DECEMBER 2006

Code: D-01 / DC-01 **Subject: MATHEMATICS - I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. If one root of the equation $2x^2 - 10x + K = 0$ is $\frac{2}{3}$ of the other root, then K is

(A) 2 **(B)** 8
(C) 10 **(D)** 12

b. The centroid of the triangle formed by the straight lines $y + x = 3, y - x = 3, y = 0$ is

(A) (0, 0) **(B)** (1, 0)
(C) (0, 1) **(D)** (1, 1)

c. The distance between the parallel lines 3x + 4y + 5 = 0 and 3x + 4y + 15 = 0 is

(A) 1 **(B)** 2
(C) 3 **(D)** 5

d. $\lim_{x \to 0} \frac{\sin mx - \sin nx}{x}$, where $m \neq n$ is equal to

(A) m **(B)** n
(C) m – n **(D)** m + n

e. If $y = \sin^2 2x$, then $\frac{dy}{dx}$ is equal to

(A) 2 sin 4x **(B)** 4 sin 2x
(C) sin 4x **(D)** 2 sin 2x

f. $\int \frac{dx}{1+\sin x}$ is equal to

(A) $\sin\frac{x}{2}+\cos\frac{x}{2}$ **(B)** $\log(1+\sin x)$

(C) tan x + sec x **(D)** tan x – sec x

g. $\int_0^{\infty} \frac{e^x}{1+e^{2x}}dx$ is equal to

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{4}$

(C) 1 **(D)** 0

h. The solution of the differential equation $\sqrt{y}\,dx+\sqrt{x}\,dy=0$ is

(A) $\sqrt{x}+\sqrt{y}=\text{const}$ **(B)** $\sqrt{xy}=\text{const}$

(C) $x\sqrt{y}+y\sqrt{x}=\text{const}$ **(D)** $\sqrt{x}/\sqrt{y}=\text{const}$

i. The value of $\sin 75^\circ-\cos 75^\circ$ is equal to

(A) 1 **(B)** $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$ **(D)** zero

j. The value of $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$ is

(A) 2π **(B)** π

(C) $\frac{\pi}{2}$ **(D)** $\frac{\pi}{4}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is double the coefficient of x^n in the expansion of $(1+x)^{2n-1}$. **(8)**

b. If $x = 1 + a + a^2 + \ldots\infty$ and $y = 1 + b + b^2 + \ldots\infty$, where $|a| < 1,\ |b| < 1$ then prove that $1 + ab + a^2b^2 + a^3b^3 \ldots\infty = \dfrac{xy}{x + y - 1}$. **(8)**

Q.3 a. If A + B + C = π, show that
$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$. **(8)**

b. If a, b, c be the sides opposite to the angles A, B, C of a triangle ABC, show that
$\dfrac{b-c}{b+c} = \dfrac{\tan\dfrac{B-C}{2}}{\tan\dfrac{B+C}{2}}$. **(8)**

Q.4 a. Derive the formula for finding the area of a triangle whose vertices are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$. **(8)**

b. Find the equation of a straight line joining the point (3, 5) to the point of intersection of the lines 4x +y = 1 and 7x – 3 y = 35. **(8)**

Q.5 a. Find the equation of the circle which passes through the centre of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and is concentric with the circle $2x^2 + 2y^2 - 8x - 12y - 9 = 0$. **(8)**

b. Find the focus, vertex, directrix and axis of the parabola $y = -4x^2 + 3x$. **(8)**

Q.6 a. Evaluate $\lim\limits_{x\to 0} \dfrac{x(3^x - 1)}{1 - \cos x}$. **(8)**

b. Find $\dfrac{dy}{dx}$, if $y = \sin^{-1}\dfrac{2\theta}{1+\theta^2}, x = \tan^{-1}\dfrac{2\theta}{1-\theta^2}$. **(8)**

Q.7 a. Derive the equation of the tangent and the normal to the curve $y^2 = 4ax$ at the point $(at^2, 2at)$. **(8)**

b. Evaluate $\int \frac{x+\sin x}{1+\cos x}dx$. **(8)**

Q.8 a. Find the volume of the solid of revolution obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about x-axis. **(8)**

b. Evaluate $\int_0^{\frac{\pi}{2}} \sin^n x dx$, for any positive integer n. **(8)**

Q.9 Solve any two of the following differential equations.

(i) $\frac{dy}{dx}=e^{3x-y}+x^2e^{-y}$.

(ii) $y-x\frac{dy}{dx}=x+y\frac{dy}{dx}$.

(iii) $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$.

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