**Code: A-06/C-04/T-04 Subject: SIGNALS & SYSTEMS**
**Time: 3 Hours Max. Marks: 100**

**NOTE: There are 11 Questions in all.**

**Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**

**Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.**

**Any required data not explicitly given, may be suitably assumed and stated.**

**Q.1 Choose the correct or best alternative in the following: (2x8)**

a. $x(n) = a^{|n|}, |a| < 1$ is

**(A)** an energy signal.
**(B)** a power signal.
**(C)** neither an energy nor a power signal.
**(D)** an energy as well as a power signal.

b. The spectrum of x (n) extends from $-\omega_0$ to $+\omega_0$, while that of h(n) extends from $-2\omega_0$ to $+2\omega_0$. The spectrum of $y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$ extends from

**(A)** $-4\omega_0$ to $+4\omega_0$. **(B)** $-3\omega_0$ to $+3\omega_0$.
**(C)** $-2\omega_0$ to $+2\omega_0$. **(D)** $-\omega_0$ to $+\omega_0$.

c. The signals $x_1(t)$ and $x_2(t)$ are both bandlimited to $(-\omega_1, +\omega_1)$ and $(-\omega_2, +\omega_2)$ respectively. The Nyquist sampling rate for the signal $x_1(t)x_2(t)$ will be

**(A)** $2\omega_1$ if $\omega_1 > \omega_2$. **(B)** $2\omega_2$ if $\omega_1 < \omega_2$.
**(C)** $2(\omega_1+\omega_2)$. **(D)** $(\omega_1+\omega_2)/2$.

d. If a periodic function f(t) of period T satisfies $f(t) = -f(t + T/2)$, then in its Fourier series expansion,

**(A)** the constant term will be zero.
**(B)** there will be no cosine terms.
**(C)** there will be no sine terms.
**(D)** there will be no even harmonics.

e. A band pass signal extends from 1 KHz to 2 KHz. The minimum sampling frequency needed to retain all information in the sampled signal is

**(A)** 1 KHz. **(B)** 2 KHz.
**(C)** 3 KHz. **(D)** 4 KHz.

f. The region of convergence of the z-transform of the signal $2^n u(n) - 3^n u(-n-1)$

**(A)** is $|z| > 1$. **(B)** is $|z| < 1$.
**(C)** is $2 < |z| < 3$. **(D)** does not exist.

g. The number of possible regions of convergence of the function $\frac{(e^{-2} - 2)z}{(z - e^{-2})(z - 2)}$ is

**(A)** 1. **(B)** 2.

**(C)** 3. **(D)** 4.

h. The Laplace transform of u(t) is A(s) and the Fourier transform of u(t) is $B(j\omega)$. Then

**(A)** $B(j\omega)=A(s)|_{s=j\omega}$. **(B)** $A(s)=\frac{1}{s}$ but $B(j\omega)\neq\frac{1}{j\omega}$.

**(C)** $A(s)\neq\frac{1}{s}$ but $B(j\omega)=\frac{1}{j\omega}$. **(D)** $A(s)\neq\frac{1}{s}$ but $B(j\omega)\neq\frac{1}{j\omega}$.

## PART I

**Answer any THREE Questions. Each question carries 14 marks.**

**Q.2** a. The signal x(t) shown below in Fig.1 is applied to the input of an
(i) ideal differentiator. (ii) ideal integrator.
Sketch the responses. **(1+4=5)**

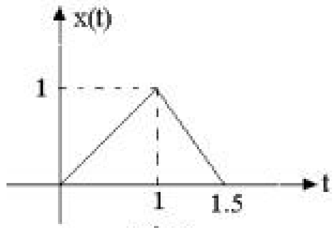


Fig.1

b. Sketch the even and odd parts of
(i) a unit impulse function (ii) a unit step function
(iii) a unit ramp function. **(1+2+3=6)**

c. Sketch the function $f(t)=u\left(\sin\frac{\pi t}{T}\right)-u\left(-\sin\frac{\pi t}{T}\right)$. **(3)**

**Q.3** a. Under what conditions, will the system characterized by $y(n)=\sum_{k=n_0}^{\infty} e^{-ak}\,x(n-k)$ be linear, time-invariant, causal, stable and memory less? **(5)**

b. Let E denote the energy of the signal x (t). What is the energy of the signal x (2t)? **(2)**

c. x(n), h(n) and y(n) are, respectively, the input signal, unit impulse response and output signal of a linear, time-invariant, causal system and it is given that $y(n-2)=x(n-n_1)*h(n-n_2)$, where * denotes convolution. Find the possible sets of values of $n_1$ and $n_2$. **(3)**

d. Let h(n) be the impulse response of the LTI causal system described by the difference equation $y(n)=a\,y(n-1)+x(n)$ and let $h(n)*h_1(n)=\delta(n)$. Find $h_1(n)$. **(4)**

**Q.4** Determine the Fourier series expansion of the waveform f (t) shown below (Fig.2) in terms of sines and cosines. Sketch the magnitude and phase spectra. **(10+2+2=14)**

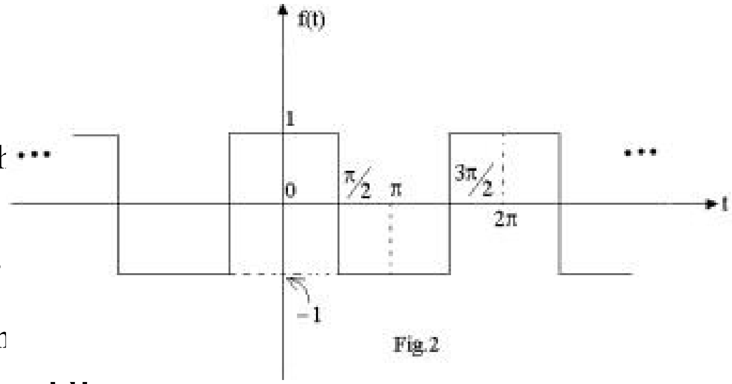

Fig.2

**Q.5** a. Sh ... hen FT $\left[\frac{dx(t)}{dt}\right] = j\omega X(\omega)$. **(3)**

b. Show,

c. Find th ... d by the relationship : $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$.

(3)

d. Using the results of parts (a) and (b), or otherwise, determine the frequency response of the system of part (c). **(6)**

**Q.6** Let $X(e^{j\omega})$ denote the Fourier Transform of the signal x (n) shown below (Fig.3).

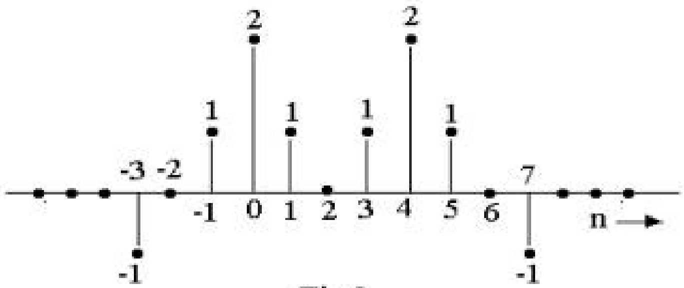

Fig.3

Without explicitly finding out $X(e^{j\omega})$, find the following :-

(i) X (1) (ii) $\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$

(iii) X(-1) (iv) the sequence y(n) whose Fourier Transform is the real part of $X(e^{j\omega})$.

(v) $\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega$. **(2+2+3+5+2=14)**

**PART II**

**Answer any THREE Questions. Each question carries 14 marks.**

**Q.7** a. If the z-transform of x (n) is X(z) with ROC denoted by $R_x$, find the z-transform of

$y(n) = \sum_{k=-\infty}^{n} x(k)$ and its ROC. **(4)**

b. (i) x (n) is a real right-sided sequence having a z-transform X(z). X(z) has two poles, one of which is at $a\,e^{j\phi}$ and two zeros, one of which is at $r\,e^{-j\theta}$. It is also known that $\sum x(n) = 1$. Determine X(z) as a ratio of polynomials in $z^{-1}$. **(6)**

(ii) If $a = \frac{1}{2}, r = 2, \theta = \phi = \pi/4$ in part (b) (i), determine the magnitude of X(z) on the unit circle. **(4)**

**Q.8** Determine, by any method, the output y(t) of an LTI system whose impulse response h(t) is of the form shown in fig.4(a),

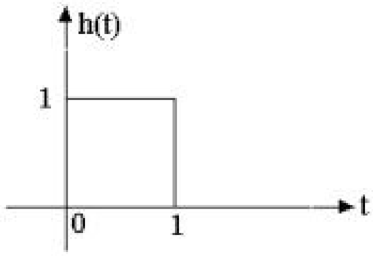


Fig.4(a)

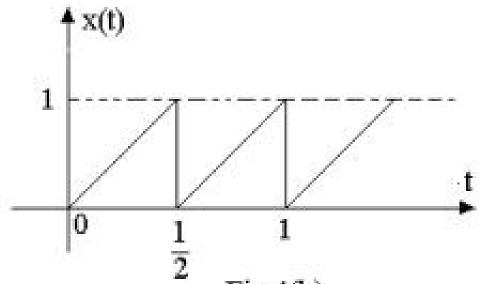


Fig.4(b)

to the periodic excitation x(t) as shown in fig.4(b). **(14)**

**Q.9** Obtain the time function f(t) whose Laplace Transform is $F(s) = \frac{s^2 + 3s + 1}{(s+1)^3 (s+2)^2}$. **(14)**

**Q.10** a. The unit impulse response of an LTI causal system is h(t). If the input to the system is a random process of mean value $\bar{x}$ and a constant power spectral density $S_O$, find the mean and mean squared values of the output y(t). **(6)**

b. The joint probability density function of two random variables X and Y is $f_{x;y}(x, y) = Ke^{-(\alpha|x| + \beta|y|)}$, $-\infty < x < \infty, -\infty < y < \infty, 0 < \alpha, \beta < 1$

Find K. Are X and Y independent? Also find the probability that $X \le \frac{1}{\alpha}$ and $Y \le \frac{1}{\beta}$. **(8)**

**Q.11** a. Define the terms variance, co-variance and correlation coefficient as applied to random variables. **(6)**

b. Given Y = mX + c, where X and Y are random variables, and m and c are constants, which may be positive or negative, find the mean value, mean squared value and the variance of Y, in terms of those of X. Also find the co-variance and the correlation coefficient of X and Y. **(8)**