

# SOLUTIONS

## IIT JEE-2009 PAPER-1

### SECTION -I

#### Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices. (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Q.21 Let  $z = \cos \theta + i \sin \theta$ . Then the value of

$$\sum_{m=1}^{15} \operatorname{Im}(z^{2m-1})$$

at  $\theta = 2^\circ$  is

- (A)  $\frac{1}{\sin 2^\circ}$       (B)  $\frac{1}{3 \sin 2^\circ}$       (C)  $\frac{1}{2 \sin 2^\circ}$       (D)  $\frac{1}{4 \sin 2^\circ}$

[Sol. **[D]**

$$z = \cos \theta + i \sin \theta = e^{i\theta}; \theta = 2^\circ$$

$$z^{2m-1} = e^{i(2m-1)\theta} = \cos(2m-1)\theta + i \sin(2m-1)\theta$$

$$S = \sum_{m=1}^{15} \operatorname{Im}(z^{2m-1}) = \sum_{m=1}^{15} \sin(2m-1)\theta$$

$$\therefore S = \sin \theta + \sin 3\theta + \sin 5\theta + \dots + \sin 29\theta$$

$$= \frac{\sin(15\theta)}{\sin \theta} \cdot \sin(15\theta) \quad (\text{given } \theta = 2^\circ)$$

$$= \frac{\sin^2 30^\circ}{\sin 2^\circ} = \frac{1}{4 \sin 2^\circ} \Rightarrow \textbf{(D) ]}$$

Q.22 The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

- (A) 55      (B) 66      (C) 77      (D) 88

Sol. **[C]**

Possible cases are

$$3 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1 \rightarrow \frac{7!}{5!} = 42$$

$$\text{or } 2 \ 2 \ 2 \ 1 \ 1 \ 1 \ 1 \rightarrow \frac{7!}{4! \cdot 3!} = 35$$

$$\therefore \text{ number of 7 digits number is } 42 + 35 = 77 \textbf{ Ans.} \Rightarrow \textbf{(C)}$$

Alternatively: coefficient of  $x^{10}$  in  $(x + x^2 + x^3)^7$   
or coefficient of  $x^3$  in  $(1 + x + x^2)^7$  ]

Q.23 Let P(3, 2, 6) be a point in space and Q be a point on the line

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

Then the value of  $\mu$  for which the vector  $\overrightarrow{PQ}$  is parallel to the plane  $x - 4y + 3z = 1$  is

- (A)  $\frac{1}{4}$  (B)  $-\frac{1}{4}$  (C)  $\frac{1}{8}$  (D)  $-\frac{1}{8}$

Sol. [A]

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

position vector of Q is  $(1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (5\mu + 2)\hat{k}$

position vector of P is  $3\hat{i} + 2\hat{j} + 6\hat{k}$

$$\Rightarrow \overrightarrow{PQ} = (2 + 3\mu)\hat{i} + (3 - \mu)\hat{j} + (4 - 5\mu)\hat{k}$$

given  $\overrightarrow{PQ}$  is parallel to the plane  $x - 4y + 3z = 1$

vector normal to plane is  $\vec{n} = \hat{i} - 4\hat{j} + 3\hat{k}$

hence  $\overrightarrow{PQ} \cdot \vec{n} = 0$

$$(2 + 3\mu) - 4(3 - \mu) + 3(4 - 5\mu) = 0$$

$$2 - 8\mu = 0 \Rightarrow \mu = 1/4 \text{ Ans. } \Rightarrow \text{ (A) ]}$$

Q.24 Let  $f$  be a non-negative function defined on the interval  $[0, 1]$ . If

$$\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, \quad 0 \leq x \leq 1,$$

and  $f(0) = 0$ , then

- (A)  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) > \frac{1}{3}$  (B)  $f\left(\frac{1}{2}\right) > \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) > \frac{1}{3}$   
 (C)  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$  (D)  $f\left(\frac{1}{2}\right) > \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$

[Sol. [C]

$$f(x) \geq 0 \quad \forall x \in [0, 1]$$

$$\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$$

differentiate both the sides

$$\text{let } y = f(x); \quad f'(x) = \frac{dy}{dx}$$

$$\sqrt{1 - (f'(x))^2} = f(x)$$

$$1 - (f'(x))^2 = (f(x))^2$$

$$1 - \left(\frac{dy}{dx}\right)^2 = y^2 \Rightarrow \left(\frac{dy}{dx}\right)^2 = 1 - y^2 \Rightarrow \frac{dy}{dx} = \pm \sqrt{1 - y^2}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \pm \int dx$$

$$\sin^{-1} y = \pm (x + C)$$

$$y = \pm \sin(x + C)$$

$$y|_{x=0} = 0 \Rightarrow C = 0$$

$$y = \pm \sin x \quad (\text{-ve sign reject } \because f(x) \geq 0)$$

$$y = \sin x$$

$$\text{now } \frac{\sin x}{x} < 1 \quad \forall x \in (0, 1]$$

$$\sin\left(\frac{1}{2}\right) < \frac{1}{2} \text{ and } \sin\left(\frac{1}{3}\right) < \frac{1}{3} \Rightarrow \quad \text{(C) ]}$$

Q.25 Let  $z = x + iy$  be a complex number where  $x$  and  $y$  are integers. Then the area of the rectangle whose vertices are the roots of the equation

$$z\bar{z}^3 + \bar{z}z^3 = 350 \quad \text{is}$$

(A) 48

(B) 32

(C) 40

(D) 80

[Sol. [A]

$$z\bar{z}(\bar{z}^2 + z^2) = 350$$

$$\text{put } z = x + iy$$

$$(x^2 + y^2)(x^2 - y^2) = 175 = 25 \cdot 7$$

$$= 35 \cdot 5$$

$$x^2 + y^2 = 25$$

$$x^2 + y^2 = 35$$

$$x^2 - y^2 = 7$$

$$x^2 - y^2 = 5$$

rejected ( $\because x$  and  $y$  are integer)

$$x^2 = 16 \Rightarrow x = \pm 4$$

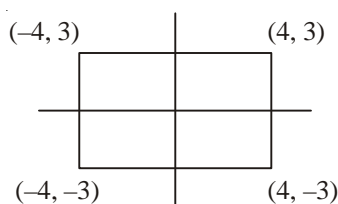
$$\therefore x = 4 ; \quad y = 3$$

$$x = -4 ; \quad y = -3$$

$$x = 4 ; \quad y = -3$$

$$x = -4 ; \quad y = 3$$

$$\therefore \text{area of rectangle} = 6 \times 8 = 48 \text{ Ans. } \Rightarrow \quad \text{(A) ]}$$



Q.26 Tangents drawn from the point  $P(1, 8)$  to the circle

$$x^2 + y^2 - 6x - 4y - 11 = 0$$

touch the circle at the points  $A$  and  $B$ . The equation of the circumcircle of the triangle  $PAB$  is

(A)  $x^2 + y^2 + 4x - 6y + 19 = 0$

(B)  $x^2 + y^2 - 4x - 10y + 19 = 0$

(C)  $x^2 + y^2 - 2x + 6y - 29 = 0$

(D)  $x^2 + y^2 - 6x - 4y + 19 = 0$

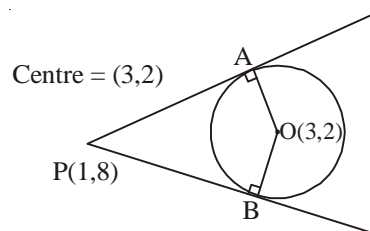
[Sol. [B]

Circumcircle of  $\Delta PAB$  will pass through the centre of circle

i.e. circle of diammetrical form

$$(x-1)(x-3) + (y-8)(y-2) = 0$$

$$x^2 + y^2 - 4x - 10y + 19 = 0 \Rightarrow \quad \text{(B) ]}$$



Q.27 The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse  $x^2 + 9y^2 = 9$  meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is

- (A)  $\frac{31}{10}$  (B)  $\frac{29}{10}$  (C)  $\frac{21}{10}$  (D)  $\frac{27}{10}$

[Sol. [D]

Equation of ellipse

$$\frac{x^2}{9} + \frac{y^2}{1} = 1$$

equation of auxillary circle  $x^2 + y^2 = 9$

equation of line AB

$$\frac{x}{3} + \frac{y}{1} = 1 \Rightarrow x + 3y = 3$$

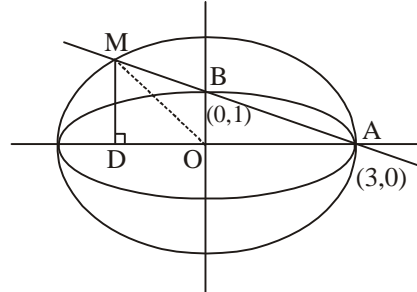
point of intersection of line AB and auxillary circle

$$(3 - 3y)^2 + y^2 = 9$$

$$10y^2 = 18y$$

$$y = 0, \frac{9}{5}$$

$$\text{Area} = \frac{1}{2}(\text{OA})(\text{MD}) = \frac{1}{2} \times 3 \times \frac{9}{5} = \frac{27}{10} \text{ Ans.} \Rightarrow \text{(D)}$$



Q.28 If  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  are unit vectors such that  $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$  and  $\vec{a} \cdot \vec{c} = \frac{1}{2}$ , then

- (A)  $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar (B)  $\vec{b}, \vec{c}, \vec{d}$  are non-coplanar  
(C)  $\vec{b}, \vec{d}$  are non-parallel (D)  $\vec{a}, \vec{d}$  are parallel and  $\vec{b}, \vec{c}$  are parallel

[Sol. [C]

$$\text{Given } (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1 \Rightarrow (\sin \theta \hat{n}_1)(\sin \phi \hat{n}_2) = 1$$

$$\therefore \sin \theta \sin \phi \cos \beta = 1 \dots (1)$$

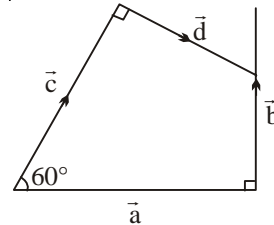
(1) is possible only if  $\theta = \phi = \frac{\pi}{2}$  and  $\beta = 0$

where  $\theta = \vec{a} \wedge \vec{b}$  ;  $\phi = \vec{c} \wedge \vec{d}$  ;  $\beta = (\vec{a} \times \vec{b}) \wedge (\vec{c} \times \vec{d})$

it will satisfy only when  $\vec{a}$  is perpendicular to  $\vec{b}$ ,  $\vec{c}$  is perpendicular to  $\vec{d}$  and  $(\vec{a} \times \vec{b}) \parallel (\vec{c} \times \vec{d})$

$$\text{simultaneously } \vec{a} \cdot \vec{c} = \frac{1}{2} \Rightarrow \theta = 60^\circ \text{ (between } \vec{a} \text{ and } \vec{c} \text{)}$$

hence  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  will be coplanar and  $\vec{b}, \vec{d}$  are non parallel **Ans.**  $\Rightarrow$  (C)]



## SECTION -II

### Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

Q.29 Let

$$L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$$

If L is finite, then

- (A)  $a = 2$                       (B)  $a = 1$                       (C)  $L = \frac{1}{64}$                       (D)  $L = \frac{1}{32}$

[Sol. [A, C]

$$L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4} \quad \left( \frac{0}{0} \right), a > 0$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{a^2 - x^2}}(2x) - \frac{x}{2}}{4x^3} = \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{a^2 - x^2}} - \frac{1}{2}}{4x^2}$$

now As  $x \rightarrow 0$ ,  $D^r \rightarrow 0$

$\therefore N^r \rightarrow 0$

$$\Rightarrow \frac{1}{\sqrt{a^2}} = \frac{1}{2} \Rightarrow a = 2 \quad \text{Ans.} \Rightarrow (A)$$

$$\text{now } L = \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{4-x^2}} - \frac{1}{2}}{4x^2} = \lim_{x \rightarrow 0} \frac{\left(2 - \sqrt{4-x^2}\right) \times \left(2 + \sqrt{4-x^2}\right)}{8x^2 \left(\sqrt{4-x^2}\right) \left(2 + \sqrt{4-x^2}\right)}$$

$$L = \frac{1}{8 \times 2 \times 4} = \frac{1}{64} \quad \text{Ans.} \Rightarrow (C)$$

Q.30 Area of the region bounded by the curve  $y = e^x$  and lines  $x = 0$  and  $y = e$  is

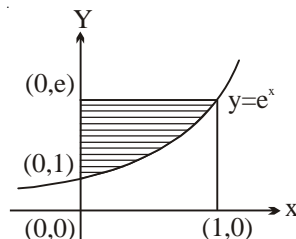
- (A)  $e - 1$                       (B)  $\int_1^e \ln(e+1-y) dy$                       (C)  $e - \int_1^e e^x dx$                       (D)  $\int_1^e \ln y dy$

[Sol. [B, C, D]

$$\text{Required area} = \int_0^1 (e - e^x) dx = e - \int_0^1 e^x dx$$

$\therefore$  option (C) is correct

$$\text{Also required area} = \int_0^e \ln y dx \dots (1) \quad (\text{on integrating along y-axis})$$



by king property in (1), we also get

$$\text{required area} = \int_1^e \ln(1+e-y) dy$$

$\therefore$  option **(B)** is also correct]

Q.31 If  $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$ , then

(A)  $\tan^2 x = \frac{2}{3}$

(B)  $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(C)  $\tan^2 x = \frac{1}{3}$

(D)  $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

[Sol. **[A, B]**

$$\frac{5}{2} \sin^4 x + \frac{5}{3} \cos^4 x = 1$$

divide by  $\cos^4 x$

$$\frac{5}{2} \tan^4 x + \frac{5}{3} = \sec^4 x \Rightarrow \frac{5}{2} \tan^4 x + \frac{5}{3} = (1 + \tan^2 x)^2$$

$$\Rightarrow 15 \tan^4 x + 10 = 6(1 + \tan^4 x + 2 \tan^2 x) \Rightarrow 9 \tan^4 x - 12 \tan^2 x + 4 = 0$$

$$\Rightarrow (3 \tan^2 x - 2)^2 = 0 \Rightarrow \tan^2 x = \frac{2}{3} \quad \text{Ans.} \Rightarrow \text{(A)}$$

$$1 + \tan^2 x = \frac{5}{3}$$

$$\cos^2 x = \frac{3}{5}; \quad \sin^2 x = \frac{2}{5}$$

$$\text{Verify: } \frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{16}{325 \times 8} + \frac{81}{27 \times 625} = \frac{5}{625} = \frac{1}{125} \quad \text{Ans.} \Rightarrow \text{(B) ]}$$

Q.32 In a triangle ABC with fixed base BC, the vertex A moves such that

$$\cos B + \cos C = 4 \sin^2 \frac{A}{2}.$$

If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, then

(A)  $b + c = 4a$

(B)  $b + c = 2a$

(C) locus of point A is an ellipse

(D) locus of point A is a pair of straight lines

[Sol. **[B, C]**

$$\cos B + \cos C = 4 \sin^2 \frac{A}{2}$$

$$2 \cos \frac{B+C}{2} \cos \frac{B-C}{2} = 4 \sin^2 \frac{A}{2}$$

$$\Rightarrow \cos \frac{B-C}{2} = 2 \sin \frac{A}{2}$$

multiply  $\cos \frac{A}{2}$

$$\Rightarrow 2 \cdot \sin \frac{B+C}{2} \cos \frac{B-C}{2} = 2 \cdot 2 \sin \frac{A}{2} \cos \frac{A}{2}$$

$$\Rightarrow \sin B + \sin C = 2 \sin A$$

$$b + c = 2a \quad (\text{Sine law}) \text{ Ans. } \Rightarrow \quad (\text{B})$$

given base is fixed so  $a = \text{constant}$

$$b + c = \text{constant}$$

locus of point A is ellipse **Ans.**  $\Rightarrow$  (C) ]

### SECTION - III

#### Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

#### Paragraph for Question Nos. 33 to 35

Let  $A$  be the set of all  $3 \times 3$  symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

Q.33 The number of matrices in  $A$  is

- (A) 12 (B) 6 (C) 9 (D) 3

[Sol. [A]

$$\text{Diagonal } (1, 1, 1) \text{ non diagonal } (1, 1) (0, 0) (0, 0) = \frac{3!}{2!} \times 1 = 3$$

$$\text{Diagonal } (1, 0, 0) (1, 1) (1, 1) (0, 0) = \frac{3!}{2!} \times 3 = 9$$

diagonal elements permute 12 ]

Q.34 The number of matrices  $A$  in  $A$  for which the system of linear equations

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is

- (A) less than 4 (B) at least 4 but less than 7  
(C) at least 7 but less than 10 (D) at least 10

[Sol. [B]

$$A_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}; \quad A_2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad A_3 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad A_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}; \quad A_5 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$|A_i| = 0$$

Q.35 The number of matrices A in A for which the system of linear equations

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent, is

(A) 0

(B) more than 2

(C) 2

(D) 1

[Sol. **[B]**

$$A_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}; A_2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}; A_4 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; A_5 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\text{Adj } A_i \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \neq 0. \text{ Then inconsistent}$$

Now

$$\text{Adj } A_1 = \begin{pmatrix} (-) & - & - \\ (-) & - & - \\ (-) & - & - \end{pmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow B$$

If any non zero then  $\text{Adj } (A_i) \cdot B \neq 0$

$$\Rightarrow \text{Adj } A_4 = \begin{pmatrix} (-) & - & - \\ (-) & - & - \\ (-) & - & - \end{pmatrix}$$

non zero

$$\text{Adj } A_5 = \begin{pmatrix} (-) & - & - \\ (-) & - & - \\ (-) & - & - \end{pmatrix}$$

non zero

$$\text{Adj } (A_3) = \begin{pmatrix} (-) & - & - \\ (-) & - & - \\ (-) & - & - \end{pmatrix}$$

non zero



### Paragraph for Question Nos. 36 to 38

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

Q.36 The probability that  $X = 3$  equals

- (A)  $\frac{25}{216}$  (B)  $\frac{25}{36}$  (C)  $\frac{5}{36}$  (D)  $\frac{125}{216}$

[Sol. [A]

$$\text{Probability six in third attempt} = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216} \text{ Ans. } \Rightarrow \text{ (A) ]}$$

Q.37 The probability that  $X \geq 3$  equals

- (A)  $\frac{125}{216}$  (B)  $\frac{25}{36}$  (C)  $\frac{5}{36}$  (D)  $\frac{25}{216}$

[Sol. [B]

$$x \geq 3$$

$$= \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^3 \frac{1}{6} + \dots \infty$$

$$= \frac{\left(\frac{5}{6}\right)^2 \frac{1}{6}}{1 - \frac{5}{6}} = \frac{25}{36} \text{ Ans. } \Rightarrow \text{ (B) ]}$$

Q.38 The conditional probability that  $X \geq 6$  given  $X > 3$  equals

- (A)  $\frac{125}{216}$  (B)  $\frac{25}{216}$  (C)  $\frac{5}{36}$  (D)  $\frac{25}{36}$

[Sol. [D]

$$\text{Probability} = \frac{x \geq 6}{x > 3}$$

$$\begin{aligned} & \frac{\left(\frac{5}{6}\right)^5 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \dots \infty}{\left(\frac{5}{6}\right)^3 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \dots \infty} \\ &= \frac{25}{36} \text{ Ans. } \Rightarrow \text{ (D) ]} \end{aligned}$$

Q.39 Match the statements/expressions In **Column I** with the open intervals In **Column II**.

Column I		Column II
(A) Interval contained in the domain of definition of non-zero solutions of the differential equation $(x - 3)^2 y' + y = 0$	(p)	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$	(q)	$\left(0, \frac{\pi}{2}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(r)	$\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	(s)	$\left(0, \frac{\pi}{8}\right)$
	(t)	$(-\pi, \pi)$

[Ans (A) p, q, s (B) p, t (C) p, q, r, t (D) s]

[Sol. (A) On solving the differential equation

$$-\int \frac{dy}{y} = \int \frac{dx}{(x-3)^2} \Rightarrow \text{you can check very easily } x \neq 3.$$

(B) Let  $x - 3 = t \Rightarrow \begin{matrix} t = -2 \text{ as } x \rightarrow 1 \\ t = 2 \text{ as } x \rightarrow 5 \end{matrix}$

$$\int_{-2}^2 (t+2)(t-1)t(t+1)(t+2)dt$$

on solving this will be completely odd hence equals to 0.

(C)  $f'(x) = 2 \cos x (-\sin x) + \cos x = 0$

$y = f(x)$  will take maximum at  $x = \frac{\pi}{6}$

(D)  $y = \tan^{-1}(\sin x + \cos x)$  will be ( $\uparrow$ ) increasing when  $\sin x + \cos x$  is  $\uparrow$ ]

Q.40 Match the conics in **Column I** with the statements/expressions in **Column II**.

Column I		Column II	
(A)	Circle	(p)	The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
(B)	Parabola	(q)	Points z in the complex plane satisfying $ z + 2  -  z - 2  = \pm 3$
(C)	Ellipse	(r)	Points of the conic have parametric representation
(D)	Hyperbola		$x = \sqrt{3} \left( \frac{1-t^2}{1+t^2} \right), y = \frac{2t}{1+t^2}$
		(s)	The eccentricity of the conic lies in the interval $1 \leq e < \infty$
		(t)	Points z in the complex plane satisfying $\operatorname{Re}(z + 1)^2 =  z ^2 + 1$
[Ans. (A) p, (B) s, t; (C) r; (D) q, s]			

[Sol.

(p)  $\frac{1}{\sqrt{h^2 + k^2}} = 2 \Rightarrow h^2 + k^2 = \frac{1}{4} \Rightarrow x^2 + y^2 = \frac{1}{4} \Rightarrow$  (A)

(q) difference of distances from points = constant  $\Rightarrow$  (D)

(r)  $x = \sqrt{3} \cos 2\theta; y = \tan 2\theta$

$\frac{x^2}{3} + \frac{y^2}{1} = 1 \Rightarrow$  ellipse  $\Rightarrow$  (C)

(s) hyperbola ( $e > 1$ ) and parabola ( $e = 1$ )  $\Rightarrow$  (B) and (D)

(t)  $\operatorname{Re}(x + iy + 1)^2 = |z|^2 + 1$

$(1 + x)^2 - y^2 = x^2 + y^2 + 1$

$x^2 + 1 + 2x - y^2 = x^2 + y^2 + 1$

$2y^2 = 2x - 1$  (parabola)  $\Rightarrow$  (B) ]

