Code: D-01 / DC-01 **Subject: MATHEMATICS - I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 11 Questions in all.

- **Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x8)

a. $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2$ is equal to

(A) $\frac{n(n+1)}{2}$. **(B)** $\frac{n(2n+1)}{2}$.

(C) $\frac{n(n+1)(2n+1)}{6}$. **(D)** $\frac{(n+1)(2n+1)}{6}$.

b. If $\log_9 x = 1.5$ then x is equal to

(A) 3 **(B)** 27
(C) 9 **(D)** 15

c. The value of $\text{Sin } 75^\circ \text{ Cos} 15^\circ + \text{Cos } 75^\circ \text{ Sin } 15^\circ$ is equal to

(A) 1. **(B)** 0.
(C) -1. **(D)** ½

d. If α, β are the roots of $ax^2 + bx + c = 0$ then $\alpha^2 + \beta^2$ is

(A) $-b/c$. **(B)** $\frac{b^2 - 2ac}{a^2}$.

(C) $b^2 - 2ac$. **(D)** $\frac{b^2 - ac}{a^2}$.

e. $\lim_{x \to 1} (x)^{\frac{1}{(x-1)}}$ is equal to

(A) 1. **(B)** 0.

(C) e. **(D)** $\frac{1}{e}$.

f. $\int \log x \, dx$ is equal to

(A) $x \log x - x + c$. **(B)** $x \log x$.

(C) $\log x$. **(D)** $\frac{1}{x} \log x$.

g. The maximum value of $y = 2 \cos 2x - \cos 4x$, $0 \le x \le \pi/2$ is

(A) -1. **(B)** $1/2$.

(C) $3/2$. **(D)** 1.

h. The equation of the line which is perpendicular to the line $3x - 4y + 7 = 0$ and passes through the point (-3, 2) is

(A) $4x + 3y + 5 = 0$. **(B)** $4x + 3y - 3 = 0$.
(C) $4x + 3y + 6 = 0$. **(D)** $3x - 4y + 6 = 0$.

PART I

Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. Prove that 7 divides $2^{3n} - 1$ for all positive integers n. **(7)**

b. Find the condition that the roots of equation $ax^2 + bx + c = 0$ are equal. **(7)**

Q.3 a. Evaluate $\tan\left(\frac{5\pi}{12}\right)$. **(6)**

b. If $x + \frac{1}{x} = 2 \cos \theta$, prove that $x^3 + \frac{1}{x^3} = 2 \cos 3\theta$. **(8)**

Q.4 a. If a, b, c are lengths of sides opposite to angles A, B, C in a triangle ABC, then show that $a^2 = b^2 + c^2 - 2bc\,Cos\,A$. **(7)**

b. Show that in a triangle ABC,
a Sin (B – C) + b Sin (C – A) + c Sin (A – B) = 0,
where a , b, c are lengths of sides opposite to angles A, B, C. **(7)**

Q.5 a. Find the condition that the points (1, 1), (3, 5) and (a, b) are collinear. **(7)**

b. Find equations of lines which pass through the point (4, 5) and make an angle 45° with the line 2x + y +1 = 0. **(7)**

Q.6 a. Find the equation of the circle concentric with the circle $x^2 + y^2 - 4x - 6y - 9 = 0$ and which passes through (-4, 5). **(7)**

b. Show that $y^2 - 8y - x + 19 = 0$ represents a parabola. Find its focus, vertex and directrix. **(7)**

PART II
Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. Find $\lim_{x \to 0} \frac{Sin\ 3x}{x}$. **(6)**

b. Examine the continuity of the function f(x) = [x],
where [x] is greatest integer $\leq x$, x being any real number. **(8)**

Q.8 a. Show that the semi verticle angle of a cone of maximum volume and a given slant height is $\tan^{-1}\sqrt{2}$. **(7)**

b. Find the equation of tangent and normal to the curve $y = x^2 - 9$ at the point where it intersects the positive x-axis. **(7)**

Q.9 a. Find a reduction formula for the integral $\int \sin^n x\,dx$. **(7)**

b. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$. **(7)**

Q.10 a. Find the area bounded by $y^2 = 4ax$ and its latus rectum. **(7)**

b. Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, about its major axis. **(7)**

Q.11 a. Solve the equation $\frac{dy}{dx}=\frac{x-y}{x+y}$. **(6)**

b. Solve the equation $x\frac{dy}{dx}-3y=x^2$. **(8)**