BE4-R3: PRINCIPLES OF MODELLING AND SIMULATION

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let X have the exponential distribution with mean β, the cdf is

$$F(x) = \begin{cases} 1-e^{-x/\beta} & , \ x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Outline the procedure to generate the desired random variate.

b) One of the principal approaches for advancing simulation clock in a discrete-event simulation is called fixed-increment time advance. For which type of problems is this approach used?

c) A telephone exchange receives 100 calls per minute on average, according to a Poisson process. What is the probability that no calls are received in an interval of five seconds?

d) There are different ways in which a system can be studied: Experiment with the Actual System v/s Experiment with a Model of the system. Illustrate the ways with suitable examples.

e) What are pseudo random numbers? Discuss the linear congruential generator.

f) Compare simulation packages with programming languages. Also list the advantages of using both types of the software.

g) Differentiate between deterministic and stochastic system. Give two examples of each system.

(7x4)

2.

a) Suppose X and Y are uniform random variables over (0, 1). Noting that $P(x^2 + y^2 \leq 1) = \frac{\pi}{4}$, describe Monte Carlo method to estimate π.

b) i) Very briefly explain why it is not a good idea to operate a queuing system under a heavy load (arrival rate close to service rate).

ii) Very briefly explain why an infinite buffer queue is said to be "unstable" when the mean arrival rate is greater than the mean service rate.

c) For the state independent M/M/1 queuing system, obtain the ratio p_{100}/p_0 when λ=1.0 and μ=2.0. Comment on potential numerical problems.

(6+6+6)

3.

a) Describe three-step approach, employed in the validation process, viz

i) Build a model that has high face validity

ii) Validate model assumptions

iii) Compare the model input-output transformations to that for real systems.

b) State the purpose of model verification. Compare and contrast the issues involved in validation and verification.

c) Differentiate between calibration and validation of a system in discrete-event simulation.

(8+4+6)

4.

a) What type of statistical analysis is carried out to gain insight into output data generated in simulation experiments?

b) Bring out the need for identification of probability distribution which characterizes the input models in simulation studies. Discuss the situation when such characterization is needed. Which probability distributions are used?

c) Differentiate between steady state and transient state of a system.

(6+6+6)

5.

a) For discrete distributions, show that the Histogram is an unbiased estimator of the (unknown) mass function i.e. E(hj)=p(xj) for all j. To make a Histogram, we break the range of values into k disjoint adjacent intervals $[b_0, b_1]$, $[b_1, b_2]$ …, $[b_{k-1}, b_k]$. Here hj denotes the proportion of X_i's that are in the j^{th} interval $[b_{j-1}, b_j]$.

b) Argue heuristically the comparable output random variables from replications using different numbers should be independent.

c) Give inverse-transform algorithm from the density

$$f(x) = \begin{cases} \frac{3x^2}{2} & , \quad -1 \le x \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(6+6+6)

6.

a) Given a mathematical model of a system, it is sometimes possible to derive information about the system by analytical means. When is it not possible?

b) In the aircraft system, suppose the control surface angle Y is made to be A times the error signal. The response of the aircraft to the control surface is found to be $I\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} = Ky$ where I, D, K are constant. Find the conditions under which the aircraft motion is oscillatory.

c) A simple model of the inventory control system would need two levels and three rates which are defined as follows:
X (Current inventory level), Y (Outstanding level of orders placed with the supplier), U (Rate of ordering from supplier), V (Rate of delivery from supplier), S (Rate of sales).
In addition, two constants need to be defined. I (Planned inventory level), T* (Average delivery time).
Draw system dynamics model of the inventory control system.

(6+6+6)

7.

a) Develop a GPSS model of a telephone system. The system is one in which a series of calls come from a number of telephone lines and the system is to connect the calls by using one of a limited number of links. Only one call can be made to any one line at a time and it is assumed that calls are lost if the called party is busy or no link is available. Each line is represented by a logic switch whose number is line number. The line is considered busy if the switch is set.

b) You are given 2n uniformly distributed random numbers $u_1, u_2, \ldots, u_{2n}$. Describe the procedure to generate normally distributed random numbers with mean μ and variance σ^2.

c) Describe frequency test and run test in the context of text of random numbers.

(7+6+5)