NARAYANA IIT ACADEMY - INDIA

IIT – JEE (2011) PAPER I QUESTION & SOLUTIONS (CODE 0)

PART I : CHEMISTRY

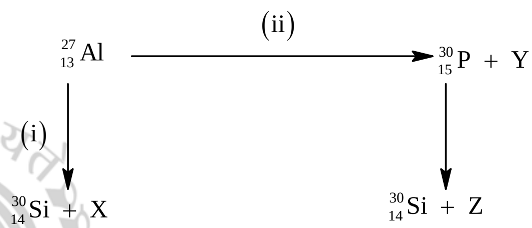
PAPER – I

SECTION – I (TOTAL MARKS: 21)

(Single Correct Answer Type)

This section contains 7 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Bombardment of aluminum by α–particle leads to its artificial disintegration in two ways, (i) and (ii) as shown. Products X, Y and Z respectively are,



(A) proton, neutron, positron
(B) neutron, positron, proton
(C) proton, positron, neutron
(D) positron, proton, neutron, neutron.

Key: **(A)**

Sol.:

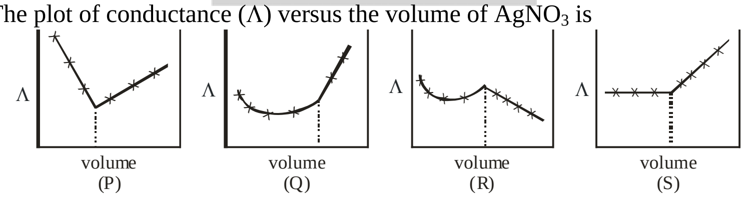
$$^{27}_{13}Al + {}^{4}_{2}He \xrightarrow{(ii)} {}^{30}_{15}P + {}^{1}_{0}n \ (Y)$$

$\downarrow$(i) $\qquad\qquad$ $\downarrow$(i)

$^{30}_{14}Si + {}^{1}_{1}H$ (X) $\qquad$ $^{30}_{14}Si + {}_{+1}e^{0}$ (Z)

Thus 'X' , "Y' and 'Z' is proton neutron and positron respectively

2. $AgNO_3$(aq.) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance (Λ) versus the volume of $AgNO_3$ is



(A) (P) $\qquad$ (B) (Q)
(C) (R) $\qquad$ (D) (S)

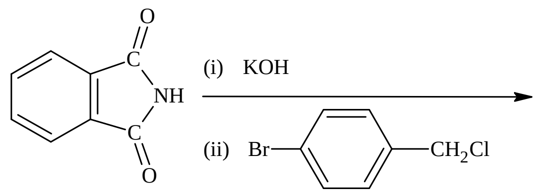
Key: (D)

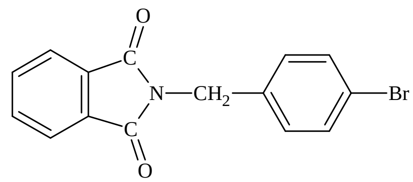
Sol.: $Ag^{+} + NO_3^{-} + K^{+} + Cl^{-} \longrightarrow AgCl\downarrow + K^{+} + NO_3^{-}$

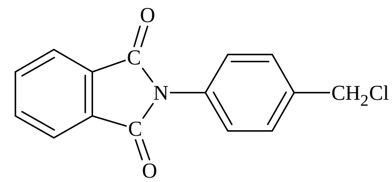
$AgNO_3(aq) + K^{+}Cl^{-}(aq) \longrightarrow AgCl(s) + K^{+}NO_3^{-}$

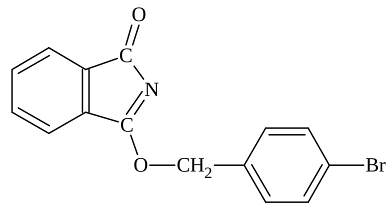
On adding $AgNO_3$ p.p.t will not started immediately until ionic product of AgCl > Ksp after it Cl^{-} will be precipited in the form of AgCl but at the same time NO_3^{-} will be added to the solution hence the number ion remain same therefore the conductance will remain same when the precipitation completed on further adding $AgNO_3$ the number of ions in the solution increases hence the conductance increases.

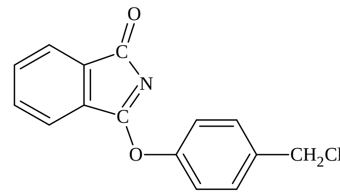
3. The major product of the following reaction is



(A) 

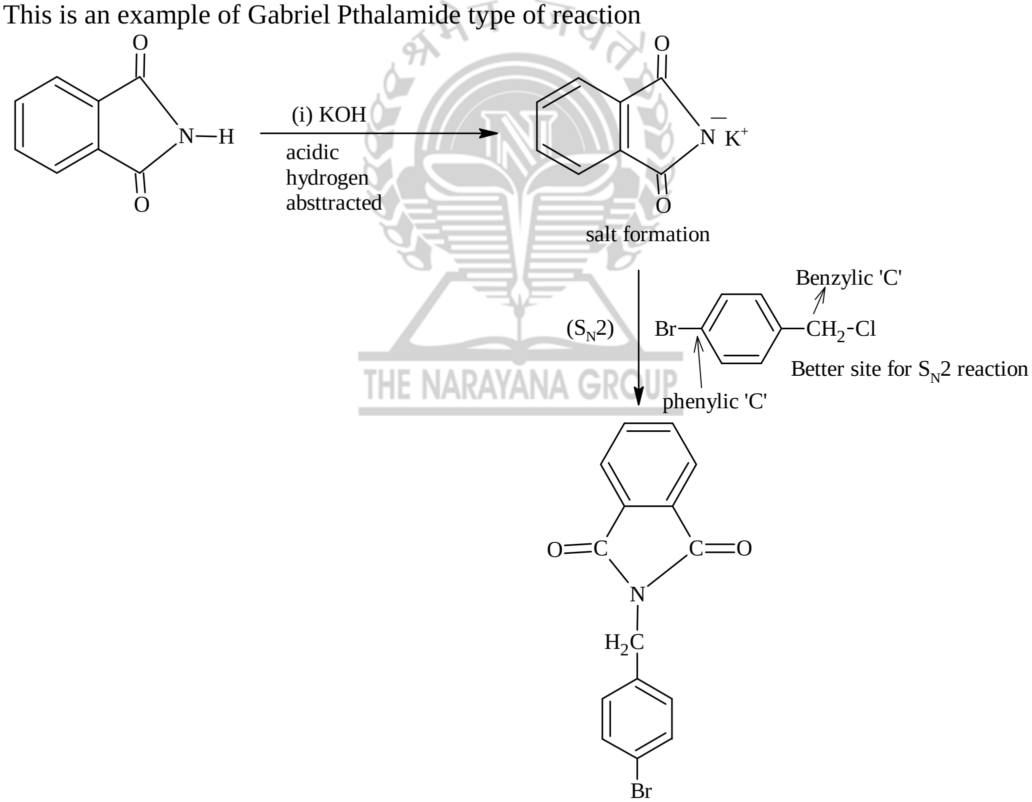
(B) 

(C) 

(D) 

Key: (A)

Sol.: This is an example of Gabriel Pthalamide type of reaction

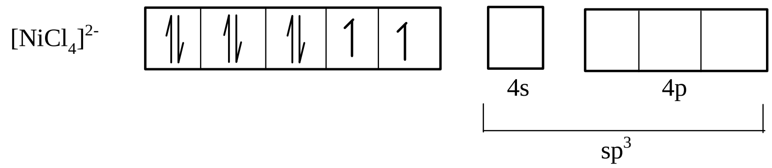


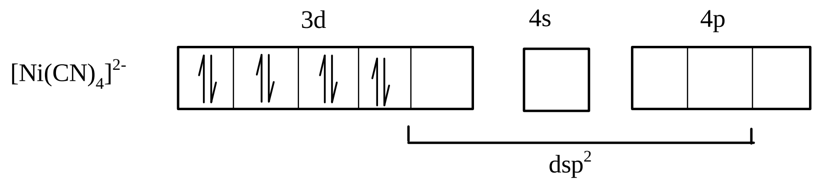
4. Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^-, CN^- and H_2O, respectively, are

(A) octahedral, tetrahedral and square planar
(B) tetrahedral, square planar and octahedral
(C) square planer, tetrahedral and octahedral
(D) octahedral, square planar and octahedral.

Key: (B)

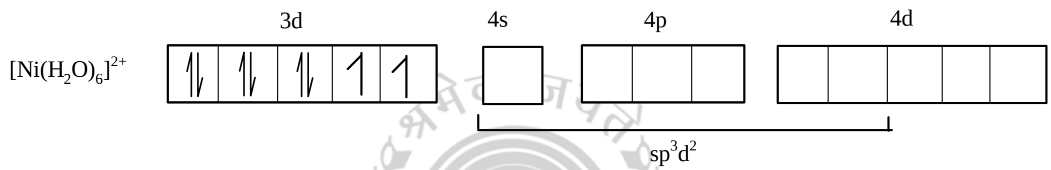
Sol.: Ni(28) = [Ar] $3d^8$, $4s^2$ and Ni^{2+} = [Ar] $3d^8$, $4s^0$





due to strong field ligand CN^-

$[Ni(H_2O)_6]^{+2}$



5. Extra pure N_2 can be obtained by heating

(A) NH_3 with CuO (B) NH_4NO_3
(C) $(NH_4)_2Cr_2O_7$ (D) $Ba(N_3)_2$.

Key: (D)

Sol.: $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2\,(g)$

6. Dissolving 120 g of urea (mol. wt. 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is

(A) 1.78 M (B) 2.00 M
(C) 2.05 M (D) 2.22 M.

Key: (C)

Sol.: Volume of solution = $\dfrac{\text{mass of solution}}{\text{density of solution}}$

$$= \frac{1000+120}{1.15} = 973.91$$

$$\text{Molarity} = \frac{\omega}{m} \times \frac{1000}{V} = \frac{1000 \times 120}{973.91 \times 60} = 2.05$$

7. Among the following compounds, the most acidic is

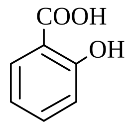
(A) p-nitrophenol (B) p-hydroxybenzoic acid
(C) o-hydroxybenzoic acid (D) p-toluic acid.

Key: (C)

Sol.:

Compound	***pk_a***
p-nitrophenol	7.15
p-hydroxybenzoic acid	4.58
o-hydroxybenzoic acid	2.98
p-toluic acid	4.37

Hence



is the most acidic one due to ortho effect.

SECTION – II (TOTAL MARKS: 16)
(Multiple Correct Answers Type)

This section contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

8. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is(are)
(A) Adsorption is always exothermic.
(B) Physisorption may transform into chemisorption at high temperature.
(C) Physisorption increases with increasing temperature but chemisorption decreases with increasing temperature.
(D) Chemisorption is more exothermic than physisorption, however it is very slow due to higher energy of activation.

Key: (A, B, D)
Sol.: According to adsorption theory

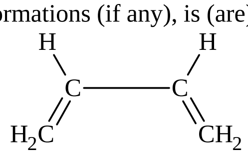
9. According to kinetic theory of gases
(A) collisions are always elastic.
(B) heavier molecules transfer more momentum to the wall of the container.
(C) only a small number of molecules have very high velocity.
(D) between collisions, the molecules move in straight lines with constant velocities.

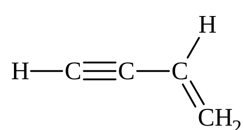
Key: (A, B, C, D)
Sol.: According to kinetic theory of gases.

10. Extraction of metal from the ore cassiterite involves
(A) carbon reduction of an oxide ore
(B) self-reduction of a sulphide ore
(C) removal of copper impurity
(D) removal of iron impurity.

Key: (A, D)

11. Amongst the given options, the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is (are)

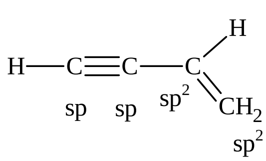
(A)


(B)


(C) $H_2C{=}C{=}O$

(D) $H_2C{=}C{=}CH_2$

Key: (B, C)
Sol.: (A) Only two conformer cisoid and transoid have all the atom in same plane. where as other conformer in different plane.
(D) The terminal H of allene will be in perpendicular to each other plane.

(B)


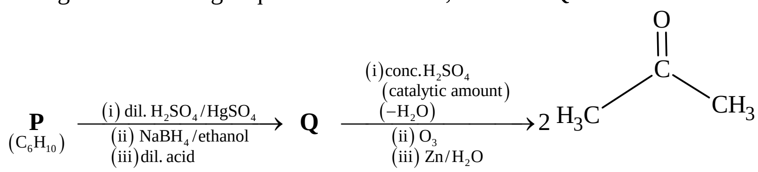
(C) $CH_2 = C = O$
sp^2 sp (absence of any atom at oxygen)

SECTION – III (TOTAL MARKS: 15)
(Paragraph Type)

This section contains 2 paragraphs. Based upon one of the paragraph 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 12 to 13

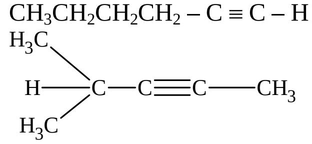
An acyclic hydrocarbon **P**, having molecular formula C_6H_{10}, gave acetone as the only organic product through the following sequence of reactions, in which Q is an intermediate organic compound.

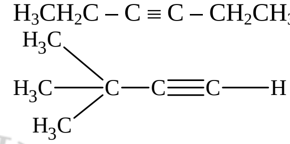


12. The structure of compound P is

(A) $CH_3CH_2CH_2CH_2 - C \equiv C - H$

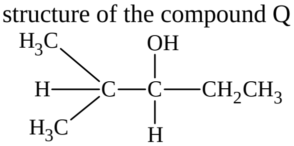
(B) $H_3CH_2C - C \equiv C - CH_2CH_3$

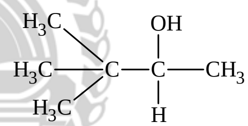
(C) 

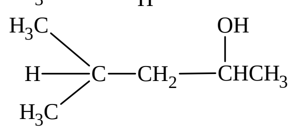
(D) 

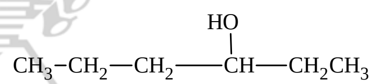
Key: (D)

13. The structure of the compound Q is

(A) 

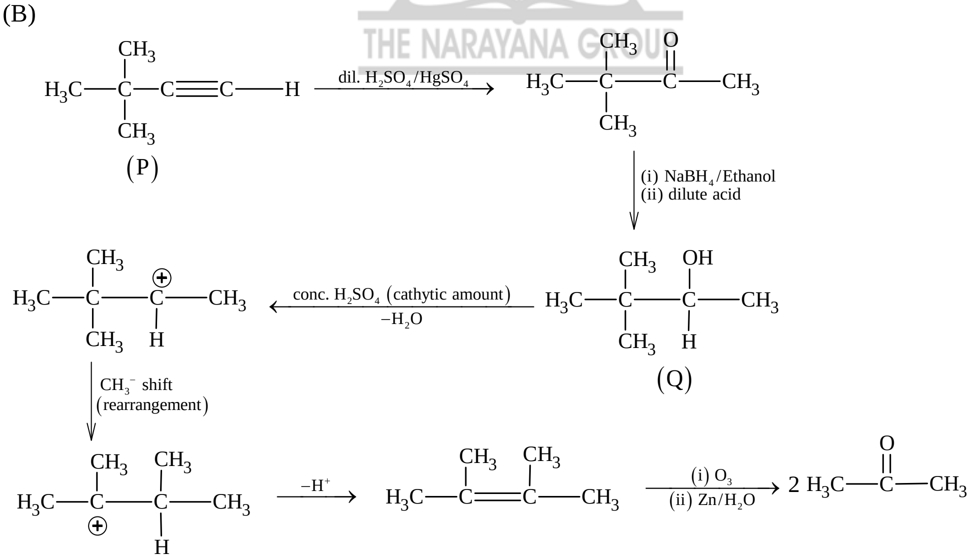
(B) 

(C) 

(D) 

Key. (B)

Sol.:



Paragraph for Question Nos. 14 to 16

When a metal rod M is dipped into an aqueous colourless concentrated solution of compound N, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives a white precipitate O. Addition of aqueous NH_3 dissolves O and gives an intense blue solution.

14. The metal rod M is

(A) Fe (B) Cu

(C) Ni (D) Co

Key: (B)

15. The compound N is

(A) $AgNO_3$ (B) $Zn(NO_3)_2$

(C) $Al(NO_3)_3$ (D) $Pb(NO_3)_2$

Key: (A)

16. The final solution contains

(A) $\left[Pb(NH_3)_4\right]^{2+}$ and $\left[CoCl_2\right]^{2-}$ (B) $\left[Al(NH_3)_4\right]^{3+}$ and $\left[Cu(NH_3)_4\right]^{2+}$

(C) $\left[Ag(NH_3)_2\right]^{+}$ and $\left[Cu(NH_3)_4\right]^{2+}$ (D) $\left[Ag(NH_3)_2\right]^{+}$ and $\left[Ni(NH_3)_6\right]^{2+}$

Key: (C)

Sol.: $\underset{(M)}{Cu} + \underset{(N)}{2AgNO_3} \longrightarrow \underset{\text{blue}}{Cu(NO_3)_2} + 2Ag\downarrow$

Blue solution contains $Cu(NO_3)_2$ and unreacted $AgNO_3$

$AgNO_3 + NaCl \longrightarrow AgCl\downarrow + NaNO_3$

$AgCl + 2NH_4OH \longrightarrow \left[Ag(NH_3)_2\right]Cl + 2H_2O$

$Cu(NO_3)_2 + 4NH_4OH \longrightarrow \underset{\text{intense blue solution}}{\left[Cu(NH_3)_4\right](NO_3)_2} + 4H_2O$

SECTION – IV (TOTAL MARKS: 28)

(Integer Answer Type)

This Section contains 7 questions. The answer to each question is a Single Integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to darkened in the ORS.

17. The maximum number of electrons that can have principal quantum number, n = 3, a spin quantum number, $m_s = -½$, is

Key: 9

Sol.: n = 3, Total number of orbitals = $n^2 = 9$

$\therefore$ No. of electrons having $m_s = -\frac{1}{2} = 9$.

18. Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromate with evolution of CO_2 gas. The number of sodium bromide molecules involve in the balanced chemical equation is

Key: 5

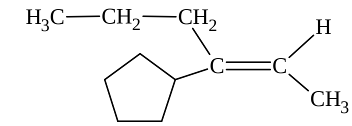
Sol.: $3Br_2 + 3CO_3^{2-} \longrightarrow 5Br^- + BrO_3^- + 3CO_2$

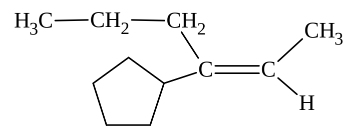
19. The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is

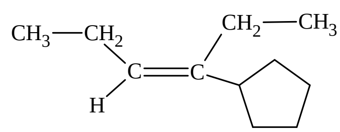
Key: 5

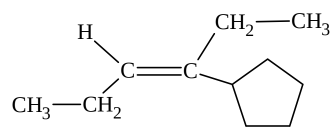
Sol.: The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentyl hexane using alcoholic KOH is 5.

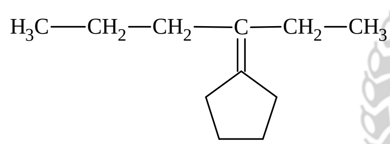
The following products will be formed











20. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

Key: 4

Sol.: $\lambda = 300 \text{ nm} = 300 \times 10^{-9} \text{ m} = 3 \times 10^{-7} \text{ m}$

$$E = h\frac{c}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{3\times10^{-7}} = 6.6\times10^{-19} J$$

$$E \text{ in eV} = \frac{6.6\times10^{-19}}{1.6\times10^{-19}} = 4.1\,\text{eV}$$

Number of metals having ϕ less than 4.1 eV
= 4.

21. A decapeptide (mol. Wt. 796) on complete hydrolysis gives glycine (mol. Wt. 75), alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is

Key. 6

Sol.:

$$-\overset{\displaystyle O}{\overset{\|}{C}}-NH- \xrightarrow{H_2O} -\overset{\displaystyle O}{\overset{\|}{C}}-OH + NH_2 -$$

Let there are n glycine unit in the compound because it is decapeptide hence 9 water molecule will be added during hydrolysis therefore total weight of the product will be

$796 + 9 \times 18 = 958$

% Wt. of glycine in the given weight of product

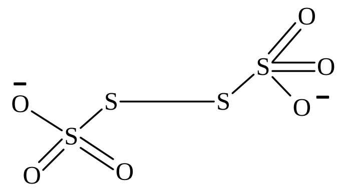
$= \dfrac{75n}{958} \times 100 = 47$

$n = \dfrac{47 \times 958}{75 \times 100} \approx 6.$

22. The difference in the oxidation number of the two types of sulphur atoms in $Na_2S_4O_6$ is

Key: 5

Sol.:



Two sulphur are in zero oxidation state and two sulphur are in +5 oxidation state. The difference in the oxidation number of the two types of sulphur atoms in $Na_2S_4O_6$ is 5.

23. To an evacuated vessel with movable piston under external pressure of 1 atm., 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. at 0°C) are introduced. Considering the ideal gas behaviour the total volume (in litre) of the gaes at 0°C is close to

Key: 7

Sol.: Let the volume of gas is V

Mole of the vapour of compound

$PV = nRT \rightarrow n = \dfrac{PV}{RT} = \dfrac{0.68 \times V}{RT}$ . . . (i)

Hence total mole of gas in vessel will be

$= \left(0.1 + \dfrac{0.689V}{RT}\right)$

There for applying gas equation

PV = nRT

$1 \times V = \left(0.1 + \dfrac{0.68V}{RT}\right)RT = 0.1RT + 0.68V$

or $0.32\ V = 0.1\ RT$

$V = \dfrac{0.1 \times 0.082 \times 273}{0.32} \approx 7$

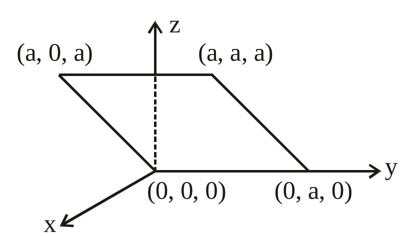
PART II : PHYSICS

SECTION – I (TOTAL MARKS: 21)

(Single Correct Answer Type)

This section contains 7 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

24. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



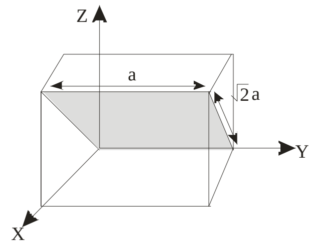
(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$

(C) E_0a^2 (D) $\dfrac{E_0a^2}{\sqrt{2}}$

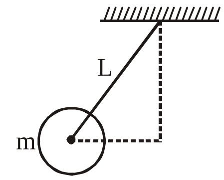
Key. (C)

Sol. $\phi = EA\cos 45^{o}$

$= E_0\left(\sqrt{2}a^2\right)\dfrac{1}{\sqrt{2}} = E_0 a^2$



25. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension the string can bear is 324N. The maximum possible value of angular velocity of ball (in radian/s) is

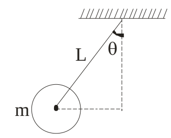


(A) 9 (B) 18
(C) 27 (D) 36

Key. (D)

Sol. $T\cos\theta = mg$ …(1)

$T\sin\theta = m(L\sin\theta)\omega^2$ …(2)



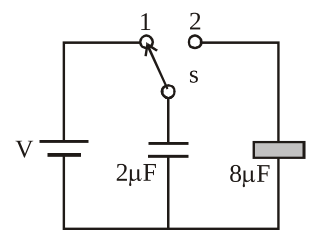
from (2)

$T = mL\omega^2$

$\Rightarrow \quad 324 = 0.5 \times 0.5 \times \omega^2$

$\Rightarrow \quad \omega = \dfrac{\sqrt{324}}{0.5} = \dfrac{18}{0.5} = 36$

26. A 2μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(A) 0% (B) 20%
(C) 75% (D) 80%

Key. (D)

Sol. $u_i = \dfrac{1}{2} \times 2 \times V^2 = V^2$

$u_f = \dfrac{(2 \times V)^2}{2 \times 10} = \dfrac{V^2}{5}$

Loss of energy $= V^2 - \dfrac{V^2}{5} = \dfrac{4V^2}{5}$

% loss $= \dfrac{\frac{4V^2}{5}}{V^2} \times 100 = \dfrac{4}{5} \times 100 = 80\%$

27. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is

(A) $\dfrac{9}{8}RT_1$ (B) $\dfrac{3}{2}RT_1$

(C) $\dfrac{15}{8}RT_1$ (D) $\dfrac{9}{2}RT_1$

Key. (A)

Sol. $W = \dfrac{nR(T_1 - T_2)}{\gamma - 1}$

$T_1(5.6)^{2/3} = T_2(0.7)^{2/3}$

$\Rightarrow\ T_2 = T_1(8)^{2/3}$

$= T_1 \times 4$

$W = \dfrac{nR \times 3T_1}{2/3} = \dfrac{9}{2}nRT_1$

But $n = \dfrac{1}{4}$

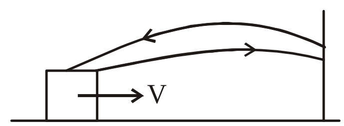
$\Rightarrow\ W = \dfrac{9}{8}RT_1$

28. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(A) 8.50 kHz (B) 8.25 kHz
(C) 7.75 kHz (D) 7.50 kHz

Key. (A)

Sol. Frequency received by total bulting



$f' = f\left[\dfrac{C}{C - V}\right]$

$\Rightarrow\ f' = 8\left[\dfrac{320}{310}\right]$

Now wall becomes source so frequency heard by driven is

$f'' = f'\left[\dfrac{C + V}{C}\right]$

$\Rightarrow\ f'' = \dfrac{8 \times 320}{310} \times \dfrac{330}{320} = \dfrac{8 \times 33}{31} = 8.52$

So, $f'' \approx 8.5\,\text{kHz}$

29. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561Å. The wavelength of the second spectral line in the Balmer series of singly–ionized helium atom is

(A) 1215 Å (B) 1640 Å
(C) 2430 Å (D) 4687 Å.

Key. (A)

Sol. $\dfrac{hc}{\lambda_1} = 13.6(1)^2\left[\dfrac{1}{2^2} - \dfrac{1}{3^2}\right]$

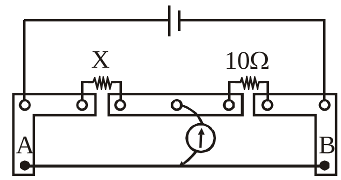
$\dfrac{hc}{\lambda_2} = 13.6(2)^2\left[\dfrac{1}{2^2} - \dfrac{1}{4^2}\right]$

Dividing

$\dfrac{\lambda_2}{\lambda_1} = \dfrac{1 \times (5/36)}{4 \times (3/16)}$

$\Rightarrow\ \lambda_2 = \dfrac{\lambda_1 \times 5 \times 16}{36 \times 4 \times 3} = \dfrac{5}{27}\lambda_1 = 1215\ \text{Å}$

30. A meter bridge is set–up as shown, to determine an unknown resistance X using standard 10 ohm resistor. The galvanometer shows null point when tapping key is a 52 cm mark. The end–corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is



(A) 10.2 ohm (B) 10.6 ohm
(C) 10.8 ohm (D) 11.1 ohm

Key. (B)

Sol. $\frac{X}{53} = \frac{10}{50}$

$X = \frac{53}{50} \times 10 = \frac{53}{5} = 10.6\ \Omega$

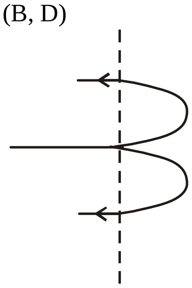
SECTION – II (TOTAL MARKS: 16)
(Multiple Correct Answers Type)

This section contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

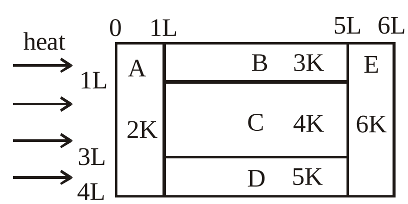
31. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi–infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true ?
(A) they will never come out of the magnetic field region
(B) they will come out traveling along parallel paths
(C) they will come out at the same time
(D) they will come out at different times.

Key. (B, D)

Sol.



32. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat Q flows only from left to right through the blocks. Then in steady state



(A) heat flow through A and E slabs are same
(B) heat flow through slab E is maximum
(C) temperature difference across slab E is smallest
(D) heat flow through C = heat flow through B + heat flow through D.

Key. (A, B, C, D)

Sol. $\frac{dQ_B}{dt} = 3K\left(\frac{A}{4}\right)\frac{\Delta T_2}{4L} = i_B$

$\frac{dQ_C}{dt} = 4K\left(\frac{2A}{4}\right)\frac{\Delta T_2}{4L} = i_C$

$\frac{dQ_D}{dt} = 5K\left(\frac{A}{4}\right)\frac{\Delta T_2}{4L} = i_D$

$i_B + i_D = i_C$

33. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge +Q. Now they are connected by a thin metal wire. Then

(A) $E_A^{inside} = 0$ (B) $Q_A > Q_B$

(C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (D) $E_A^{on\ surface} < E_B^{on\ surface}$.

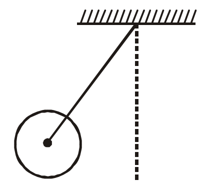
Key. (A, B, C, D)

Sol. $Q_A + Q_B = 2Q$

$\frac{Q_A}{R_A} = \frac{Q_B}{R_B} \Rightarrow \sigma_A R_A = \sigma_B R_B$

So, A, B, C, D all are correct

34. A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius R (< L) is attached at its centre to the free end of the rod. Consider two ways the disc is attached: (case A) The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod–disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true ?



(A) restoring torque in case A = restoring torque in case B
(B) restoring torque in case A < restoring torque in case B
(C) angular frequency for case A > angular frequency for case B
(D) angular frequency for case A < angular frequency for case B.

Key. (A, D)

Sol. Resorting torque is same in both cases

$$\alpha = \frac{\tau}{I} = -\omega^2\theta$$

In case A the moment of inertia is more as compared to B, so $\omega_B > \omega_A$

SECTION – III (TOTAL MARKS: 15)

(Paragraph Type)

This section contains 2 paragraphs. Based upon one of the paragraph 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 35 to 36

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ω_p, which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

35. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_p.

(A) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\frac{m\varepsilon_0}{Ne}}$

(C) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$.

Key. (C)

Sol. Let $\omega_P \propto e^a \varepsilon_0^b N^C m^d$

Putting dimensions

$$[T^{-1}] \propto [Q]^a \left[\frac{Q^2}{ML^3T^{-2}}\right]^b [L^{-3}]^c [M]^d$$

Solving $\omega_P \propto \sqrt{\frac{Ne^2}{m\varepsilon_0}}$.

$\therefore$ Answer is (C)

36. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(A) 800 nm (B) 600 nm

(C) 300 nm (D) 200 nm.

Key. (B)

Sol. $\omega_p = \sqrt{\frac{Ne^2}{m\varepsilon_0}} = \sqrt{\frac{4 \times 10^{27}}{10^{-30} \times 10^{-11}}} \times 1.6 \times 10^{-19}$ (from previous question)

$= 1.6 \times 10^{-19} \times 2 \times 10^{34} = \times 1.6 \times 2 \times 10^{15}$

$f = \frac{\omega_p}{2\pi} = \frac{1.6 \times 2 \times 10^{15}}{2 \times 3.14} = 0.5 \times 10^{15}$

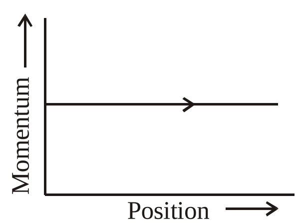
$c = f\lambda$

$\Rightarrow \quad \lambda = \frac{3 \times 10^8}{0.5 \times 10^{15}} = 6 \times 10^{-7} = 600\,\text{nm}$

$\therefore$ Answer is (B)

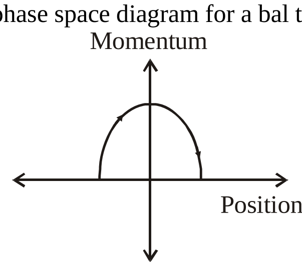
Paragraph for Question Nos. 37 to 39

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one–dimension. For such system, phase space is a plane is which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x (t) vs p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

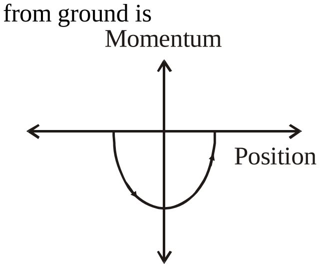


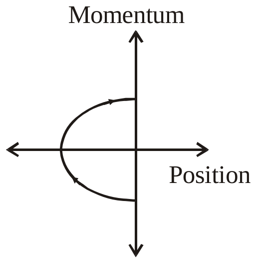
37. The phase space diagram for a bal thrown vertically up from ground is

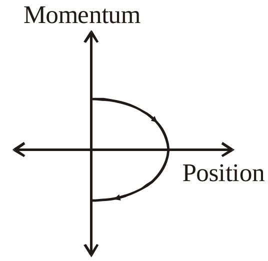
(A)



(B)



(C) 

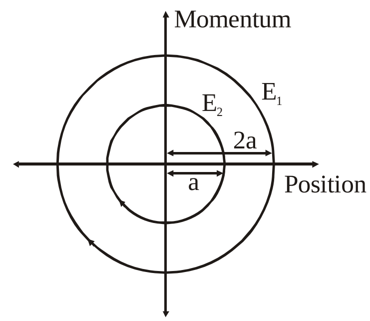
(D) 

Key. (D)

Sol. The momentum is initially positive and then negative.

$\therefore$ Answer is (D)

38. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



(A) $E_1 = \sqrt{2}\,E_2$ (B) $E_1 = 2\,E_2$

(C) $E_1 = 4\,E_2$ (D) $E_1 = 16\,E_2$.

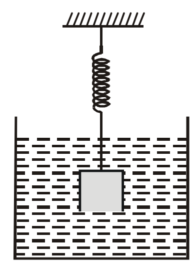
Key. (C)

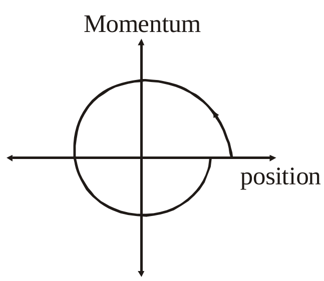
Sol. From diagram $(\text{amplitude of oscillator})_1 = 2 \times (\text{amplitude of oscillator})_2$

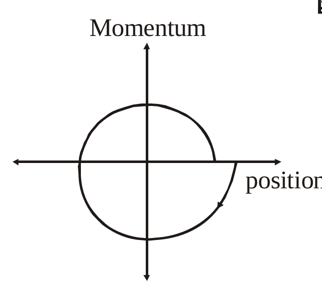
$\Rightarrow$ $E_1 = 4E_2$

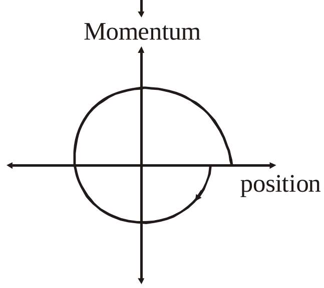
$\therefore$ Answer is (C)

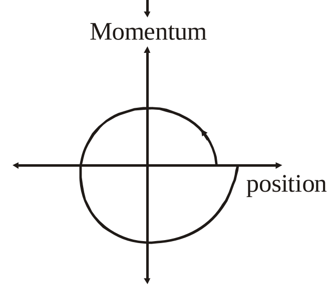
39. Consider the spring mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A) 

(B) 

(C) 

(D) 

Key. (B)

Sol. From diagram, initial position is positive meaning thereby that the block starts from above the mean position. As it comes down, momentum first increases and then decreases (in negative direction).

$\therefore$ Answer is (B)

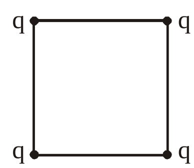
SECTION – IV (TOTAL MARKS: 28)
(Integer Answer Type)

This Section contains 7 questions. The answer to each question is a Single Integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to darkened in the ORS.

40. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is γ. The system of charges and planer film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{\frac{1}{N}}$, where k is a constant. Then N is

Key. 3.

Sol. For equilibrium of a charge



$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(\frac{1}{2}+\sqrt{2}\right) = C\gamma a$$

Where C is a constant

$\Rightarrow \quad a^3 \propto \frac{q^2}{\gamma}$

$\therefore \quad N = 3.$

41. A block is moving on an inclined plane making an angle 45º with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10 μ, then N is

Key. 5.

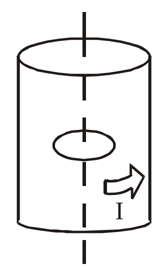
Sol. $(mg \sin\theta + \mu\, mg \cos\theta) = 3\,(mg \sin\theta - \mu\, mg \cos\theta)$

$\Rightarrow \quad 1 + \mu = 3\,(1 - \mu)$

$\Rightarrow \quad \mu = 0.5$

$\Rightarrow \quad N = 5.$

42. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire–loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300\,t)$ where I_0 is constant. If the magnetic moment of the loop is $N\,\mu_0\, I_0 \sin(300\,t)$, then N is



Key. 6.

Sol. From Ampere's Law

$$B_{in} = \frac{\mu_0 I}{L}$$

$\therefore \quad \varepsilon_{ind} = -\frac{d\phi}{dt} = -\frac{\mu_0}{L}\cdot \pi(r)^2\frac{dI}{dt}$

$\therefore \quad I_{ind} = \frac{\varepsilon_{ind}}{R}$

$\therefore \quad |\vec{M}| = I_{ind}\cdot \pi r^2 = 6\mu_0 I_0 \sin(300t)$

$\Rightarrow \quad N = 6.$

43. Steel wire of length L at 40ºC is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40ºC to 30ºC to regain its original length L. The coefficient of linear thermal expansion of the steel is 10^{-5}/ºC, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the value of m in kg is nearly

Key. 3.

Sol. For equilibrium,

$$mg = YA\frac{\ell}{A}$$

and $\ell = L\alpha\Delta t$

$$\Rightarrow \quad m = \frac{\gamma A\alpha\Delta t}{g}$$

$$= \frac{10^{11}\times\pi(1\times10^{-3})^2\times10^{-5}\times10}{10}$$

$$\cong 3\text{kg}.$$

44. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The momentum of inertia of the system about the diagonal of the square is N × 10^{-4} kg–m^2, then N is

Key. 9

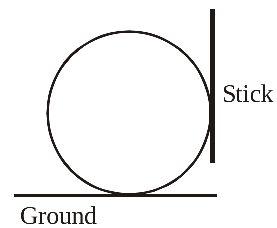
Sol.

$$I = 2\times\frac{2}{5}mR^2 + 2\left[\frac{2}{5}mR^2 + md^2\right]$$

$$= 9\times10^{-4}\,\text{kg m}^2$$

$\Rightarrow$ N = 9.

45. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applied a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is

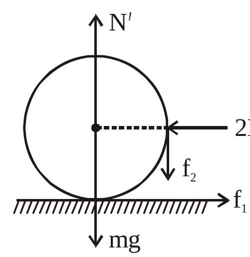


Key. 4.

Sol. Equations of motion are

$$2 - f_1 = 2\times0.3 \quad \ldots(i)$$

and $(f_1 - f_2)0.5 = 2(0.5)^2\cdot\frac{0.3}{0.5} \quad \ldots(ii)$



solving f_2 = 0.8 N

$\Rightarrow$ μ = 0.8/2 = 0.4.

$\therefore$ P = 4.

46. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

Key. 1.

Sol.

$$\left|\frac{dN}{dt}\right| = \lambda N = \lambda\frac{M}{M_0}$$

M_0 = Mass of an atom

$\Rightarrow$ M = 1 mg.

PART III : MATHEMATICS

SECTION – I (TOTAL MARKS: 21)
(Single Correct Answer Type)

This section contains 7 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

47. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$

(B) $-3\hat{i} - 3\hat{j} - \hat{k}$

(C) $3\hat{i} - \hat{j} + 3\hat{k}$

(D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Key: (C)

Sol.: $\vec{v} = x\vec{a} + y\vec{b}$

$= x(\hat{i} + \hat{j} + \hat{k}) + y(\hat{i} - \hat{j} + \hat{k})$

$\vec{v} = (x+y)\hat{i} + (x-y)\hat{j} + (x+y)\hat{k}$

$\frac{\vec{v}.\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}}$

$\frac{x+y-(x-y)-(x+y)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$-x + y = 1$

$x - y = -1$

$y = x + 1$

$\vec{v} = (x+1+x)\hat{i} - \hat{j} + (x+x+1)\hat{k}$

$\vec{v} = (2x+1)\hat{i} - \hat{j} + (2x+1)\hat{k}$

$2x + 1 = 3$

$x = 1$

Put $x = 1$ in $\vec{v}$

48. Let the straight line x = b divide the area enclosed by $y = (1 - x)^2$, y = 0 and x = 0 into two parts R_1 ($0 \le x \le b$) and R_2 ($b \le x \le 1$) such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

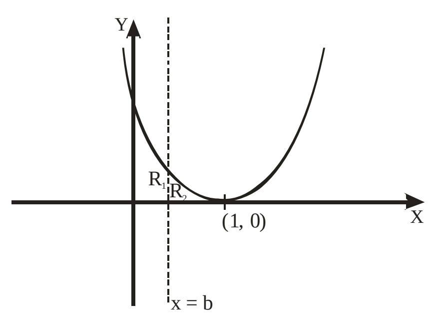
Key: (B)

Sol.: $\int_0^b (1-x)^2 dx - \int_b^1 (1-x)^2 dx = \frac{1}{4}$

$\int_0^1 (1-x)^2 dx - 2\int_b^1 (1-x)^2 dx = \frac{1}{4}$

$\left[\frac{(x-1)^3}{3}\right]_0^1 - 2\left[\frac{(x-1)^3}{3}\right]_b^1 = \frac{1}{4}$

$\frac{2}{3}\left[(x-1)^3\right]_b^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$



$= -(b-1)^3 = \frac{1}{8}$

$(b-1)^3 = -\frac{1}{8}$

$b-1 = -\frac{1}{2}$

$b = \frac{1}{2}$

49. A straight line L through the point (3, -2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$
(B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$
(D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

Key: (B)

Sol.: Two line can be drawn at an angle 60º with $\sqrt{3}x + y = 1$ and their slope will be.

120º + 60º = 180º

and 120º – 60º = 60º

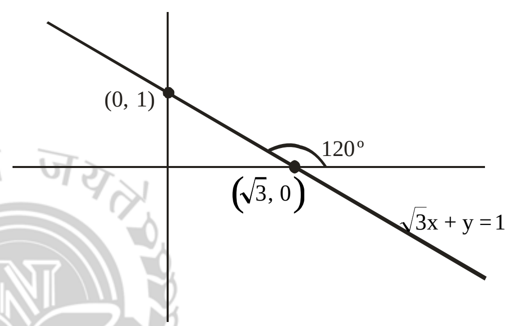
As line *L* intersects x-axis

So m = 60º

And line will be

$y + 2 = \sqrt{3}(x - 3)$

$\Rightarrow y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$



50. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

(A) $\frac{1}{4}\ln\frac{3}{2}$
(B) $\frac{1}{2}\ln\frac{3}{2}$
(C) $\ln\frac{3}{2}$
(D) $\frac{1}{6}\ln\frac{3}{2}$

Key: (A)

Sol.: Put $x^2 = t$

$\therefore \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t}{\sin t + \sin(\ln 6 - t)} dt$

$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t) + \sin(t)} dt$

$2I = \frac{1}{2}\int_{\ln 2}^{\ln 3} dt = \frac{1}{2}\ln\frac{3}{2}$

$I = \frac{1}{4}\ln\frac{3}{2}$

51. Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$

The x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) 6

Key: (C)

Sol.: $(2x)^{\ln 2} = (3y)^{\ln 3}$

$\ln 3y = (\ln 2x)\frac{\ln 2}{\ln 3}$

$\ln 2 \ln 2x = \ln 3 \ln 3y$ … (i)

$\ln x \ln 3 = \ln y \ln 2$ … (ii)

$\ln 3 + \ln y = \ln 3y = \frac{\ln 2}{\ln 3}\ln 2x$

$\ln y = \frac{\ln 2(\ln 2x) - (\ln 3)^2}{\ln 3}$

$= \frac{(\ln 2)^2 + \ln x \times \ln 2 - (\ln 3)^2}{\ln 3}$ … (iii)

from (ii) $\ln y = \frac{\ln 3}{\ln 2} \times \ln x$

$\frac{\ln 3}{\ln 2}\ln x = \frac{(\ln 2)^2 + \ln 2 \times \ln x - (\ln 3)^2}{\ln 3}$

$\ln x\left(\frac{\ln 3}{\ln 2} - \frac{\ln 2}{\ln 3}\right) = \frac{(\ln 2)^2 - (\ln 3)^2}{\ln 3}$

$\ln x\left(\frac{(\ln 3)^2 - (\ln 2)^2}{\ln 2 \ln 3}\right) = \left(\frac{(\ln 2)^2 - (\ln 3)^2}{\ln 3}\right)(-1)$

$\ln x = \ln\frac{1}{2}$

$x = \frac{1}{2}$

52. Let P = {θ : sinθ - cosθ = $\sqrt{2}$ cosθ} and Q = {θ : sinθ + cosθ = $\sqrt{2}$ sinθ} be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$

(C) $P \not\subset Q$ (D) P = Q

Key: (D)

Sol.: $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\left(\sqrt{2}+1\right)\cos\theta = \sin\theta$

$\tan\theta = \sqrt{2}+1$

$\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\left(\sqrt{2}-1\right)\sin\theta = \cos\theta$

$\tan\theta = \frac{1}{\sqrt{2}-1}$

$= \sqrt{2}+1$

Hence P = Q

53. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with α > β. If $a_n = \alpha^n - \beta^n$ for n ≥ 1, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

(A) 1 (B) 2

(C) 3 (D) 4

Key: (C)

Sol.: $\alpha^2 - 6\alpha - 2 = 0$

$\beta^2 - 6\beta - 2 = 0$

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8\left(\alpha^2 - 2\right) - \beta^8\left(\beta^2 - 2\right)}{2\left(\alpha^9 - \beta^9\right)}$$

$$\frac{\alpha^8 \times 6\alpha - \beta^8 \times 6\beta}{2(\alpha^9 - \beta^9)} = 3.$$

SECTION – II (TOTAL MARKS: 16)

(Multiple Correct Answers Type)

This section contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

54. Let M and N be two 3 × 3 non-singular skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to

(A) M^2 (B) $-N^2$

(C) $-M^2$ (D) MN

Key: Wrong

Sol.: Statement is wrong.

55. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

Key: (B, D)

Sol.: Eccentricity of the ellipse $e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Focus (±ae, 0)

Focus $(\pm\sqrt{3}, 0)$

$e_H = \frac{2}{\sqrt{3}}, \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ will pass through the points $(\pm\sqrt{3}, 0)$

$$\frac{3}{a^2} - 1 = 0$$

$\Rightarrow$ $a^2 = 3$

$$b^2 = 3\left(\frac{4}{3} - 1\right) = 1$$

Hence hyperbola is

$$\frac{x^2}{3} - y^2 = 1$$

$$x^2 - 3y^2 = 3$$

its focus is (±ae, 0)

$$\left(\pm\sqrt{3}.\frac{2}{\sqrt{3}}, 0\right)$$

(±2, 0)

56. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$

(C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

Key: (A, D)

Sol.: Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$

$\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

Let vector $\vec{c}$ is coplanar with $\vec{a}$ & $\vec{b}$

$\vec{c}=\lambda\vec{a}+\mu\vec{b}$

$\vec{c}=(\lambda+\mu)\hat{i}+(\lambda+2\mu)\hat{j}+(2\lambda+\mu)\hat{k}$

$\vec{c}\perp$ to the $\hat{i}+\hat{j}+\hat{k}$

$\lambda + \mu + \lambda + 2\mu + 2\lambda + \mu = 0$

$4\lambda + 4\mu = 0$

$\lambda + \mu = 0$

Hence $\vec{c}=\lambda(\vec{a}-\vec{b})$

$=\lambda(-\hat{j}+\hat{k})$

if $\lambda = \pm 1$

then $\vec{c}=-\hat{j}+\hat{k}$ & $\hat{j}-\hat{k}$

57. Let $f : R \to R$ be a function such that $f(x + y) = f(x) + f(y)$, $\forall x, y\in R$
If f(x) is differentiable at x = 0, then

(A) f(x) is differentiable only in a finite interval containing zero

(B) f(x) is continuous $\forall x\in R$

(C) $f'(x)$ is constant $\forall x\in R$

(D) f(x) is differentiable except at finitely many points.

Key: (B, C)

Sol.: x = y = 0

$\Rightarrow$ f(0) = 0

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(x+h)-f(x+0)}{h}$$

$$=\lim_{h\to 0}\frac{f(x)+f(h)-f(x)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=f'(0)=k$$

(Given f(x) is differentiable x = 0

$\Rightarrow$ $f'(0) = k$(let)

f(x) = kx + c

f(0) = 0

$\Rightarrow$ c = 0,

f(x) = kx

Hence clearly f(x) is continuous & has constant derivative $\forall x \in R$.

SECTION – III (TOTAL MARKS: 15)

(Paragraph Type)

This section contains 2 paragraphs. Based upon one of the paragraph 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 58 to 59

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears, then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears, then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

58. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$

(C) $\frac{19}{30}$ (D) $\frac{11}{30}$

Key: (B)

Sol.: Required probability

$$= \frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right]+\frac{1}{2}\left[\frac{3}{10}\times 1+\frac{1}{10}\times\frac{1}{3}+\frac{6}{10}\times\frac{2}{3}\right]$$

$$= \frac{23}{30}$$

59. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$

(C) $\frac{15}{23}$ (D) $\frac{12}{23}$

Key: (D)

Sol.: Required probability

$$= \frac{\frac{4}{10}}{\frac{23}{30}} = \frac{12}{23}$$

Paragraph for Question Nos. 60 to 62

Let a, b and c be three real numbers satisfying

$$[a \;\; b \;\; c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0 \;\; 0 \;\; 0] \;...\; (E)$$

60. If the point P(a, b, c), with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12

(C) 7 (D) 6

Key: (D)

61. Let ω be a solution of $x^3 - 1 = 0$ with Im $(\omega) > 0$. If a = 2 with b and c satisfying (E), then the value of $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$ is equal to

(A) -2 (B) 2

(C) 3 (D) -3

Key: (A)

62. Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is

(A) 6 (B) 7

(C) $\frac{6}{7}$ (D) ∞

Key: (B)

60-62. $[a\ b\ c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix} = [0\ 0\ 0]$

$$\left.\begin{array}{l} a+8b+7c=0 \\ \Rightarrow 9a+2b+3c=0 \\ 7a+7b+7c=0 \end{array}\right\} \ldots (*)$$

60. P(a, b, c) lies on the plane
$2x + y + z = 1 \Rightarrow 2a + b + c = 1$
Now in (*), if we put $c = \lambda$
We get $a = -\frac{\lambda}{7}, b = -\frac{6\lambda}{7}$
Now $2a + b + c = 1 \Rightarrow \lambda = -7$
So a = 1, b = 6, c = -7
Hence 7a + b + c = 6.

61. If we put a = 2 in (*), we get
b = 12, c = -14
So $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$
$= \frac{3}{\omega^2}+\frac{1}{\omega^{12}}+\frac{3}{\omega^{-14}}$
$= \frac{3}{\omega^2}+\frac{1}{1}+\frac{3}{\omega}$
$= 1 + 3\left[\frac{1}{\omega^2}+\frac{1}{\omega}\right] = 1 + 3(\omega + \omega^2)$
$= 1 + 3[-1] = -2$.

62. If we put b = 6, we get
a = 1, c = -7
So quadratic equation becomes
$x^2 + 6x - 7 = 0 \Rightarrow (x + 7)(x - 1) = 0$
So, $\alpha = -7, \beta = 1$
$\Rightarrow \frac{1}{\alpha}+\frac{1}{\beta} = \frac{6}{7}$
Now, $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n$
$= \frac{1}{1-\frac{6}{7}} = 7$.

SECTION – IV (TOTAL MARKS: 28)

(Integer Answer Type)

This Section contains 7 questions. The answer to each question is a Single Integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to darkened in the ORS.

63. The positive integer value of n > 3 satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

Key: **7**

Sol.: $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\frac{2\pi}{n}} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$

$\sin\left(\frac{2\pi}{n}\right)\sin\left(\frac{3\pi}{n}\right) = \sin\frac{\pi}{n}.\sin\frac{3\pi}{n} + \sin\frac{\pi}{n}.\sin\frac{2\pi}{n}$

$\sin\frac{2\pi}{n}\left[\sin\frac{3\pi}{n} - \sin\frac{\pi}{n}\right] = \sin\frac{\pi}{n}.\sin\frac{3\pi}{n}$

$\sin\frac{2\pi}{n}\left[2\cos\frac{2\pi}{n}.\sin\frac{\pi}{n}\right] = \sin\frac{\pi}{n}.\sin\frac{3\pi}{n}$

$\sin\frac{\pi}{n}.\sin\frac{4\pi}{n} - \sin\frac{\pi}{n}\sin\frac{3\pi}{n} = 0$

$\sin\frac{\pi}{n}.2.\cos\frac{7\pi}{2n}.\sin\frac{\pi}{2n} = 0$

$2\sin\frac{\pi}{n}\cos\frac{7\pi}{2n}\sin\frac{\pi}{2n}$ **= 0**

$\Rightarrow \cos\frac{7\pi}{2n} = 0 \Rightarrow n = 7$ $\left(\begin{array}{l}\text{As } \sin\frac{\pi}{n} \neq 0 \\ \sin\frac{\pi}{2n} \neq 0\end{array}\right)$

64. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integr n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

Key: (9)

Sol.: $S_p = \sum_{i=1}^{p} a_i = \frac{p}{2}(2a_1 + (p-1)d)$

$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}\left[2a_1 + (5n-1)d\right]}{\frac{n}{2}\left[2a_1 + (n-1)d\right]}$

$= \frac{5(6 + (5n-1)d)}{(6 + (n-1)d)} = k$ (constant)

$\Rightarrow 30 + 25nd - 5d = 6k + knd - kd$

$\Rightarrow (25 - k)dn + (k - 5)(d - 6) = 0 \ \forall\ 1 \le n \le 20$

$\Rightarrow (25 - k)d = 0$

k = 25 (here we have neglected d = 0)

then d = 6 so, $a_2 = a_1 + d = 9$

65. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with a > 0 is

Key: (8)

Sol.: a > 0, $\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8} \ge 1$ (Applying AM ≥ GM)

$\Rightarrow a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10} \ge 8$

So, the required minimum value is 8.

66. Let $f : [1, \infty) \to [2,\infty)$ be a differentiable function such that f(1) = 2. If $6\int_1^x f(t)\,dt = 3x\, f(x) - x^3$ for all $x \geq 1$, then the value of f(2) is

Key: (6)

Sol.: $f : [1, \infty) \to [2, \infty)$, given f(1) = 2

$$6\int_1^x f(t)\,dt = 3x.f(x) - x^3 \quad \forall\, x \geq 1$$

Differentiating w.r.t. x we get,

$6[f(x)] = 3\, f(x) + 3x\, f'(x) - 3x^2$

$\Rightarrow f(x) = xf'(x) - x^2$

$\Rightarrow \dfrac{xf'(x) - f(x)}{x^2} = 1$

$\Rightarrow \dfrac{d}{dx}\left(\dfrac{f(x)}{x}\right) = 1 \;\Rightarrow\; \dfrac{f(x)}{x} = x + c$

$\Rightarrow f(x) = x^2 + cx$

Now as $f(1) = 2 \Rightarrow c = 1$ (however by putting x = 1 in the given relation, f(1) can't be 2)

So, $f(x) = x^2 + x$

$\Rightarrow f(2) = 6.$

67. Let $f(\theta) = \sin\left(\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$. Then the value of $\dfrac{d}{d(\tan\theta)}(f(\theta))$ is.

Key: (1)

Sol.: $f(\theta) = \sin\left(\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right), -\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$

Let $\tan^{-1}\dfrac{\sin\theta}{\sqrt{\cos 2\theta}} = \alpha \Rightarrow \tan\alpha = \dfrac{\sin\theta}{\sqrt{\cos 2\theta}}$

$\Rightarrow f(\theta) = \sin\alpha = \dfrac{\sin\theta}{\sqrt{\cos 2\theta + \sin^2\theta}} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\Rightarrow \dfrac{d}{d(\tan\theta)}(f(\theta)) = \dfrac{d}{d(\tan\theta)}(\tan\theta) = 1.$

68. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\dfrac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangnets at P and at the end points of the latus rectum. Then $\dfrac{\Delta_1}{\Delta_2}$ is

Key: (2)

Sol.:
$$\frac{\Delta_1}{\Delta_2} = \frac{\frac{1}{2}\left|\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}\right|}{\frac{1}{2}\left|\begin{vmatrix} at_2t_3 & a(t_2+t_3) & 1 \\ at_3t_1 & a(t_3+t_1) & 1 \\ at_1t_2 & a(t_1+t_2) & 1 \end{vmatrix}\right|} = \frac{2\left|\begin{vmatrix} t_1^2 & t_1 & 1 \\ t_2^2 - t_1^2 & t_2 - t_1 & 0 \\ t_3^2 - t_2^2 & t_3 - t_2 & 0 \end{vmatrix}\right|}{\left|\begin{vmatrix} t_2t_3 & t_2+t_3 & 1 \\ t_3(t_1-t_2) & t_1 - t_2 & 0 \\ t_1(t_2-t_3) & t_2 - t_3 & 0 \end{vmatrix}\right|}$$

(Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$ both in numerator and denominator)

$$= \frac{2\,|(t_2 - t_1)(t_3 - t_2)(t_1 - t_3)|}{|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|} = 2.$$

(The result is true for any three points lying on the parabola).

69. If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the minimum value of $|2z - 6 + 5i|$ is

Key: (5)

Sol.: $|z - 3-2i| \le 2 \Rightarrow$ z will be lie on boundary or inside the circle having centre at (3, 2) and radius 2 units.

The value of $|z - 3 + \frac{5i}{2}|$ will be minimum when z = 3.

So, the minimum value of $|2z - 6 + 5i|$ will be 5.