Code: AE06/AC04/AT04 **Subject: SIGNALS & SYSTEMS**
Time: 3 Hours **JUNE 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The Fourier transform of the exponential signal $e^{j\omega_0 t}$ is

(A) a constant. **(B)** a rectangular gate.
(C) an impulse. **(D)** a series of impulses.

b. The auto-correlation function of a rectangular pulse of duration T is

(A) a rectangular pulse of duration T
(B) a rectangular pulse of duration 2T.
(C) a triangular pulse of duration T.
(D) a triangular pulse of duration 2T

c. The system characterized by the equation $y(t) = ax(t) + b$ is

(A) linear for any value of b. **(B)** linear if $b > 0$.
(C) linear if $b < 0$ **(D)** non-linear.

d. For a random variable x having the PDF shown in the Fig.1, the mean and the variance are, respectively,

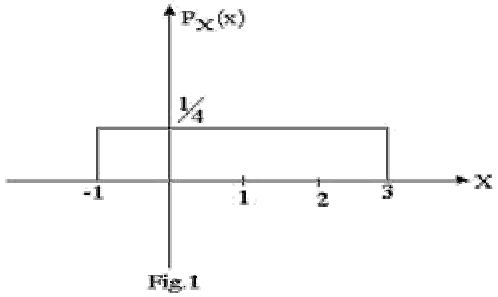

Fig.1

(A) 1/2 and 2/3 **(B)** 1 and 4/3
(C) 1 and 2/3 **(D)** 2 and 4/3

e. $\delta(at) = \frac{1}{a}\delta(t)$ is a property of the unit impulse $\delta(t)$ known as

(A) Time-scaling property **(B)** Frequency- scaling property.
(C) Magnitude -scaling property **(D)** Frequency -selective property.

f. The signal defined by the equation u(t-a) =0 for t< a and u(t-a) =1 for t≥ a is

(A) unit step function
(B) a pulse function
(C) the ramp function
(D) a shifted unit step function at t =a

g. A function having frequency f is to be sampled. For the signal to be recovered from its samples, the sampling time T should be

(A) T = 1/(2f)
(B) T > 1/(2f)
(C) T < 1/(2f)
(D) T ≥ 1/(2f)

h. The function (sin x) / x is a

(A) sine wave
(B) inverse –sine wave
(C) sinc function
(D) Can't define

i. If f(t) = δ(t-a) , then F(s) is equal to

(A) 0
(B) 1
(C) e^{as}
(D) e^{-as}

j. The discrete-time signal x (n) shown Fig.2 is periodic with fundamental period

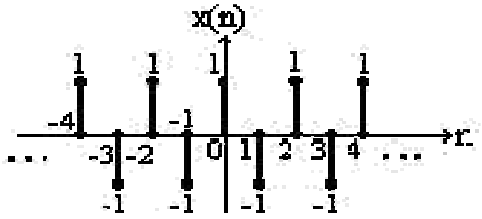

Fig. 2

(A)6
(B) 4
(C) 2
(D) 0

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. For the signal shown in Fig.3. **(8)**
Sketch (i) x(t-2) (ii) x(-t+2) (iii) x(2t) (iv) x(t/2)

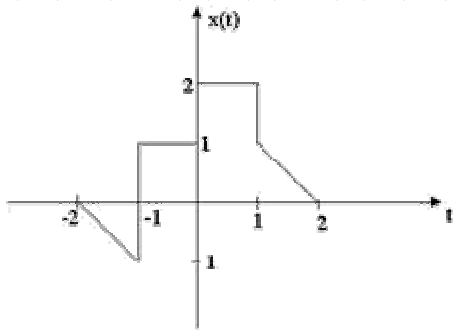

Fig.3

b. Determine whether the system having input x(t) and output y(t) and described by relationship $y(t)= x(2t)$ is
(i) memory-less, (ii) stable, (iii) causal (iv) linear and (v) time-invariant or not. **(5)**

c. Differentiate between an Energy and a Power signal. **(3)**

Q.3 a. Consider an LTI system with input x(n) and the unit impulse response h(n) specified as: $x(n) = 2^n u(-n)$ and $h(n) = u(n)$. Determine y(n). **(8)**

b. Consider the difference equation $y(n) - 0.5\, y(n-1) = x(n)$. Obtain and plot h(n) using only time domain methods. **(5)**

c. State and prove commutative, associative and distributive properties of LTI systems. **(3)**

Q.4 a. Determine the Fourier series expansion of the waveform f (t) shown in Fig.4 in terms of sine and cosine functions. Sketch the magnitude and phase spectra. **(10)**

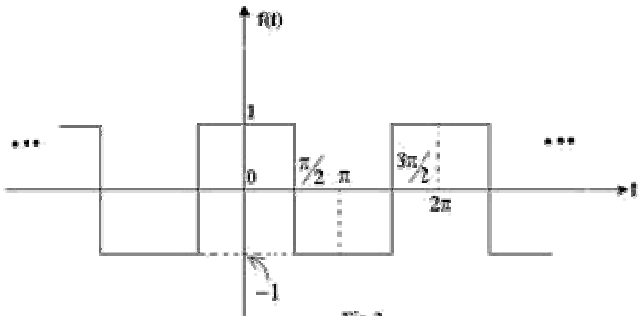

Fig. 4

b. State and prove following properties of Continuous Time Fourier Series:
(i) Differentiation (ii) Time scaling (iii) Time Reversal **(6)**

Q.5 a. For the signal $x(n) = a^{|n|}$.

(i) Plot the signal for $0 < a < 1$.
(ii) Find Fourier Transform $X(e^{jw})$.
(iii) Plot the resultant $X(e^{jw})$. **(6)**

b. Consider a causal LTI system characterized by the difference equation $y(n) - 0.75\, y(n-1) + 0.125\, y(n-2) = x(n)$. Obtain h(n).
Also, find the output y(n) if $x(n) = \left(\frac{1}{2}\right)^n u(n)$. **(10)**

Q.6 a. Explain the concept of
(i) Non-linear phase (ii) Group delay (iii) Continuous-time ideal low pass filter (iv) First – order continuous –time system. **(8)**

b. Define sampling and aliasing. For a signal x(t), calculate Nyquist rate and Nyquist interval. x (t) = 3cos 50πt + 10 sin300πt – cos100πt. **(8)**

Q.7 a. Obtain Laplace Transform of the signal $x(t) = e^{-2t}u(t) + e^{-t}\cos 3t\, u(t)$. **(8)**

b. Find the Laplace transform of $t \sin w_o t\, u(t)$. **(8)**

Q.8 a. Obtain z-transform for

(i) $x(n) = a^n u(n)$

(ii) $x(n) = -a^n u(-n-1)$. **(4)**

b. Find the inverse Z – Transform of $x(z) = \dfrac{z}{(z-1)(z-2)(z-3)}$

with ROC (i) $|z| > 3$ (ii) $|z| < 2$ (iii) $1 < |z| < 2$ **(6)**

c. State and prove initial value and final value theorems for Laplace Transform. **(6)**

Q.9 a. Write short notes on:

(i) White noise (ii) Variance and Co-variance **(8)**

b. A random variable V = b + x; where x is a Gaussian distributed random variable with mean 0 and variance σ^2 with ' b' a constant. Show that V is a Gaussian distributed random variable with mean b and variance σ^2. **(8)**