v + x\frac{dv}{dx}$

$v^2+2v = v + x\frac{dv}{dx}$

$v^2+v = x\frac{dv}{dx}$

$\frac{dx}{x} = \frac{dv}{v^2+v}$

$\frac{y}{x(y+x)}$

$x(y+x)\frac{dy}{dx} \cdot y(y+x\frac{dy}{dx}+2x)$

$(xy+x^2-xy)\frac{dy}{dx} = y^2+2xy$

$$\int \frac{dx}{x} = \int \frac{dv}{v^2+v}$$

$$\int \frac{dx}{x} = \int \frac{dv}{\cancel{v^2+v}\left(v+\frac{1}{2}\right)^2 - \frac{1}{4}}$$

$$\log x + \log c = \frac{1}{2 \times \frac{1}{2}} \log \left| \frac{v+\frac{1}{2}-\frac{1}{2}}{v+\frac{1}{2}+\frac{1}{2}} \right|$$

$$xc = \frac{v}{v+1}$$

$$xc = \frac{\frac{y}{x}}{\frac{y}{x}+1}$$

$$c = \frac{y}{x(y+x)}$$

$$y = c(xy + x^2)$$

$y(1) = 1$

$1 = c(1+1)$

$\frac{1}{2} = c$

$y = \frac{1}{2}(xy + x^2)$

$2y = xy + x^2$

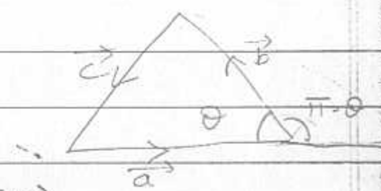
ans:- $x^2 - 2y + xy = 0$.

$\vec{a} + \vec{b} + \vec{c} = 0$ $\quad |\vec{a}| = 3 \quad |\vec{b}| = 5 \quad |\vec{c}| = 7$

$\vec{a}\cdot\vec{b} + |\vec{b}|^2 + \vec{c}\cdot\vec{b} = 0$ (taking dot pdt with $\vec{b}$) - ①

$\vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} + |\vec{c}|^2 = 0$ (" " " $\vec{c}$) - ②

$|\vec{a}|^2 + \vec{b}\cdot\vec{a} + \vec{c}\cdot\vec{a} = 0$ (" " " $\vec{a}$) - ③



(1) + (3) − (2)

$$\Rightarrow \quad |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$$

$$49 = 9 + 25 + 2\times 3\times 5 \cos(\pi-\theta)$$

$$15 = -30\cos\theta$$

$$\cos\theta = \frac{-1}{2}$$

$$\cancel{\theta = \frac{\pi}{3}}$$

~~but~~ ($\angle^{le}$ bet $\vec{a}$ & $\vec{b}$ = $\pi - \theta$)

$$\therefore \quad \theta = \frac{2\pi}{3}$$

$$\boxed{\therefore \angle^{le} \text{ bet } \vec{a} \,\&\, \vec{b} = \frac{2\pi}{3}}$$

11) E – Event that ~~it~~ card drawn is an even no.

F – event that card has no. > 3.

$S = \{1, 2, 3, 4 \ldots 12\}$

$E = \{2, 4, 6, 8, 10, 12\}$

$F = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$P(E) = \frac{6}{12} = \frac{1}{2}$

$P(F) = \frac{9}{12} = \frac{3}{4}$

$E \cap F = \{4, 6, 8, 10, 12\}$

$P(E \cap F) = \frac{5}{12}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{\frac{5}{\cancel{12}\,3}}{\frac{3}{\cancel{4}}}$$

$$= \frac{5}{9}$$

$\boxed{\text{Probability} = \frac{5}{9}}$

10) $\int_0^1 \frac{2x}{1+x^2}\,dx$

$= \int_0^1 \frac{1}{1+x^2}\,d(1+x^2)$

$= \log(1+x^2)\Big|_0^1$

$= \log 2$

9) $\int \frac{2}{3} \frac{\cos x}{\sin^2 x} = \int \frac{2}{3} \cot x \operatorname{cosec} x \, dx$

$= -\frac{2}{3} \int -\cot x \operatorname{cosec} x \, dx$

$= -\frac{2}{3} \operatorname{cosec} x + C$

8) $|3A| = 3^3 |A|$ (3x3)

$= 27 |A|$

$= 27 \times 4$

$|3A| = 108$

7) $\begin{vmatrix} \sin 30^\circ & \cos 30^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{vmatrix}$

$= \sin 30^\circ \cos 60 + \cos 30^\circ \sin 60$

$= \sin (30^\circ + 60^\circ)$

$= \sin 90 = \boxed{1}$

6) $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$

$|A| = 0$

$\Rightarrow 12 - 8x - 2x - 2 = 0$

$12 - 10x = 0$

$x = 1$

5) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$= \frac{2\pi}{3} + \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= \frac{2\pi}{3} + \frac{\pi}{3}$

$= \pi$

4) $a * b = 2a + b - 3$

$3 * 4 = 2 \times 3 + 4 - 3$

$= 6 + 1$

$2 \times 1 = 7$

3) DR's of $\overrightarrow{PQ}$ = $(4, -1), (1, -5), (-2-4)$

DR^s of $\overrightarrow{PQ}$ $3, -4, -6.$

2) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$ (θ = $\angle^{le}$ bet $\vec{a}$ & $\vec{b}$)

$3 = 2 \times \sqrt{3} \cos\theta$

$\frac{\sqrt{3}}{2} = \cos\theta$

$= \frac{\pi}{6},$

1) $\vec{a} - \vec{b} = -2\hat{i} + \hat{j} + 4\hat{k}$

$\vec{a}$ unit vector $= \frac{\vec{a} - \vec{b}}{|\vec{a} - \vec{b}|}$

$= \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{4+1+16}}$

$= \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{21}}$

unit vector $= -2\hat{i} + 1\,\hat{} \ldots$

P.T.O