

Important Formulas in QA

1. If an equation (i.e. $f(x) = 0$) contains all positive coefficients of any powers of x , it has no positive roots.

Eg: $x^3+3x^2+2x+6=0$ has no positive roots

2. For an equation, if all the even powers of x have same sign coefficients and all the odd powers of x have the opposite sign coefficients, then it has no negative roots.
3. For an equation $f(x)=0$, the maximum number of positive roots it can have is the number of sign changes in $f(x)$; and the maximum number of negative roots it can have is the number of sign changes in $f(-x)$
4. Complex roots occur in pairs, hence if one of the roots of an equation is $2+3i$, another has to be $2-3i$ and if there are three possible roots of the equation, we can conclude that the last root is real. This real root could be found out by finding the sum of the roots of the equation and subtracting $(2+3i)+(2-3i)=4$ from that sum.

5. ✓..... For a cubic equation $ax^3+bx^2+cx+d=0$

- Sum of the roots = $-b/a$
- Sum of the product of the roots taken two at a time = c/a
- Product of the roots = $-d/a$

- ✓..... For a bi-quadratic equation $ax^4+bx^3+cx^2+dx+e = 0$

- Sum of the roots = $-b/a$
- Sum of the product of the roots taken three at a time = c/a
- Sum of the product of the roots taken two at a time = $-d/a$
- Product of the roots = e/a

6. If an equation $f(x)=0$ has only odd powers of x and all these have the same sign coefficients or if $f(x) = 0$ has only even powers of x and all these have the same sign coefficients, then the equation has no real roots in each case (except for $x=0$ in the second case)

7. Consider the two equations

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

Then,

✓..... If $a_1/a_2 = b_1/b_2 = c_1/c_2$, then we have infinite solutions for these equations.

✓..... If $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, then we have no solution.

✓..... If $a_1/a_2 \neq b_1/b_2$, then we have a unique solution.

8. Roots of $x^2 + x + 1 = 0$ are 1, w , w^2 where $1 + w + w^2 = 0$ and $w^3 = 1$

9. $|a| + |b| = |a + b|$ if $a \cdot b \geq 0$

else, $|a| + |b| > |a + b|$

10. The equation $ax^2 + bx + c = 0$ will have max. value when $a < 0$ and min. value when $a > 0$. The max. or min. value is given by $(4ac - b^2)/4a$ and will occur at $x = -b/2a$

11.

✓..... If for two numbers $x + y = k$ (a constant), then their PRODUCT is MAXIMUM if $x = y (=k/2)$. The maximum product is then $(k^2)/4$.

✓..... If for two numbers $x \cdot y = k$ (a constant), then their SUM is MINIMUM if $x = y (= \sqrt{k})$. The minimum sum is then $2 \cdot \sqrt{k}$.

12. Product of any two numbers = Product of their HCF and LCM. Hence product of two numbers = LCM of the numbers if they are prime to each other.

13. For any 2 numbers a, b where $a > b$

✓..... $a > AM > GM > HM > b$ (where AM, GM, HM stand for arithmetic, geometric, harmonic means respectively)

✓..... $(GM)^2 = AM * HM$

14. For three positive numbers a, b, c

✓..... $(a + b + c) * (1/a + 1/b + 1/c) \geq 9$

15. For any positive integer n

✓..... $2 \leq (1 + 1/n)^n \leq 3$

16. $a^2 + b^2 + c^2 \geq ab + bc + ca$

If $a=b=c$, then the case of equality holds good.

17. $a^4 + b^4 + c^4 + d^4 \geq 4abcd$ (Equality arises when $a=b=c=d=1$)

18. $(n!)^2 > n^n$

19. If $a + b + c + d = \text{constant}$, then the product $a^p * b^q * c^r * d^s$ will be maximum if $a/p = b/q = c/r = d/s$

20. If n is even, $n(n+1)(n+2)$ is divisible by 24

21. $x^n - a^n = (x-a)(x^{n-1} + x^{n-2} + \dots + a^{n-1})$ Very useful for finding multiples.
For example $(17-14=3)$ will be a multiple of $17^3 - 14^3$

22. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ to infinity

Note: $2 < e < 3$

23. $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ to infinity [Note the alternating sign. Also note that the logarithm is with respect to base e]

24. $(m+n)!$ is divisible by $m! \cdot n!$

25. When a three digit number is reversed and the difference of these two numbers is taken, the middle number is always 9 and the sum of the other two numbers is always 9.

26. Any function of the type $y=f(x)=\frac{ax-b}{bx-a}$ is always of the form $x=f(y)$

27.

✓..... The sum of first n natural numbers = $\frac{n(n+1)}{2}$

✓..... The sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$

✓..... The sum of cubes of first n natural numbers is $\frac{(n(n+1)/2)^2}{4}$

✓..... The sum of first n even numbers = $n(n+1)$

✓..... The sum of first n odd numbers = n^2

28. If a number ' N ' is represented as $a^x \cdot b^y \cdot c^z \dots$ where $\{a, b, c, \dots\}$ are prime numbers, then

✓..... the total number of factors is $(x+1)(y+1)(z+1) \dots$

✓..... the total number of relatively prime numbers less than the number is $N * (1-1/a) * (1-1/b) * (1-1/c) \dots$

✓..... the sum of relatively prime numbers less than the number is $N/2 * N * (1-1/a) * (1-1/b) * (1-1/c) \dots$

✓..... the sum of factors of the number is $\{a^{(x+1)}\} * \{b^{(y+1)}\} * \dots / (x * y * \dots)$

29.

✓..... Total no. of prime numbers between 1 and 50 is 15

✓..... Total no. of prime numbers between 51 and 100 is 10

✓..... Total no. of prime numbers between 101 and 200 is 21

30.

✓..... The number of squares in $n*m$ board is given by $m*(m+1)*(3n-m+1)/6$

✓..... The number of rectangles in $n*m$ board is given by ${}^{n+1}C_2 * {}^{m+1}C_2$

31. If 'r' is a rational no. lying between 0 and 1, then, r^r can never be rational.

32. Certain nos. to be remembered

$$\checkmark \dots\dots 2^{10} = 4^5 = 32^2 = 1024$$

$$\checkmark \dots\dots 3^8 = 9^4 = 81^2 = 6561$$

$$\checkmark \dots\dots 7 * 11 * 13 = 1001$$

$$\checkmark \dots\dots 11 * 13 * 17 = 2431$$

$$\checkmark \dots\dots 13 * 17 * 19 = 4199$$

$$\checkmark \dots\dots 19 * 21 * 23 = 9177$$

$$\checkmark \dots\dots 19 * 23 * 29 = 12673$$

33. Where the digits of a no. are added and the resultant figure is 1 or 4 or 7 or 9, then, the no. could be a perfect square.

34. If a no. 'N' has got k factors and a^l is one of the factors such that $l \geq k/2$, then, a is the only prime factor for that no.

35. To find out the sum of 3-digit nos. formed with a set of given digits

This is given by (sum of digits) * (no. of digits-1)! * 1111...1 (i.e. based on the no. of digits)

Eg) Find the sum of all 3-digit nos. formed using the digits 2, 3, 5, 7 & 8.

$$\text{Sum} = (2+3+5+7+8) * (5-1)! * 11111 \text{ (since 5 digits are there)}$$

$$= 25 * 24 * 11111$$

$$= 6666600$$

36. Consider the equation $x^n + y^n = z^n$

As per Fermat's Last Theorem, the above equation will not have any solution whenever $n \geq 3$.

37. Further as per Fermat, where 'p' is a prime no. and 'N' is co-prime to p, then, $N^{(p-1)} - 1$ is always divisible by p.

38. 145 is the 3-digit no. expressed as sum of factorials of the individual digits i.e.

$$145 = 1! + 4! + 5!$$

39.

✓..... Where a no. is of the form $a^n - b^n$, then,

•□□ The no. is always divisible by $a - b$

•□□ Further, the no. is divisible by $a + b$ when n is even and not divisible by $a + b$ when n is odd

✓..... Where a no. is of the form $a^n + b^n$, then,

•□□ The no. is usually not divisible by $a - b$

•□□ However, the no. is divisible by $a + b$ when n is odd and not divisible by $a + b$ when n is even

40. The relationship between base 10 and base 'e' in log is given by $\log_{10}N = 0.434 \log_e N$

41. WINE and WATER formula

Let Q - volume of a vessel, q - qty of a mixture of water and wine be removed each time from a mixture, n - number of times this operation is done and A - final qty of wine in the mixture, then,

$$A/Q = (1 - q/Q)^n$$

Eg) The price of a T.V set is increased by 40 % of the cost price and then is decreased by 25% of the new price. On selling, the profit made by the dealer was Rs.1000. At what price was the T.V sold?

Applying the formula,

$$\text{Final difference\%} = 40 - 25 - (40 \times 25 / 100) = 5 \%$$

So if 5 % = 1,000

Then, 100 % = 20,000.

Hence, C.P = 20,000

& S.P = 20,000 + 1000 = 21,000

44. Where the cost price of 2 articles is same and the mark up % is same, then, marked price and NOT cost price should be assumed as 100.

45.

✓..... Where 'P' represents principal and 'R' represents the rate of interest, then, the difference between 2 years' simple interest and compound interest is given by $P \times (R/100)^2$

✓..... The difference between 3 years' simple interest and compound interest is given by $(P \times R^2 \times (300+R))/100^3$

46.

✓..... If A can finish a work in X time and B can finish the same work in Y time then both of them together can finish that work in $(X \times Y) / (X+Y)$ time.

✓..... If A can finish a work in X time and A & B together can finish the same work in S time then B can finish that work in $(XS)/(X-S)$ time.

✓..... If A can finish a work in X time and B in Y time and C in Z time then all of them working together will finish the work in $(XYZ) / (XY + YZ + XZ)$ time

✓..... If A can finish a work in X time and B in Y time and A, B & C together in S time then

•□□ C can finish that work alone in $(XYS) / (XY-SX-SY)$

•□□ B+C can finish in (SX)/(X-S); and

•□□ A+C can finish in (SY)/(Y-S)

47. In case 'n' faced die is thrown k times, then, probability of getting atleast one more than the previous throw = ${}^n C_5/n^5$

48.

✓..... When an unbiased coin is tossed odd no. (n) of times, then, the no. of heads can never be equal to the no. of tails i.e. $P(\text{no. of heads}=\text{no. of tails}) = 0$

✓..... When an unbiased coin is tossed even no. (2n) of times, then,
 $P(\text{no. of heads}=\text{no. of tails}) = 1 - ({}^{2n} C_n/2^{2n})$

49. Where there are 'n' items and 'm' out of such items should follow a pattern, then, the probability is given by $1/m!$

Eg)1. Suppose there are 10 girls dancing one after the other. What is the probability of A dancing before B dancing before C?

Here $n=10$, $m=3$ (i.e. A, B, C)

Hence, $P(A>B>C) = 1/3!$

$= 1/6$

Eg)2. Consider the word 'METHODS'. What is the probability that the letter 'M' comes before 'S' when all the letters of the given word are used for forming words, with or without meaning?

$P(M>S) = 1/2!$

$= 1/2$

50. CALENDAR

- ✓..... Calendar repeats after every 400 years.
- ✓..... Leap year- it is always divisible by 4, but century years are not leap years unless they are divisible by 400.
- ✓..... Century has 5 odd days and leap century has 6 odd days.
- ✓..... In a normal year 1st January and 2nd July and 1st October fall on the same day. In a leap year 1st January 1st July and 30th September fall on the same day.
- ✓..... January 1, 1901 was a Tuesday.

51.

- ✓..... For any regular polygon, the sum of the exterior angles is equal to 360 degrees, hence measure of any external angle is equal to $360/n$ (where n is the number of sides)
- ✓..... For any regular polygon, the sum of interior angles $= (n-2) * 180$ degrees
So measure of one angle is $(n-2)/n * 180$
- ✓..... If any parallelogram can be inscribed in a circle, it must be a rectangle.
- ✓..... If a trapezium can be inscribed in a circle it must be an isosceles trapezium (i.e. oblique sides equal).

52. For an isosceles trapezium, sum of a pair of opposite sides is equal in length to the sum of the other pair of opposite sides (i.e. $AB+CD = AD+BC$, taken in order)

53.

- ✓..... For any quadrilateral whose diagonals intersect at right angles, the area of the quadrilateral is

$0.5 * d_1 * d_2$, where d_1, d_2 are the length of the diagonals.

- ✓..... For a cyclic quadrilateral, area = $\sqrt{(s-a) * (s-b) * (s-c) * (s-d)}$, where $s = (a + b + c + d)/2$

Further, for a cyclic quadrilateral, the measure of an external angle is equal to the measure of the interior opposite angle.

- ✓..... Area of a Rhombus = Product of Diagonals/2

54. Given the coordinates (a, b); (c, d); (e, f); (g, h) of a parallelogram, the coordinates of the meeting point of the diagonals can be found out by solving for

$$[(a + e)/2, (b + f)/2] = [(c + g)/2, (d + h)/2]$$

55. Area of a triangle

✓..... $1/2 \cdot \text{base} \cdot \text{altitude}$

✓..... $1/2 \cdot a \cdot b \cdot \sin C$ (or) $1/2 \cdot b \cdot c \cdot \sin A$ (or) $1/2 \cdot c \cdot a \cdot \sin B$

✓..... $\text{root}(s \cdot (s-a) \cdot (s-b) \cdot (s-c))$ where $s = (a+b+c)/2$

✓..... $a \cdot b \cdot c / (4 \cdot R)$ where R is the circumradius of the triangle

✓..... $r \cdot s$, where r is the inradius of the triangle

56. In any triangle

✓..... $a = b \cdot \cos C + c \cdot \cos B$

✓..... $b = c \cdot \cos A + a \cdot \cos C$

✓..... $c = a \cdot \cos B + b \cdot \cos A$

✓..... $a/\sin A = b/\sin B = c/\sin C = 2R$, where R is the circumradius

✓..... $\cos C = (a^2 + b^2 - c^2) / 2ab$

✓..... $\sin 2A = 2 \sin A \cdot \cos A$

✓..... $\cos 2A = \cos^2(A) - \sin^2(A)$

57. The ratio of the radii of the circumcircle and incircle of an equilateral triangle is 2:1

58. Appollonius Theorem

In a triangle ABC, if AD is the median to side BC, then

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \text{ or } 2(AD^2 + DC^2)$$

59.

✓..... In an isosceles triangle, the perpendicular from the vertex to the base or the angular bisector from vertex to base bisects the base.

✓..... In any triangle the angular bisector of an angle bisects the base in the ratio of the other two sides.

60. The quadrilateral formed by joining the angular bisectors of another quadrilateral is always a rectangle.

61. Let W be any point inside a rectangle ABCD, then, $WD^2 + WB^2 = WC^2 + WA^2$

62. Let a be the side of an equilateral triangle, then, if three circles are drawn inside this triangle such that they touch each other, then each circle's radius is given by $a/(2*(\sqrt{3}+1))$

63.

✓..... Distance between a point (x_1, y_1) and a line represented by the equation $ax + by + c=0$ is given by $|ax_1+by_1+c|/\sqrt{a^2+b^2}$

✓..... Distance between 2 points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

64. Where a rectangle is inscribed in an isosceles right angled triangle, then, the length of the rectangle is twice its breadth and the ratio of area of rectangle to area of triangle is 1:2.