Code: AE01/AC01/AT01 **Subject: MATHEMATICS-I**

Time: 3 Hours **Max. Marks: 100**

DECEMBER 2007

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2 x 10)

a. The value of the limit $\lim_{(x,y)\to(0,0)} \frac{x+\sqrt{y}}{\sqrt{(x^2+y)}}$ is

(A) limit does not exist **(B)** 0
(C) 1 **(D)** -1

b. If $u = x^y$ then the value of $\frac{\partial u}{\partial y}$ is equal to

(A) 0 **(B)** $x^y \log(x)$
(C) xy^{x-1} **(D)** yx^{y-1}

c. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

(A) u **(B)** 2u
(C) 3u **(D)** 0

d. The value of integral $\int_0^2\int_1^3\int_1^2 xy^2 z\,dx\,dy\,dz$ is equal to

(A) 22 **(B)** 26
(C) 5 **(D)** 25

e. The solution of the differential equation $(y+x)^2\frac{dy}{dx} = a^2$ is given by

(A) $y + x = a\tan\left(\frac{y-c}{a}\right)$

(B) $y - x = \tan\left(\frac{y-c}{a}\right)$

(C) $y - x = a\tan(y-c)$

(D) $a(y - x) = \tan\left(y - \frac{c}{a}\right)$

f. The solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$ is

(A) $y = ae^{x} + be^{2x} + \frac{1}{2}e^{3x}$ **(B)** $y = ae^{-x} + be^{-2x} + \frac{1}{2}e^{3x}$

(C) $y = ae^{x} + be^{-2x} + \frac{1}{2}e^{3x}$ **(D)** $y = ae^{-x} + be^{2x} + \frac{1}{2}e^{3x}$

g. If 3x+2y+z= 0, x+4y+z=0, 2x+y+4z=0, be a system of equations then

(A) system is inconsistent
(B) it has only trivial solution
(C) it can be reduced to a single equation thus solution does not exist
(D) Determinant of the coefficient matrix is zero.

h. If λ is an eigen value of a non-singular matrix A then the eigen value of A^{-1} is

(A) 1/ λ **(B)** λ
(C) –λ **(D)** –1/ λ

i. The product of the eigen values of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$ is

(A) 3 **(B)** 8
(C) 1 **(D)** –1

j. The value of the integral $\int x^2 J_1(x)dx$ is

(A) $x^2 J_1(x) + c$ **(B)** $x^2 J_{-1}(x) + c$

(C) $x^2 J_2(x) + c$ **(D)** $x^2 J_{-2}(x) + c$

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. Find the extreme value of the function f(x,y,z) = 2x + 3y + z such that $x^2+y^2=5$ and x + z =1 **(8)**

b. Show that the function $f(x,y)=\begin{cases}(x+y)\sin\left(\dfrac{1}{x+y}\right), & x+y\neq 0\\ 0, & x+y=0\end{cases}$ is continuous at (0,0) but its partial derivatives of first order do not exist at (0,0). **(8)**

Q.3 a. Evaluate the integral $\iiint_T z\,dx\,dy\,dz,$ where T is region bounded by the cone $x^2\tan^2\alpha+y^2\tan^2\beta=z^2$ and the planes z=0 to z=h in the first octant. **(8)**

b. Show that the approximate change in the angle A of a triangle ABC due to small changes $\delta a,\ \delta b,\ \delta c$ in the sides a, b, c respectively, is given by $\delta A=\dfrac{a}{2\Delta}(\delta a-\delta b\cos C-\delta c\cos B)$ where Δ is the area of the triangle. Verify that $\delta A+\delta B+\delta C=0$ **(8)**

Q.4 a. If $x+y=2e^{\theta}\cos\phi\ and\ x-y=2ie^{\theta}\sin\phi$ Show that

$$\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^2 u}{\partial\phi^2}=4xy\frac{\partial^2 u}{\partial x\partial y}$$

(8)

b. Using the method of variation of parameter method, find the general solution of the differential equation $y''+16y=32\sec 2x$ **(8)**

Q.5 a. Find the general solution of the equation $y''-4y'+13y=18e^{2x}\sin 3x$. **(8)**

b. Find the general solution of the equation $x^3\dfrac{d^3y}{dx^3}+5x^2\dfrac{d^2y}{dx^2}+5x\dfrac{dy}{dx}+y=x^2+\ln x$. **(8)**

Q.6 a. Solve $(1+y^2)dx=(\tan^{-1}y-x)dy$ **(8)**

b. The set of vectors $\{x_1, x_2\}$, where $x_1=(1,3)^T$, $x_2=(4,6)^T$ is a basis in R^2. Find a linear transformations T such that $Tx_1=(-2,2,-7)^T$ and $Tx_2=(-2,-4,-10)^T$ **(8)**

Q.7 a. Show that the matrix A is diagonalizable. $A = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$. Hence, obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix. **(8)**

b. Investigate the values of λ for which the equations

$$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0,\ (\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0,$$

$$2x+(3\lambda+1)y+3(\lambda-1)z=0$$

are consistent, and hence find the ratios of x:y:z when λ has the smallest of these values. **(8)**

Q.8 a. Find the first five non-vanishing terms in the power series solution of the initial value problem

$$(1-x^2)y''+2xy'+y=0,\ y(0)=1,\ y'(0)=1.$$ **(11)**

b. Show that $\int xJ_0^2(x)dx=\frac{1}{2}x^2\left[J_0^2(x)+J_1^2(x)\right]$ **(5)**

Q.9 a. Show that $J_{5/2}(x)=\sqrt{\frac{2}{n\pi}}\left[\frac{1}{x^2}(3-x^2)\sin x-\frac{3}{x}\cos x\right]$ **(8)**

b. Show that $\int_{-1}^{1} P_m(x)P_n(x)dx=\begin{cases} 0, & m\neq n \\ \frac{2}{2n+1}, & m=n \end{cases}$. **(8)**