Code: DE01 / DC01   **Subject: MATHEMATICS - I**
Time: 3 Hours   **Max. Marks: 100**

NOTE: There are 9 Questions in all.
- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1   **Choose the correct or best alternative in the following:**
          (2x10)

a.   The number of terms in the sequence $\frac{5}{2}, 5, 10 \ldots\ldots\ldots, 640$ are

   (A) 8   (B) 9
   (C) 10   (D) 6

b.   First three terms in the expansion of $\left(1 - 2x^3\right)^{11/2}$ are

   (A) $1 + 11x^3 + \frac{99}{2}x^6 + \ldots\ldots\ldots$ (B) $1 + \frac{11}{2}x^3 + 99x^6 + \ldots\ldots$

   (C) $1 - \frac{11}{2}x^3 - \frac{99}{2}x^6 + \ldots\ldots$ (D) $1 - 11x^3 + \frac{99}{2}x^6 + \ldots\ldots$

c.   Value of $\tan 105°$ is

   (A) $-\left(2 + \sqrt{3}\right)$ (B) $2 - \sqrt{3}$
   (C) $-2 + \sqrt{3}$ (D) $2 + \sqrt{3}$

d.   If $\cos A = \frac{4}{5}$, then the value of cos 2A is

   (A) $\frac{3}{25}$ (B) $\frac{1}{25}$

   (C) $\frac{2}{25}$ (D) $\frac{7}{25}$

e. The value of 'x' such that PQ = QR, where P, Q and R are (6, -1), (1, 3) and (x, 8) respectively is given by

(A) 5, –3   (B) 3, 5
(C) 2, 5   (D) 2, 3

f. Slope of the line passing through the points $\left(\frac{5}{2},3\right)$ & $\left(0,\frac{3}{4}\right)$ is

(A) $\dfrac{9}{10}$ (B) $\dfrac{3}{5}$

(C) $\dfrac{9}{5}$ (D) $\dfrac{10}{9}$

g. $\underset{x\to3}{\mathrm{Lim}}\dfrac{x^2-4x+3}{x^2-2x-3}$ is equal to

(A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$

(C) $\dfrac{1}{2}$ (D) $-\dfrac{1}{3}$

h. If $y=\sin^{-1}\left(3x-4x^3\right)$ then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{3}{\sqrt{1-x^2}}$ (B) $\dfrac{-3}{\sqrt{1-x^2}}$

(C) $\dfrac{2}{1+x^2}$ (D) $\dfrac{-1}{\sqrt{1-x^2}}$

i. $\int \sin 3x \sin 2x\, dx$ is equal to

(A) $\dfrac{1}{2}\left(\cos x+\dfrac{\cos 5x}{5}\right)$ (B) $\dfrac{-1}{2}\left(\cos x+\dfrac{\sin 5x}{5}\right)$

(C) $\dfrac{1}{2}\left(\sin x-\dfrac{\sin 5x}{5}\right)$ (D) $\dfrac{1}{2}\left(\sin x+\dfrac{\sin 5x}{5}\right)$

j. Order and degree of the differential equation $\dfrac{d^3y}{dx^3} + \left(\dfrac{d^2y}{dx^2}\right)^3 + \dfrac{dy}{dx} + 4y = \sin x$ is given by

(A) 3, 2   (B) 2, 3
(C) 1, 3   (D) 3, 1

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If 5 times the 5th term of an A.P. is equal to the 10 times the 10th term, find the 15th term of the A.P. **(8)**

    b. If S_n denotes the sum of n terms of a G.P., prove that $\left(S_{10} - S_{20}\right)^2 = S_{10}\left(S_{30} - S_{20}\right)$. **(8)**

Q.3 a. Show that $\dfrac{\sin A - \sin 3A + \sin 5A - \sin 7A}{\cos A - \cos 3A - \cos 5A + \cos 7A} = \cot 2A$. **(8)**

    b. If in the triangle ABC, A = $60°$, prove that $\dfrac{1}{c+a} + \dfrac{1}{a+b} = \dfrac{3}{a+b+c}$. **(8)**

Q.4 a. Find the equation of the straight line which passes through the intersection of the lines $x + y - 3 = 0$ and $2x - y = 0$ and is inclined at an angle of $45°$ with x-axis. **(8)**

    b. Show that $9x^2 + 4y^2 - 54x - 56y + 241 = 0$ represents an ellipse. Find its centre, vertices, foci, eccentricity, directrices, latusrectum and equations of major and minor axis. **(8)**

Q.5 a. Find the equation of the circle which passes (4, 1) & (6, 5) and having centre on the line 4x+y =16. **(8)**

    b. Find the value of $\underset{x \to b}{\text{Lim}} \dfrac{e^{ax} - e^{ab}}{x - b}$ **(8)**

Q.6 a. Differentiate $y = \tan x$ w.r.t. 'x' from first principle. **(6)**

    b. Differentiate $y = x^{\sin x} + \left(\sin x\right)^x$ w.r.t 'x'. **(10)**

Q.7 a. Prove that straight line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point where the curve crosses the axis of y. **(8)**

b. Find the volume generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x-axis. **(8)**

Q.8 a. Prove that $\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2}\log 2$. **(10)**

b. Solve $\int \frac{\left(\sin^{-1} x\right)^3}{\sqrt{1-x^2}} \, dx$. **(6)**

Q.9 a. Solve $3e^x \tan y \, dx + \left(1 - e^x\right)\sec^2 y \, dy = 0$. **(8)**

b. Solve $\left(1 + x^2\right)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to the initial condition y(0) = 0. **(8)**