Directions for Questions 65 to 73: Answer the questions independently of each other.

65. A sprinter starts running on a circular path of radius r metres. Her average speed (in metres/minute) is πr during the first 30 seconds, $\pi r/2$ during next one minute, $\pi r/4$ during next 2 minutes, $\pi r/8$ during next 4 minutes, and so on. What is the ratio of the time taken for the nth round to that for the previous round?

1. 4   2. 8   3. 16   4. 32

Sol. The distances run by the sprinter in 30 sec, 1 min, 2 min, 4 min and so on are

$$\pi r \times \frac{1}{2} m,\ \frac{\pi r}{2} \times 1,\ \frac{\pi r}{4} \times 2,\ \frac{\pi r}{8} \times 4$$

and so on respectively i.e. we can observe that the distances travelled in the given times has been constant.

Let nth round be 2nd round then (n – 1)th round will be 1st round

Time taken to run first round = $\frac{1}{2} + 1 + 2 + 4 = 7.5$ min

Time taken to run 2nd round = 8 + 16 + 32 + 64 = 120 min

$\therefore$ Required ratio = $\frac{120}{7.5} = 16:1$. **Ans.(3)**

66. Consider the sequence of numbers $a_1, a_2, a_3, \ldots$ to infinity where $a_1 = 81.33$ and $a_2 = -19$ and $a_j = a_{j-1} - a_{j-2}$ for $j \geq 3$. What is the sum of the first 6002 terms of this sequence?

1. −100.33   2. −30.00   3. 62.33   4. 119.33

Sol. $a_1 = 81.33$

$a_2 = -19$

$a_3 = a_2 - a_1$

$a_4 = a_3 - a_2 = -a_1$

$a_5 = a_4 - a_3 = -a_2$

$a_6 = -a_2 + a_1$

$a_7 = a_1$, $a_8 = a_{2.}$

Repeated loop a1 to a6 (6 terms) has sum “0”

$\Rightarrow \quad a_{6002} = a_1 + a_2$

$= 81.33 - 19$

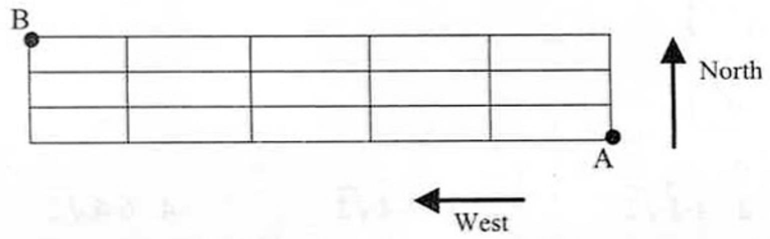
$= 62.33$. **Ans.(3)**

67. The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is

1. 4   2. 15   3. 0   4. 18

Sol. Since $15^{23} + 23^{23}$ is of the form $a^m + b^n$ where mondn are odd numbers, then it is definitely divisible by a + b. Apply this concept $15^{23} + 23^{23}$ will always be divisible by 19. Hence remainder = 0. **Ans.(3)**

68. In the adjoining figure, the lines represent one-way roads allowing travel only northwards or only westwards. Along how many distinct routes can a car reach point B from point A?



1. 15 2. 56 3. 120 4. 336

Sol. If we have m lines in north direction and n lines in west direction then the total number of ways to move from one end to the diagonally opposite end is given by $(m + n - 2)\ C_{n-1}$

$\therefore$ Required answer is $(6 + 4 - 2)C_{4-1} = {}^{8}C_{3} = 56$. **Ans.(2)**

Remember Question # 28 of MA Excel Sheet # 16 of PT " m parallel roads running from North to South....."

69. Let C be a circle with centre P_0 and AB be a diameter of C. Suppose P_1 is the mid point of the line segment P_0B, P_2 is the mid point of the line segment P_1B and so on. Let $C_1, C_2, C_3, \ldots$ be circles with diameters P_0P_1, P_1P_2, P_2P_3 … respectively. Suppose the circles $C_1, C_2, C_3, \ldots$ are all shaded. The ratio of the area of the unshaded portion of C to that of the original circle C is

1. 8:9 2. 9:10 3. 10:11 4. 11:12

Sol. Let the diameter of circle C

AB = x

Now $P_0B = x/2$

As P_1 is the mid point

$P_0P_1 = x/4 = P_1B$

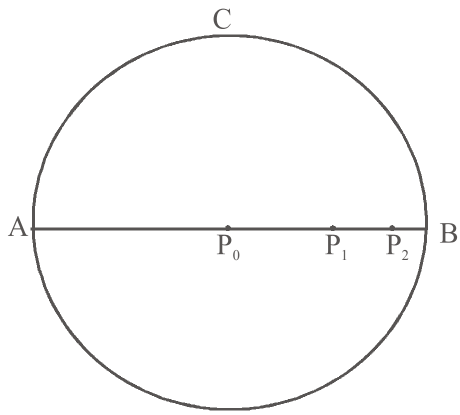
$\therefore$ Radius of $C_1 = x/8$

and $P_1P_2 = x/8 = P_2B$

radius of $C_2 = P_1P_2 = x/16$

and Radius of $C_3 = P_2P_3 = x/32$

Now area of $C_1 + C_2 + C_3 + \ldots\ldots\ldots\ldots =$



$$\pi\left(\frac{x}{8}\right)^2 + \pi\left(\frac{x}{16}\right)^2 + \pi\left(\frac{x}{32}\right)^2 + \ldots\ldots\ldots\ldots\infty$$

$$= \pi x^2 \left(\frac{1}{64} + \frac{1}{256} + \frac{1}{1024} + \ldots\ldots\infty\right)$$

This is an infinite G.P with common ratio 1/4

$$= \pi x^2 \times \frac{\frac{1}{64}}{1-\frac{1}{4}} = \frac{\pi x^2}{48}$$

Area of unshaded region $= \dfrac{\pi x^2}{4} - \dfrac{\pi x^2}{48} = \dfrac{11\times\pi x^2}{48}$

Ratio of unshaded portion to original circle $= \dfrac{11\times\pi x^2}{48} : \dfrac{\pi x^2}{4} = 11:12$. **Ans.(4)**

70. Let $u = (\log_2 x)^2 - 6\log_2 x + 12$ where x is a real number. Then the equation $x^u = 256$, has

1. no solution for x
2. exactly one solution for x
3. exactly two distinct solutions for x
4. exactly three distinct solutions for x

Sol. We have u = $(\log_2 x)^2 - 6\log_2 x + 12$

$\Rightarrow$ put $\log_2 x = y \Rightarrow x = 2^y$

$\Rightarrow x^u = 256 \Rightarrow x^u = 2^8 \Rightarrow 2^{uy} = 2^8 \Rightarrow uy = 8 \Rightarrow u = 8/y$

$\Rightarrow u = y^2 - 6y + 12 \Rightarrow \dfrac{8}{y} = y^2 - 6y + 12$

$\Rightarrow 8 = y^3 - 6y^2 + 12y \Rightarrow y^3 - 6y^2 + 12y - 8 = 0$

$\Rightarrow (y - 2)(y^2 - 4y + 4) = 0$ either $y - 2 = 0$ or $(y - 2)^2 = 0 \Rightarrow y = 2$

Hence equation has excactly one solution for x. **Ans.(2)**

A easy question, Similar questions discussed in PT's Classroom

71. A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent stripes have the same colour is

1. 12×81
2. 16×192
3. 20×125
4. 24×216

Sol. The flags have to be arranged in a vertical order as shown below

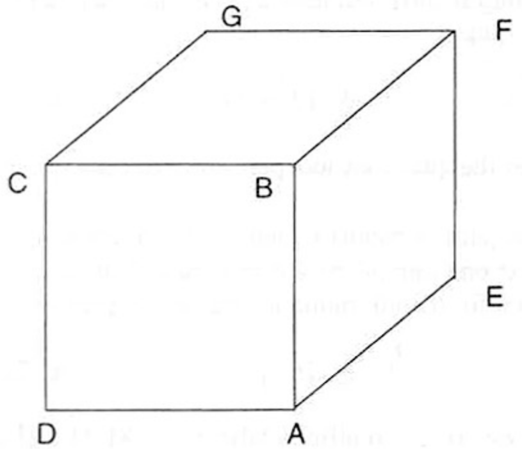
6
5
6
3
2
1

For the first place we can use anyone of the 4 flags in 4 ways.

For the second place we can use only 3 of the remaining flags and similarly for the 3rd, 4th, 5th and 6th place we can use 3 flags.

$\therefore$ Required number of ways = $4 \times 3^4 = 12 \times 81$. **Ans.(1)**

72. If the lengths of diagonals DF, AG and CE of the cube shown in the adjoining figure are equal to the three sides of a triangle, then the radius of the circle circumscribing that triangle will be



1. equal to the side of the cube
2. $\sqrt{3}$ times the side of the cube
3. $\frac{1}{\sqrt{3}}$ times the side of the cube
4. impossible to find from the given information

Sol. Let the side of cube = a

$\therefore$ DF = AG = CE = $a\sqrt{3}$

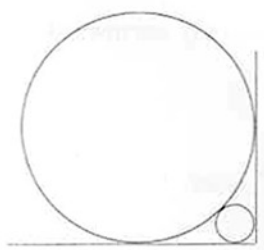
Now these sides form an equilateral triangle of side $a\sqrt{3}$

Area of equilateral $\Delta = \frac{3\sqrt{3}}{4} \times a^2$

Circumradius of the triangle = $\frac{(\text{side})^3}{4A} = \frac{(a\sqrt{3})^3}{4 \times \frac{3\sqrt{3}}{4} a^2} = a$. **Ans.(1)**

An absolute sitter, verbatim questions present in Chapter # 15 of PT's study material.

73. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smaller circle?



1. $3-2\sqrt{2}$
2. $4-2\sqrt{2}$
3. $7-4\sqrt{2}$
4. $6-4\sqrt{2}$

Sol. In triangle O_2B_1A

$x^2 = r^2 + r^2 \Rightarrow x = r\sqrt{2}$

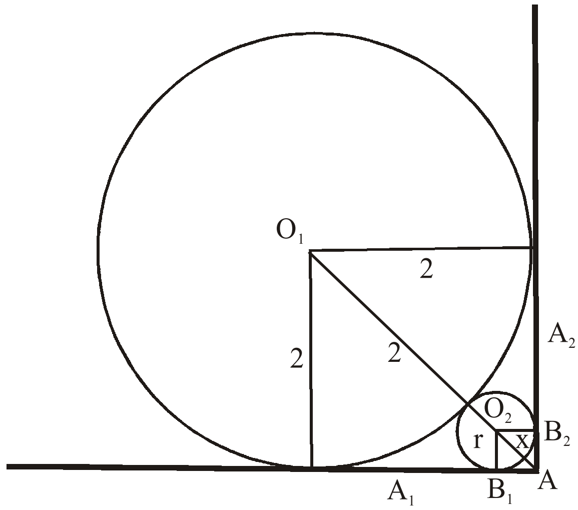
In triangle A_1O_1A

$2^2 + 2^2 = (2 + r + x)^2$

$\Rightarrow 2\sqrt{2} = 2 + r + x$

$2\sqrt{2} - 2 = r(1 + \sqrt{2})$

$r = \frac{2\sqrt{2}-2}{1+\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 2(2+1-2\sqrt{2}) = 6-4\sqrt{2}$. **Ans.(4)**



A very simple question, lots of similar questions are present in MA excel sheet# 13