AMIETE – ET/CS/IT (OLD SCHEME)

JUNE 2009 **Code:**

AE01/AC01/AT01 **Subject: MATHEMATICS-I**

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. A square matrix A is called orthogonal if

(A) $A = A^2$ **(B)** $A' = A^{-1}$

(C) $AA^{-1} = I$ **(D)** $AA' = 0$

b. If every minor or order r of a matrix A is zero, then rank of A is

(A) greater than r **(B)** equal to r

(C) less than or equal to r **(D)** less than r

c. For any square matrix A, $A'A$ is

(A) Hermitian **(B)** Skew Hermitian

(C) Symmetric **(D)** Skew Symmetric

d. If $u = x^y$, then $\frac{\partial u}{\partial x}$ is

(A) 0 **(B)** yx^{y-1}

(C) $x^y \log x$ **(D)** $x^y \log y$

e. $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)}$ equals

(A) 1 **(B)** -1

(C) zero **(D)** none of these

f. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ is equal to

(A) $\pi/4$ **(B)** $\pi/2$
(C) 1 **(D)** 0

g. $\int_0^2\int_1^3\int_1^2 xy^2 z\,dz\,dy\,dx$ is equal to

(A) 13 **(B)** 39
(C) 1 **(D)** 26

h. The particular integral of $(D^2+a^2)y = \sin ax$ is

(A) $-\frac{x}{2a}\cos ax$ **(B)** $\frac{x}{2a}\cos ax$
(C) $-\frac{ax}{2}\cos ax$ **(D)** $\frac{ax}{2}\cos ax$

i. $\int_{-1}^{1} P_n^2(x)\,dx$ is equal to

(A) $\frac{2}{n+1}$ **(B)** $\frac{2}{2n+1}$
(C) $\frac{1}{2n+1}$ **(D)** $\frac{1}{2n-1}$

j. $\lim_{n\to\infty} \left\{\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}\,\ldots\ldots\ldots\ldots\ldots\ldots+\frac{1}{3n}\right\}$

(A) $\log_e 2$ **(B)** $2\log_e 2$
(C) $\log_e 3$ **(D)** $2\log_e 3$

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. Show that the given function are discontinuous at all the point (2,2).

$$f(x,y)=\begin{cases}\dfrac{x^2+xy+x+y}{x+y}, & (x,y)\neq(2,-2)\\ 4, & (x,y)=(2,-2)\end{cases}$$

(8)

b. Find the percentage error in the computed areas of an ellipse when an error of 1% is made in measuring the major and minor axes. **(8)**

Q.3 a. Find the minimum value of $x^2+y^2+z^2$ subject to the condition xyz= a^3. **(8)**

b. Evaluate the integral $\iint_R xy\ dxdy$, where R is the region bounded by the x axis, the line y = 2x and the parabola $y=\dfrac{x^2}{4a}$. **(8)**

Q.4 a. Solve $3\dfrac{dy}{dx}+xy=xy^{-2}$. **(8)**

b. The initial value problem governing the current 'i' flowing in a series RL circuit when a voltage v(t) = t is applied, is given by $iR+L\dfrac{di}{dt}=t;\ t\geq 0, i(0)=0$. When R and L are constants, find the current 'i' at the time t. **(8)**

Q.5 a. Find the general solution of the equation $y''+3y'+2y=2e^x$. Using the method of variation of parameters. **(8)**

b. Find the general solution of equation $y''+4y'+3y=x\sin 2x$. **(8)**

Q.6 a. Find the inverse of A by Gauss-Jordan method, where $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$. **(8)**

b. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **(8)**

Q.7 a. Given that $A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$, show that , $(I-A)(I+A)^{-1}$ is a unitary matrix. **(8)**

b. Test for consistency and solve

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

(8)

Q.8 a. Solve in the series the equation $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$. **(8)**

b. $\int J_3(x)dx = c - J_2(x) - \frac{2}{x}J_1(x)$. **(8)**

Q.9 a. Show that $\int_0^1 xJ_n(\alpha x)\, J_n(\beta x)\, dx = \begin{bmatrix} 0, & \alpha \neq \beta \\ \frac{1}{2}[J_{n+1}(\alpha)]^2, & \alpha = \beta \end{bmatrix}$,

where α, β are the roots of $J_n(x) = 0$. **(8)**

b. Change the order of integration in $I = \int_1^0 \int_{x^2}^{2-x} xy\ dxdy$ and hence evaluate the same. **(8)**