PERCENTAGE

A percentage *is a* ratio *expressed in terms of a* unit *being 100. A percentage is usually denoted by the symbol "%"*

- To express a% as a fraction, divide it by 100 $\Rightarrow$ a% = a/100
- To express a fraction as %, multiply it by 100 $\Rightarrow$ a/b = [(a/b) $\times$ 100]%
- x% of y is given by $\frac{x}{100}$ y

Conversion of fractions to percentage:

$\frac{1}{1}$ = 100% $\quad \frac{1}{2}$ = 50% $\quad \frac{1}{3}$ = 33.33% $\quad \frac{1}{4}$ = 25% $\quad \frac{1}{5}$ = 20% $\quad \frac{1}{6}$ = 16.66%

$\frac{1}{7}$ = 14.28% $\quad \frac{1}{8}$ = 12.5% $\quad \frac{1}{9}$ = 11.1% $\quad \frac{1}{10}$ = 10% $\quad \frac{1}{11}$ = 9.09% $\quad \frac{1}{12}$ = 8.33%

Percentage Increase/Decrease

- X increased by 10% is given by x + 0.1x = 1.1x
  Similarly 20% more of x = x + 0.2x = 1.2x
  10% less of x = x – 0.1x = 0.9x
  20% less of x = x – 0.2x = 0.8x
- If x is **n** times of y, it means x is (n – 1) × 100% more than y.
- Percentage Increase = [Increase / Original value] $\times$ 100%
- Percentage Decrease = [Decrease / Original value] $\times$ 100%
- Percentage Change = [Change / Original value] $\times$ 100%
- If A is x% more / less than B, then B is $\frac{100\ x}{100 \pm x}$ %.less/more than A.

If any number (quantity) is changed (increased/decreased) by p%, then

New quantity = Original quantity $\times \left(\frac{100 + p^*}{100}\right)$

* p is (–) ve, when the original quantity is reduced by p%.
New value = original value + increase
Or New value = original value – decrease

Percentage change in product of two quantities

Consider a product of two quantities A = a $\times$ b

If a and b change (increase or decrease) by a certain percentage say x & y respectively, then the overall %age change in their product is given by the formula:

$$x + y + \frac{xy}{100}$$

This formula also holds true if there are successive changes as in the case of population increase or decrease. But when there are either more than 2 successive changes or there is a product of more than 2 quantities as in the case of volume, **then we have to apply the same formula twice.**

This formula can be used for following questions:

- If A is successively increased by X% and Y%, find the percentage increase
- If there is successive discount of X% and Y%, find the total discount.
- If there is X% increase and y% decrease, find the total change is X-Y-$\frac{XY}{100}$
- If the sides of a rectangle increases by X% and Y%, Find the percentage increase in its area

Population Increase/Decrease

Let the present population of a town be "p" and let there be an increase/decrease at X% per annum. Then

(i) Population after n years = $p[1 + (X/100)]^n$

(ii) Population n years ago = $p/[1 + (X/100)]^n$

[X is positive if population is increasing annually and negative if decreasing]

Income Comparison

(i) If A's income is r% more than B's then B's income is $[r / (r + 100)] \times 100$ % less than A's

(ii) If A's income is r% less than B's then B's income is $[r /(100 – r)] \times 100$% more than A's

Mixture problems:

If x% of a quantity is taken by the first person, y% of the remaining quantity is taken by the second person, and z% of the remaining is taken by the third person and if A is left, then initial quantity was

$$= \frac{A \times 100 \times 100 \times 100}{(100 - x)\,(100 - y)\,(100 - z)}$$

The same concept we can use, if we add something, then the initial quantity was

$$= \frac{A \times 100 \times 100 \times 100}{(100 + x)\,(100 + y)\,(100 + z)}$$

Profit, Loss and Discount

1. Gain or profit = S.P – C.P
2. Profit % = $\frac{S.P - C.P}{C.P} \times 100$ (S.P. is sold price, C.P. is cost price)
3. Discount = M.P – S.P (M.P is marked price)
4. Discount % = $\frac{M.P - S.P}{M.P} \times 100$
5. If the product is constant, and if one quantity increases / decreases by x%, then the other quantity decreases / increases by $\frac{100\ x}{100 \pm x}$%.
6. If the price of an item increases by x%, the consumption has to be reduced by $\frac{100}{100 + x}$% to keep the expenditure constant.
7. If two articles are sold at the same price, and on the first one a shopkeeper makes a profit of p% and on the other suffers a loss of p%, overall he will suffer a loss and it is given by

$$\textbf{Loss} = \frac{p^2}{100}\%$$