PROGRESSION

Arithmetic Progression (A.P.):

It is a series in which any two consecutive terms have a common difference and the next term can be derived by adding that common difference to the previous term.

Therefore $T_{n+1} - T_n$ = constant and called common difference (d) for all n $\in$ N.

If in an A. P. **a** = first term,

d = common difference = $\mathbf{T_n - T_{n-1}}$

$\mathbf{T_n}$ = n^{th} term (Thus T_1 = first term, T_2 = second term, T_{10} tenth term and so on.)

l = last term,

$\mathbf{S_n}$ = Sum of n terms.

a, a + d, a + 2d, a + 3d,... are in A.P.

The n^{th} term of an A.P is given by the formula

$$\boxed{T_n = a + (n - 1)\, d}$$

Note: If the last term of the A.P. consisting of n terms be ℓ,

$$\ell = a + (n - 1)\, d$$

The sum of first n terms of an AP is usually denoted by S_n and is given by the following formula:

$$\boxed{\mathbf{S_n} = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)}$$

Where '*l*' is the last term of the series.

Properties of an AP:

I. If each term of an AP is increased, decreased, multiplied or divided by the same non-zero number, the resulting sequence is also an AP.

For example: For A.P. 3, 5, 7, 9, 11…

If you add a constant let us say 1 to each term, you get	4, 6, 8, 10, 12......	This is an A.P. with a common difference of 2.
If you multiply each term by a constant let us say 2, you get	6, 10, 14, 18, 22…..	Again this is an A.P. with a common difference of 4.

II. In an AP, the sum of terms equidistant from the beginning and end is always same and equal to the sum of first and last terms as shown in example below.

III. Three numbers in AP are taken as a – d, a, a + d.
For 4 numbers in AP are taken as a – 3d, a – d, a + d, a + 3d
For 5 numbers in AP are taken as a – 2d, a – d, a, a + d, a + 2d

IV. Three numbers a, b, c are in A.P. if
2b = a + c.
or $b = \frac{a+c}{2}$ and b is called the Arithmetic mean of a & c

Geometric Progression:

A series in which each preceding term is formed by multiplying it by a constant factor is called a Geometric Progression G. P. The constant factor is called the common ratio and is formed by dividing any term by the term which precedes it.

In other words, a sequence, $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called a geometric progression

If $\frac{a_{n+1}}{a_n}$ = constant for all n $\in$ N.

The General form of a G. P. with n terms is a, ar, $ar^2 \ldots ar^{n-1}$

Thus, if **a** = the first term

r = the common ratio,

$\mathbf{T_n}$ = nth term and

$\mathbf{S_n}$ = sum of n terms

General term of a GP is $T_n = ar^{n-1}$

Sum of first n terms of G.P:

a. $\mathbf{S_n} = \frac{a(r^n - 1)}{r - 1}$ where r > 1

b. $\mathbf{S_n} = \frac{a(1 - r^n)}{1 - r}$ where r < 1

c. $\mathbf{S_n}$ = na where r = 1

Sum of infinite G.P:

If a G.P. has **infinite terms** and **– 1 < r < 1 or $|x|$ < 1**,

Sum of infinite G.P is $S_\infty = \frac{a}{1-r}$.

Geometric mean:

Three non-zero numbers a, b, c are in G.P. if

$b^2 = ac$ or $b = \sqrt{ac}$

b is called the geometric mean of a & c

Harmonic Progression (H.P):

1. A series of quantities is said to be in a harmonic progression when their reciprocals are in arithmetic progression.

   e.g. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ and $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$ are in HP as their reciprocals

   3, 5, 7, …, and a, a + d, a + 2d…….. are in AP.

2. T_n of the HP is …… $\frac{1}{a+(n-1)d}$

3. In order to solve a question on HP, one should form the corresponding AP.

A comparison between AP and GP:

Description	AP	GP
Principal characteristic	Common Difference (d)	Common Ratio (r)
n^{th} term	$T_n = a + (n - 1)d$	$T_n = ar^{(n-1)}$
Mean	$A = (a + b)/2$	$G = (ab)^{1/2}$
Sum of First n Terms	$S_n = n/2\,[2a + (n - 1)d] = n/2\,[a + \ell]$	$S_n = a(1 - r^n)/(1 - r)$
'm'th mean	$a + [m(b - a)/(n + 1)]$	$a(b/a)^{m/(n+1)}$

Arithmetic – Geometric progression:

$a + (a + d)r + (a + 2d)r^2 + (a + 3d)r^3 + \ldots\ldots$ Is the form of Arithmetic geometric progression (A.G.P). One part of the series is in Arithmetic progression and the other part is in Geometric progression.

The sum of first n terms of the series is $S_n = \frac{a}{1-r} + dr\frac{(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{(1-r)}$.

The sum of a series is $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Relation between AM, GM and HM:

For two positive numbers a and b

A = Arithmetic mean = $\frac{a+b}{2}$

G = Geometric Mean = $\sqrt{ab}$

H = Harmonic Mean = $\frac{2ab}{a+b}$

Multiplying A and H, we get

A x H = $\frac{a+b}{2} \times \frac{2ab}{a+b}$ = ab = G^2

<u>NOTE:</u> AM $\geq$ GM $\geq$ HM