DECEMBER 2008 **Code: DE01 / DC01**

Subject: MATHEMATICS - I

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. If one root of the $x^2 + ax + 2b^2 = 0$ be double of the other, then

(A) a=2b **(B)** b=2a
(C) a=3b **(D)** b=3a

b. $\frac{\sin 3A + \sin A}{\cos 3A + \cos A}$ is equal to

(A) tan A **(B)** tan 2A
(C) cot A **(D)** cot 2A

c. The point (x, y) lies on the line joining (2, 1) and (–6, –3) if

(A) x = 2y **(B)** y = 2x
(C) x = y **(D)** x+y = 0

d. The equation of the straight line which passes through (3, 5) and is parallel to 2x+3y = 7 is

(A) 3x – 2y = 9 **(B)** 2x + 3y = 19
(C) 3x – 2y = –1 **(D)** 2x + 3y = 21

e. The equation of the circle passing through the origin and making intercepts –2 and 3 on x-axis and y-axis respectively is

(A) $x^2 + y^2 - 2x + 3y = 0$ **(B)** $x^2 + y^2 + 2x - 3y = 0$

(C) $x^2 + y^2 - 2x - 3y = 0$ **(D)** $x^2 + y^2 + 2x + 3y = 0$

f. If $y = \log(\sec x + \tan x)$, then dy/dx is equal to

(A) $\frac{1}{\sec x + \tan x}$ **(B)** $\sec x$
(C) $\tan x$ **(D)** $\sec x - \tan x$

g. The value of $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$ is equal to

(A) 1 **(B)** 2
(C) 0 **(D)** none of these

h. $\int \frac{dx}{1+\sin x}$ is equal to

(A) $\log (1 + \sin x)$ **(B)** $\frac{\log(1+\sin x)}{\cos x}$
(C) $\sec x - \tan x$ **(D)** $\tan x - \sec x$

i. The area bounded by the axis of x and the curve $y = 1 - x^2$ is

(A) $1/3$ **(B)** $2/3$
(C) $4/3$ **(D)** 1

j. The order and the degree of the differential equation $x^2\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$ are

(A) 3, 2 **(B)** 2, 3
(C) 2, 4 **(D)** 3, 4

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. The coefficients of second, third and fourth terms in the expansion of $(1+x)^n$ are in A.P.; find the value of n. **(8)**

b. If the sum of first n terms of an A.P. is zero, show that the sum of next m terms is $-\frac{a(n+m)m}{n-1}$, if a be the first term of the A.P. **(8)**

Q.3 a. If $A+B+C=\pi$, show that

$$\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$ **(8)**

b. In any triangle ABC, show that
$a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$ **(8)**

Q.4 a. Determine the ratio in which $3x-5y+8=0$ divides the join of (4, 3) and (8, 7). Also find the coordinates of that point. **(8)**

b. Find the equation of a straight line passing through the point of intersection of $5x-3y=1$ and $2x+3y=23$ and perpendicular to the line $x-2y=3$. **(8)**

Q.5 a. Find the equation of a circle whose centre is (3, – 4) and passes through the intersection of the straight lines $3x + 4y = 0$ and $4x + 3y = 0$. **(8)**

b. Find the vertex, focus, latus rectum and directrix of the parabola $y^2=4x+4y$. **(8)**

Q.6 a. Find the differential coefficient of tan x from first principle. **(8)**

b. Find $\frac{dy}{dx}$, if $y=\tan^{-1}\left[\frac{\cos x+\sin x}{\cos x-\sin x}\right]$. **(8)**

Q.7 a. Find the points at which the function $y = (x-1)(x-2)(x-3)$ has maximum and minimum values. **(8)**

b. Evaluate $\int\sqrt{\frac{2+x}{2-x}}dx$. **(8)**

Q.8 a. Evaluate $\int_0^{\frac{\pi}{4}}\log(1+\tan\theta)\,d\theta$. **(8)**

b. Find the volume of the solid generated by the revolution of the semi-circle of radius a, about its bounding diameter. **(8)**

Q.9 Solve the following differential equations:-

(i) $\left(x^2 - yx^2\right)dy + \left(y^2 + xy^2\right)dx = 0$.

(ii) $\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x$. **(2 x 8 = 16)**