Code: A-01/C-01/T-01 **JUNE 2006** **Subject: MATHEMATICS-I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The value of limit $\lim_{(x,y)\to(0,0)} \frac{2x^2 y}{x^4 + y^2}$ is

(A) 0 **(B)** 1
(C) 2 **(D)** does not exist

b. If $u = \frac{y^3 - x^3}{y^2 + x^2}$, then $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ equals

(A) 0 **(B)** u
(C) 2u **(D)** 3u

c. Let $f(x,y) = x \sin y + e^x \cos y, x = t^2 + 1, y = t^2$. Then the value of $\left(\frac{\partial f}{\partial t}\right)_{t=0}$ is

(A) $e+1$ **(B)** 0
(C) $e-1$ **(D)** e^2+1

d. The value of $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx$ is

(A) 1 **(B)** $1/3$
(C) $2/3$ **(D)** 3

e. The solution of $(D^2 + 2D + 2)y = 0, y(0) = 0, y'(0) = 1$ is

(A) $e^x \sin x$ **(B)** $e^{-x} \cos x$
(C) $e^{-x} \sin x$ **(D)** $e^x \cos x$

f. The solution of $y' + y \tan x = \cos x, y(0) = 0$ is

(A) sin x **(B)** cos x
(C) x sin x **(D)** x cos x

g. Let $v_1 = (1,1,0,1)$, $v_2 = (1,1,1,1)$, $v_3 = (4,4,1,1)$ and $v_4 = (1,0,0,1)$ be elements of R^4. The set of vectors $\{v_1, v_2, v_3, v_4\}$ is

(A) linearly independent **(B)** linearly dependent
(C) null **(D)** none of these

h. The eigenvalues of the matrix $\begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ are

(A) $-1,2$ and 1 **(B)** 0, 1 and 2
(C) –1, –2 and 4 **(D)** 1, 1 and –1

i. Let P_0, P_1, P_2 be the Legendre polynomials of order 0, 1, and 2, respectively. Which of the following statement is correct?

(A) $P_2(x) = 3xP_1(x) + \frac{1}{2}P_0(x)$ **(B)** $P_2(x) = \frac{3}{2}xP_1(x) + \frac{1}{2}P_0(x)$

(C) $P_2(x) = \frac{3}{2}xP_1(x) + P_0(x)$ **(D)** $P_2(x) = \frac{1}{2}xP_1(x) + \frac{3}{2}P_0(x)$

j. Let J_n be the Bessel function of order n. Then $\int \frac{1}{x}J_2(x)dx$ is equal to

(A) $xJ_1(x) + C$ **(B)** $\frac{1}{x}J_1(x) + C$

(C) $-xJ_1(x) + C$ **(D)** $-\frac{1}{x}J_1(x) + C$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Consider the function f (x, y) defined by

$$f(x,y) = \begin{cases} (x^2 + y^2)\sin\frac{1}{\sqrt{x^2 + y^2}}, & \text{if } (x,y) \neq (0,0); \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Find $f_x(0,0)$ and $f_y(0,0)$.

Is $f(x, y)$ differentiable at (0, 0)? Justify your answer. **(8)**

b. Find the extreme values of $f(x, y, z) = x^2 + 2xy + z^2$ subject to the constraints $g(x, y, z) = 2x + y = 0$ and $h(x, y, z) = x + y + z = 1$. **(8)**

Q.3 a. Find all critical points of $f(x, y) = (x^2 + y^2) e^{4x+2x^2}$ and determine relative extrema at these critical points. **(8)**

b. Find the second order Taylor expansion of $f(x, y) = \sin[(x^2 + 1)y]$ about the point $(0, \pi/2)$. **(4)**

c. Change the order of integration in the following double integral and evaluate it : $\int_0^1 \int_y^1 x^2 e^{xy} \, dx dy$. **(4)**

Q.4 a. Solve the differential equation $\frac{dy}{dx} + y = xy^3$. **(4)**

b. Solve the differential equation $\frac{y^{3/2} + 1}{x^{1/2}} dx + (3x^{1/2} y^{1/2} - 1) dy = 0$. **(6)**

c. Find the general solution of the differential equation $x^2 y'' + xy' + 4y = 2x \ln x$. **(6)**

Q.5 a. Find the general solution of the differential equation $16y'' + 8y' + y = 48x\, e^{-x/4}$. **(8)**

b. Find the linear Taylor series polynomial approximation to the function $f(x, y) = 2x^3 + 3y^3 - 4x^2 y$ about the point (1, 2). Obtain the maximum absolute error for the polynomial approximation in the region $\{ |x - 1| < 0.01$, $|y - 2| < 0.1 \}$. **(8)**

Q.6 a. Find the general solution of the differential equation $x^3 y''' - x^2 y'' + 2xy' - 2y = x^3$. **(9)**

b. Show that the eigenvalues of a Hermitian matrix are real. **(7)**

Q.7 a. Using Frobenius method, find two linearly independent solutions of the differential equation $2x(1+x)y'' + (1+x)y' - 3y = 0$. **(10)**

b. Solve the following system of equations by matrix method:

$$
\begin{aligned}
5x + 3y + 14z &= 4 \\
y + 2z &= 1 \\
2x + y + 6z &= 2 \\
x + y + 2z &= 0
\end{aligned}
$$

(6)

Q.8 a. Express the polynomial $7x^4 + 6x^3 + 3x^2 + x - 6$ in terms of Legendre polynomials. **(8)**

b. Let J_α be the Bessel function of order α. Show $\sqrt{\frac{\pi x}{2}}\; J_{3/2}(x) = \frac{\sin x}{x} - \cos x$. **(8)**

Q.9 a. If A is a diagonalizable matrix and f (x) is a polynomial, then show that f (A) is also diagonalizable. **(7)**

b. Let $A = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$. Find the matrix P so that $P^{-1}AP$ is a diagonal matrix. **(9)**