**Code: D-01 / DC-01** **Subject: MATHEMATICS - I**

**December 2005**

**Time: 3 Hours** **Max. Marks: 100**

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

**Q.1 Choose the correct or best alternative in the following: (2x10)**

a. The equation of the straight line which makes equal intercepts on the axes and passes through the point (1, 2) is

**(A)** $x + y = 3$ **(B)** $x + 2y = 5$
**(C)** $x - y = 1$ **(D)** $2x + y = 4$

b. Area of the triangle whose vertices are (a, b) (a, a + b), (-a, -a + b) is

**(A)** $a^2b^2$ **(B)** $a^2 + b^2$
**(C)** $a^2$ **(D)** $b^2$

c. $\lim_{x \to 0} \frac{1-\cos x}{x^2}$ is

**(A)** 1 **(B)** $\frac{1}{2}$
**(C)** $\frac{1}{4}$ **(D)** Zero

d. The point on the curve $y^2 = 4x$ at which the tangent to the curve is parallel to $y = x$ is

**(A)** (0, 0) **(B)** $(2, 2\sqrt{2})$
**(C)** (4, 4) **(D)** (1, 2)

e. $\int \frac{\sin^3 x - \cos^3 x}{\sin^2 x \cos^2 x} dx$ is equal to

**(A)** tan x – cot x **(B)** tan x + cot x
**(C)** sec x + cosec x **(D)** sec x - cosec x

f. $\int_0^{\frac{\pi}{2}} \text{Sin}^3 x \, dx$ is equal to

**(A)** $\frac{2}{3}$ **(B)** $\frac{3}{2}$
**(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{4}$

g. Solution of differential equation $\frac{dy}{dx} = e^{x-y}$ is

**(A)** $e^x + e^y$ = const **(B)** $e^x - e^y$ = const
**(C)** $e^x \cdot e^y$ = const **(D)** $e^x / e^y$ = const

h. Period of Sin (2x + 3) is

**(A)** $2\pi$ **(B)** $\frac{3\pi}{2}$
**(C)** $\pi$ **(D)** $\frac{\pi}{2}$

i. The value of Sin $105^0$ + Cos $105^0$ is

**(A)** $\frac{\sqrt{3}}{2}$ **(B)** $\frac{1}{\sqrt{3}}$
**(C)** $\frac{1}{2}$ **(D)** $\frac{1}{\sqrt{2}}$

j. If pth, $(2p)^{th}$ and $(3p)^{th}$ terms of a G.P. are x, y, z respectively, then x, y, z are in

**(A)** A.P. **(B)** H.P.
**(C)** G.P. **(D)** None of these

**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

**Q.2** a. Find the term independent of x in the expansion of $(x - \frac{1}{x})^{12}$. **(8)**

b. If the $p^{th}$, $q^{th}$ and $r^{th}$ terms of an A.P. are x, y, z respectively, show that $x(q-r) + y(r-p) + z(p-q) = 0$. **(8)**

**Q.3** a. If $A + B + C = \pi$, show that

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$ **(8)**

b. In any triangle ABC, show that

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

**(8)**

**Q.4** a. Find the equation of a straight line when p is the length of perpendicular on it from the origin and the inclination of this perpendicular to the x – axis is $\alpha$. **(8)**

b. Find the equation of the straight line which passes through the intersection of the straight lines $2x - 3y + 4 = 0$ and $3x + 4y + 5 = 0$ and is perpendicular to the straight line $6x - 7y + 8 = 0$. **(8)**

**Q.5** a. Show that $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle. Find its centre and radius. **(6)**

b. Find the vertex, focus, latus rectum and directrix of the parabola. $x^2 = 4x - y$. **(10)**

**Q.6** a. Evaluate $\lim_{x \to 0} \frac{a^x - 1}{x}$, by using the fact that $\lim_{t \to 0} (1+t)^{1/t} = e$. **(8)**

b. Differentiate $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ with respect to x. **(8)**

**Q.7** a. Find the points at which the function
$y = 3\sin^2 x + 4\cos^2 x$

has maximum and minimum values in the interval $\left[0, \frac{\pi}{2}\right]$
**(8)**

b. Evaluate $\int \frac{dx}{a\cos x + b\sin x}$, where a, b are not both zero. **(8)**

**Q.8** a. Find the area common to the circles $x^2 + y^2 - 2ax = 0$ and $x^2 + y^2 - 2ay = 0$. **(10)**

b. Evaluate $\int_0^1 \frac{x^3}{(1+x^8)}\,dx$. **(6)**

**Q.9** Solve following the differential equations

(i) $ydx - xdy = \sqrt{(x^2 + y^2)}dx$. **(8)**

(ii) $\text{Cos}^2 x \frac{dy}{dx} + y = \tan x$. **(8)**