**47.** If $f(x) = x^3 - 4x + p$, and $f(0)$ and $f(1)$ are of opposite signs, then which of the following is necessarily true?

1. $-1 < p < 2$ 2. $0 < p < 3$ 3. $-2 < p < 1$ 4. $-3 < p < 0$

**Sol.** f(0) = p

f(1) = p – 3 $\Rightarrow$ p (p – 3) < 0 $\Rightarrow$ 0 < p < 3. **Ans.(2)**

***A simple question present in PT's Maths Excel Sheet # 04.***

**48.** Suppose $n$ is an integer such that the sum of the digits of $n$ is 2, and $10^{10} < n < 10^{11}$. The number of different values for $n$ is

1. 11 2. 10 3. 9 4. 8

**Sol.** Sum of the digits of n is 2.

For $10 < n < 10^2$ total different possible values of n = 11, 20 i.e., 2

For $10^2 < n < 10^3$ total different possible values of n = 101, 110, 200 i.e., 3

$\vdots$

For $10^{10} < n < 10^{11}$ total different possible values of n = 10000000001, 10000000010, 10000000100, 10000001000, 10000010000, 10000100000, 10001000000, 10010000000, 10100000000, 11000000000, 20000000000 i.e., 11. **Ans.(1)**

**49.** If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$ then $r$ cannot take any value except

1. 1/2 2. −1 3. 1/2 or −1 4. −1/2 or −1

**Sol.** $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$

By option, if $r = \frac{1}{2}$

$\Rightarrow$ 2a – b – c = 0

2b – c – a = 0

2c – a – b = 0

$\Rightarrow$ 2(a + b + c ) – (a + b + c ) – (a + b + c) = 0

Similarly r = –1 is also satisfied. **Ans.(3)**

***Remember guys doing this question, in the class of Ratio, Proportion and Variation. A very simple question.***

**50.** Let $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots.}}}}$

What is the value of $y$?

1. $\frac{\sqrt{13}+3}{2}$ 2. $\frac{\sqrt{13}-3}{2}$

3. $\frac{\sqrt{15}+3}{2}$ 4. $\frac{\sqrt{15}-3}{2}$

**Sol.** $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+....}}}}$

$$y = \frac{1}{2+\frac{1}{3+y}} \Rightarrow y = \frac{3+y}{2y+7}$$

$2y^2 + 7y = 3 + y \Rightarrow 2y^2 + 6y - 3 = 0$

$y = \dfrac{-6 \pm \sqrt{36+4\cdot 2\cdot 3}}{4} = \dfrac{-6 \pm \sqrt{60}}{4} = \dfrac{\sqrt{15}-3}{2}$. **Ans.(4)**

***This question was asked in PT's PracCat # 05.***

**51.** Let $f(x) = ax^2 - b|x|$, where $a$ and $b$ are constants. Then at $x = 0$, $f(x)$ is

1. maximized whenever $a > 0, b > 0$
2. maximized whenever $a > 0, b < 0$
3. minimized whenever $a > 0, b > 0$
4. minimized whenever $a > 0, b < 0$

**Sol.** $f(x) = ax^2 - b|x|$

if $x > 0$ $f(x) = ax^2 - bx$

$f'(x) = 2ax - b$, $f''(x) = 2a$

So if $a > o$ and $b < 0$, $f''(x) > 0$ and $f(x)$ will be minimum at $x = 0$

For $x < 0$

$f(x) = ax^2 + bx$

$f'(x) = 2ax + b$, $f''(x) = 2a$

In this case also when $a > 0$, $b < 0$ $f''(x) > 0$ or $f(x)$ will be minimum value at $x = 0$. **Ans.(4)**

**52.** Two boats, traveling at 5 and 10 kms per hour, head directly towards each other. They begin at a distance of 20 kms from each other. How far apart are they (in kms) one minute before they collide?

1. $1/12$
2. $1/6$
3. $1/4$
4. $1/3$

**Sol.** Required distance = Relative speed of boats × time

$= (5 + 10) \times \dfrac{1}{60} = \dfrac{15}{60} = \dfrac{1}{4}$. **Ans.(3)**

***A very simple question present in PT's practice exercise # 01, Lecture # 10.***

**53.** Each family in a locality has at most two adults, and no family has fewer than 3 children. Considering all the families together, there are more adults than boys, more boys than girls, and more girls than families. Then the minimum possible number of families in the locality is

1. 4
2. 5
3. 2
4. 3

**Sol.** From the conditions of the question, we have

Adults > Boys > Girls > Families

Going by options,

| Number of Families | Maximum Number of Adults | Minimum Number of Children | Boys | Girls | |
|---|---|---|---|---|---|
| 2 | 4 | 6 | 3 | 3 (Families < Girls) | **Not possible** (Since Boys = Girls) |
| 3 | 6 | 9 | 5 | 4 (Families < Girls) | |
| 4 | 8 | 12 | - | | |

. **Ans.(4)**

**54.** In NutsAndBolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 minutes after production of every 1000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 minutes after production of every 1500 bolts. If both the machines start production at the same time, what is the minimum duration required for producing 9000 pairs of nuts and bolts?

1. 130 minutes 2. 135 minutes 3. 170 minutes 4. 180 minutes

**Sol.** Time required to make 9000 nuts = $=\left(\frac{1000}{100}+5\right) \times 9 - 5 = 130$ min.

Time required to make 9000 bolts = $=\left(\frac{1500}{75}+10\right) \times 6 - 10 = 170$ min

So required time = 170 min. **Ans.(3)**

**55.** A rectangular sheet of paper, when halved by folding it at the mid point of its longer side, results in a rectangle, whose longer and shorter sides are in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle is 2, what is the area of the smaller rectangle?

1. $4\sqrt{2}$ 2. $2\sqrt{2}$ 3. $\sqrt{2}$ 4. None of the above

**Sol.** Let the longer side of the rectangle = a

and the shorter side of the rectangle = b.

After folding, longer side = b and shorter side = a/2

Then, by the condition given in question

$$\frac{a}{b}=\frac{b}{a/2} \Rightarrow \frac{a^2}{2}=b^2$$

Again b = 2 (given in question)

$\Rightarrow \frac{a^2}{2}=4 \Rightarrow a=2\sqrt{2}$ . So, area of smaller rectangle = $\frac{2\sqrt{2}}{2}\times 2=2\sqrt{2}$ . **Ans.(2)**

***Questions with similar concepts were already taught in the class, should be a sitter for PT students.***