DipIETE – ET / CS (OLD SCHEME)

JUNE 2009

Code: DE01 / DC01 **Subject: MATHEMATICS - I**

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

a. If $\sin\theta + \cos\theta = x$, then the value of $\sin^6\theta + \cos^6\theta$ is equal to

(A) $\frac{1}{4}$ **(B)** $\frac{1}{4}\left(1+6x^2\right)$

(C) $\frac{1}{4}\left(1+6x^2-3x^4\right)$ **(D)** $\frac{1}{2}\left(5-3x^2\right)$

b. If $\tan\theta = \frac{m^2-n^2}{2mn}$ then the value of cosec θ is equal to

(A) $\frac{m^2+n^2}{mn}$ **(B)** $\frac{m^2-n^2}{m^2+n^2}$

(C) $\frac{m^2+n^2}{m^2-n^2}$ **(D)** $\frac{m^2+mn}{m^2-n^2}$

c. The value of definite integral $\int_{-a}^{a} |x|\,dx$ is equal to

(A) a **(B)** a^2

(C) 0 **(D)** 2a

d. If (3, –4) and (–6, 5) are the extremities of the diagonal of a parallelogram and (–2, 1) is the third vertex, then the fourth vertex is

(A) (–1, 0) **(B)** (0,–1)
(C) (–1, 1) **(D)** None of these.

e. If the circle $x^2+y^2+2x+3y+1=0$ cuts $x^2+y^2+4x+3y+2=0$ in A and B, then the equation of the circle on AB as diameter is

(A) $x^2+y^2+x+3y+3=0$
(B) $2x^2+2y^2+2x+6y+1=0$
(C) $x^2+y^2+x+6y+1=0$
(D) None of these.

f. If the rth, (r+1)th and (r+2)th terms in the expansion of $(1+x)^{14}$ are in A.P., then the value of r is given by

(A) 8 **(B)** 6
(C) 7 **(D)** 9

g. If $\sin y = x\sin(a+y)$, then $\frac{dy}{dx}$ equals to

(A) $\sin(a+y)$ **(B)** $\sin^2 a$
(C) $\frac{\sin^2(a+y)}{\sin a}$ **(D)** $\frac{\sin^2 a}{\sin(a+y)}$

h. The curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ touches the line $\frac{x}{a}+\frac{y}{b}=2$ at the points (a,b) for n =

(A) 1 **(B)** 2
(C) 3 **(D)** all non-zero values of *n*.

i. The value of $I=\int_{-1}^{1} e^{|x|}dx$ is equal to

(A) (e–1) **(B)** 2(e–1)
(C) 3(e–1) **(D)** 2(1–e)

j. The solution of $ye^y dx = \left(y^3+2xe^y\right)dy$ is

(A) $x^2+y^2e^{-y}=cy^2$ **(B)** $x-y^2e^{-y}=cy^2$
(C) $x+y^2e^{-y}=cy^2$ **(D)** None of these.

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. In a triangle ABC, a, b, c are the sides of triangle and A, B, C are the angles, then find the value of $\frac{c - b\cos A}{b - c\cos A}$ in terms of angles. **(5)**

b. In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in A.P, then find the length of the third side. **(5)**

c. If $\sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2} = \tan^{-1}x$, then find the value of *x*. **(6)**

Q.3 a. If the third term in the expansion of $\left(x + x^{\log_{10}x}\right)^5$ is 10^6 then find the value of *x*. **(8)**

b. If $y = \left[x - \left(x^2/2\right) + \left(x^3/3\right) - \left(x^4/4\right) + \ldots\ \infty\right]$ and $|x| < 1$, then $x = \left[y + \left(y^2/2!\right) + \left(y^3/3!\right) + \left(y^4/4!\right) + \ldots\infty\right]$ **(8)**

Q.4 a. The function

$$f(x) = \begin{cases} \dfrac{x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \dfrac{2b^2 - 4b}{x^2}, & x \ge \sqrt{2} \end{cases}$$

is continuous for $0 \le x < \infty$ then find the most suitable values of *a* and *b*. **(8)**

b. If *f(x)* is twice differentiable such that $f''(x) = -f(x)$ and $f'(x) = g(x)$, $h(x) = [f(x)]^2 + [g(x)]^2$, then find the value of *h*(10) if *h*(5) = 11. **(8)**

Q.5 a. A, B are two points (3, 4) and (5, -2); find the point P such that PA = PB and the area of triangle PAB = 10. **(8)**

b. If p and p' are the perpendicular from the origin on the straight lines whose equations are $x \sec\theta - y \operatorname{cosec}\theta = a, x\cos\theta + y\sin\theta = a\cos 2\theta$ prove that $4p^2 + p'^2 = a^2$. **(8)**

Q.6 a. Let A be the centre of the circle $x^2+y^2-2x-4y-20=0$. Suppose the tangents at the points B (1,7) and D (4, –2) on the circle meet at the point C. Find the area of quadrilateral ABCD. **(8)**

b. If the normal at the end of a latus rectum of an ellipse passes through one extremity of a minor axes, show that eccentricity of the curve is given by $e^4 + e^2 - 1 = 0$. **(8)**

Q.7 a. Prove that the sum of intercepts on the coordinate axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant. **(8)**

b. Show that the semi-vertical angle of the cone of maximum volume of given slant height is $\tan^{-1}\sqrt{2}$. **(8)**

Q.8 a. Prove that $\int_0^1 x\left(\tan^{-1} x\right)^2 dx = \frac{\pi}{4}\left(\frac{\pi}{4} - 1\right) + \frac{1}{2}\log 2$. **(8)**

b. If $U_n = \int_0^{\pi/2} x\left(\sin^n x\right)dx \; (n > 1)$ then prove that $U_n = \frac{n-1}{n}U_{n-2} + \frac{1}{n^2}$. Deduce that $U_5 = \frac{149}{225}$ **(8)**

Q.9 a. Find the volume formed by the revolution of the loop of the curve $y^2(a+x) = x^2(a-x)$ about x-axis. **(8)**

b. Solve $x\sin x\frac{dy}{dx} + \left(x\cos x + \sin x\right)y = \sin x$ **(8)**