PART A - PHYSICS

1. The transverse displacement $y(x, t)$ of a wave on a string is given by

$$y(x, t) = e^{-[ax^2 + bt^2 + 2\sqrt{(ab)}\, xt]}$$

This represents a :

(1) Wave moving in –x direction with speed $\sqrt{(b / a)}$

(2) Standing wave of frequency $\sqrt{b}$

(3) Standing wave of frequency $(1 / \sqrt{b})$

(4) Wave moving in +x direction with speed $\sqrt{(a / b)}$

1. **(1)** $y(x, t) = e^{-[ax^2 + bt^2 + 2\sqrt{(ab)}\, xt]}$

$$= e^{-(\sqrt{a}\,x + \sqrt{b}\,t)^2}$$

Coeff of x and t have same sign $\Rightarrow$ wave is moving in $-x$ direction

v = (coeff of t / coeff of x) = $\sqrt{(b / a)}$

2. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm

Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale. The diameter of wire from the above data is :

(1) 0.052 cm (2) 0.026 cm (3) 0.005 cm (4) 0.52 cm

2. **(1)** LC = (1 / 200) mm

$\therefore$ Reading $= 0 + 52 \times 10^{-2}$

$= 0.52$ mm $= 0.052$ cm

3. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is:

(1) g (2) $(2 / 3)\, g$ (3) $(g / 3)$ (4) $(3 / 2)\, g$

3. **(2)** $mg - T = ma$ .... (1)

and $TR = I\alpha \Rightarrow TR = (mR^2 / 2)\,(a / R)$

$\Rightarrow T = (ma / 2)$ .... (2)

from (1) & (2) $a = (2g / 3)$

4. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = $0.03\ Nm^{-1}$)

(1) 0.2π mJ (2) 2π mJ (3) 0.4π mJ (4) 4π mJ

4. **(3)** $W = \Delta U = S \times 2\Delta A = 0.03 \times 2 \times 4\pi(25 - 9) \times 10^{-4} \approx 0.4\ \pi$ mJ

5. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc:

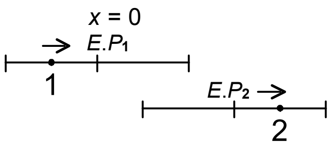
(1) continuously decreases (2) continuously increases
(3) first increases and then decreases (4) remains unchanged

5. **(3)** $\therefore$ $I\omega$ = constant

As the insect moves towards the centre, I decreases $\Rightarrow$ ω increases.
When the insect moves from centre to the rim, I increases $\Rightarrow$ ω decreases.

6. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is:

(1) $(\pi / 3)$ (2) $(\pi / 4)$ (3) $(\pi / 6)$ (4) $(\pi / 2)$

6. **(1)** 

At maximum separation, relative velocity is zero

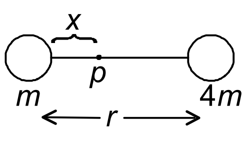
$\Rightarrow$ *particle-1* at $x = (-A / 2)$ moving along +ve x-direction and
particle-2 at $x = [X_0 + (A / 2)]$ moving along +ve x-direction.

$\therefore$ Phase for *particle-1* = $(-\pi / 6)$ and phase for *particle-2* = $(+\pi / 6)$

$\therefore$ $\Delta\phi = (\pi / 3)$

7. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is:

(1) $-(4Gm / r)$ (2) $-(6Gm / r)$ (3) $-(9Gm / r)$ (4) Zero

7. (3) 

Field to be zero, $(Gm / x^2) = [g(4m) / (r - x)^2]$ $\Rightarrow$ $x = (r / 3)$

So potential at p

$$= \frac{-Gm}{\left(\frac{r}{3}\right)} - \frac{-G(4m)}{\left(\frac{2r}{3}\right)} = (-GM / r)(3 + 6) = (-9Gm / r)$$

8. Two identical charged spheres suspended from a common point by two massless strings of length L are initially a distance $d(d << L)$apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them,

(1) $v \propto x^{-1}$ (2) $v \propto x^{1/2}$ (3) $v \propto x$ (4) $v \propto x^{-1/2}$

8. **(4)** $F = (kq^2 / x^2) \approx mg\,(x / 2L)$ $\Rightarrow q = cx^{1.5}$. $\therefore dq / dt = 1.5c\sqrt{x}\,v$.
Since dq / dt = constant, $v \propto x^{-1/2}$.

9. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1} m^{-1} due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is:

(1) 0.75 mV (2) 0.50 mV (3) 0.15 mV (4) 1 mV

9. **(3)** $\varepsilon = 5 \times 10^{-5} \times 1.5 \times 2 = 0.15$ mV

10. An object, moving with a speed of 6.25 ms^{-1}, is decelerated at a rate given by:

$(dv / dt) = -2.5\sqrt{v}$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be:

(1) 2 s (2) 4 s (3) 8 s (4) 1 s

10. **(1)** $(dv / dt) = -2.5\sqrt{v}$ $\Rightarrow$ $(dv / v) = -2.5\,dt$

$\Rightarrow \int \frac{dv}{v} = -2.5 \int dt$ $\Rightarrow$ $2_{6.25}^{0}\left[\sqrt{v}\right] = 2.5\,t$

$\Rightarrow t = (2 \times 2.5) / 2.5) = 2$ sec

11. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is:

(1) $(\pi / 4)\sqrt{LC}$ (2) $2\pi\sqrt{LC}$ (3) $\sqrt{LC}$ (4) $\pi\sqrt{LC}$

11. **(1)** for $L.C$ oscillation $T.P = 2\pi\sqrt{(LC)}$. Time at which energy will be equal
$T = (T.P / 8) = [2\pi\sqrt{(LC)} / 8] = (\pi / 4)\sqrt{(LC)}$

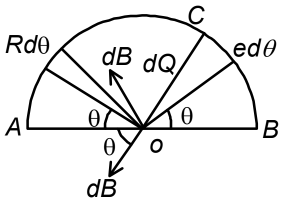
12. Let the $x - z$ plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\,\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is:

(1) 45° (2) 60° (3) 75° (4) 30°

12. **(1)** **Note :-** In question plane should be x - y not x - z. As boundary
Angle made with z-axis, $\cos\theta = (10 / 20) = (1 / 2)$ $\Rightarrow \theta = 60°$
Using Snell's law, $\sqrt{2}\,.\sin 60° = \sqrt{3}\,.\sin r$ $\Rightarrow \sin r = (1 / \sqrt{2})$ $\Rightarrow r = 45°$.

13. A current I flows in an infinitely long wire with cross section in the form of a semi-circular ring of radius R. The magnitude of the magnetic induction along its axis is:
(1) $(\mu_0 I / 2\pi^2 R)$ (2) $(\mu_0 I / 2\pi R)$ (3) $(\mu_0 I / 4\pi R)$ (4) $(\mu_0 I / \pi^2 R)$

13. **(4)**



Resultant B will be along AB. $dB' = 2\, dB \cos\theta = \frac{2\mu_0}{2\pi R}\left(\frac{I}{\pi R} Rd\theta\right)\cos\theta$

$$\int dB' = \frac{\mu_0 I^2}{\pi^2 R}\int_0^{\pi/2}\cos\theta\, d\theta \qquad \Rightarrow B' = \frac{\mu_0 I^2}{\pi^2 R}$$

14. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by:
(1) $[(\gamma - 1) / 2\gamma R] Mv^2 K$ (2) $[\gamma Mv^2 / 2R] K$
(3) $[(\gamma - 1) / 2R] Mv^2 K$ (4) $[(\gamma - 1) / 2(\gamma + 1)R] Mv^2 K$

14. **(3)** K.E $= (1 / 2)\, (n.\, M)\, V^2 = n\, .\, (R / (\gamma - 1)).\, \Delta T$ $\therefore \Delta T = [((\gamma - 1)\, MV^2) / 2R]$

15. A mass M, attached to a horizontal spring executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of (A_1 / A_2) is:
(1) $[(M + m) / M]$ (2) $[M / (M + m)]^{1/2}$
(3) $[(M + m) / M]^{1/2}$ (4) $[M / (M + m)]$

15. **(3)** When only M mass attached with spring. Then $(1 / 2)\, KA_1^2 = (1 / 2)\, MV_1^2$
$\Rightarrow V_1 = \sqrt{(K / M)}.\, A_1$ ...(1)
using C.L.M $\Rightarrow MV_1 = (m + M)V_2$
$\Rightarrow V_2 = (M / (m + M))\, V_1$ ...(2)
for $(m + M)$ mass with spring :-
$(1 / 2)\, KA_2^2 = (1 / 2)\, (m + M)\, V_2^2$
$\therefore V_2 = \sqrt{(K / (m + M))}\, A_2$ ...(3)
for (1), (2) and (3)
$\sqrt{(K / (m + M))}\, A_2 = M / (m + M)\, .\, \sqrt{(K / M)}.\, A_1$
$\therefore (A_1 / A_2) = ((M + m) / M)^{1/2}$

16. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to:
(1) 7.5×10^{-3} m (2) 9.6×10^{-3} m (3) 3.6×10^{-3} m (4) 5.0×10^{-3} m

16. **(3)** $V_2 = \sqrt{(V_1^2 + 2gh)} = \sqrt{((0.4)^2 + 2 \times 10 \times 2 \times 10^{-1})} = \sqrt{(4.16)}\ ms^{-1}$
By continuity, $A_1 V_1 = A_2 V_2$
$\Rightarrow d_1^2 V_1 = d_2^2 V_2$

$$\Rightarrow d_2 = d\sqrt{(V_1 / V_2)} = 8 \times 10^{-3} \sqrt{\frac{0.4}{\sqrt{4.16}}} \approx 3.6 \times 10^{-3} \text{ m}$$

17. This question has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement – 1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.
Statement – 2 : The state of ionosphere varies from hour to hour, day to day and season to season.
(1) Statement – 1 is true, Statement – 2 is true, Statement – 2 is the correct explanation of Statement – 1.
(2) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not the correct explanation of Statement – 1.
(3) Statement – 1 is false, Statement – 2 is true.
(4) Statement – 1 is true, Statement – 2 is false.

17. **(1)**

18. Three perfect gases at absolute temperatures T_1 and T_2 and T_3 are mixed. the masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is :

(1) $\dfrac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$ (2) $\dfrac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(3) $\dfrac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ (4) $\dfrac{(T_1 + T_2 + T_3)}{3}$

18. **(1)** **Note :** It needs to be asssumed that all the three gases have the same atomicity.
$\Sigma (3/2)\, n_i \,.\, (3/2)\, kT_i = (n_1 + n_2 + n_3)\, [(3/2)\, kT]$ (internal energy constant)
$\therefore\ T = [(n_1T_1 + n_2T_2 + n_3T_3) / (n_1 + n_2 + n_3)]$

19. A pulley of radius 2 m is rotated about its axis by a force $F = (20\,t - 5\,t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is :

(1) more than 3 but less than 6 (2) more than 6 but less than 9

(3) more than 9 (4) less than 3

19. **(1)** $\alpha = (FR / I) = [(20t - 5t^2)2 / 10] = 4t - t^2$

$\omega = \int_0^t \alpha\, dt = [2t - (t^3 / 3)]$ $\omega = 0$ $\Rightarrow$ $t = 6$ sec

$\therefore\ \theta = \int_0^6 \omega\, dt = 36$ rad. $\therefore$ Rotations $= 36 / 2\pi$ which is slightly less than 6.

20. A resistor R and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that light up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed ($\log_{10} 2.5 = 0.4$)

(1) $1.7 \times 10^5\ \Omega$ (2) $2.7 \times 10^6\ \Omega$ (3) $3.3 \times 10^7\ \Omega$ (4) $1.3 \times 10^4\ \Omega$

20. **(2)** $q = CE\,(1 - e^{-t/RC})$

$V = (q / C) = E\,(1 - e^{-t/RC})$

$V = 120$V at $t = 5$ sec

$\Rightarrow$ $120 = 200\,((1 - e^{-5/R \times 2 \times 10^{-6}})$

$\Rightarrow$ $R \approx 2.7 \times 10^6\ \Omega$

21. A Carnot engine operating between temperatures T_1 and T_2 has efficiency (1 / 6). When T_2 is lowered by 62 K, its efficiency increases to (1 / 3). Then T_1 and T_2 are, respectively :

(1) 372 K and 330 K (2) 330 K and 268 K

(3) 310 K and 248 K (4) 372 K and 310 K

21. **(4)** $T_1 > T_2$

$\eta = [1 - (T_2 / T_1)]$ $\Rightarrow$ $(5 / 6) = (T_2 / T_1)$

Also $(2 / 3) = [T_2 / 62) / T_1]$ $\Rightarrow$ $2T_2 \,.\, (6 / 5) = 3T_2 \,.\, 186$

$\Rightarrow$ $186 = [\,3 - (12 / 5)\,T_2]$ $\Rightarrow$ $[(5 \times 186) / 3\,] = T_2$

$\Rightarrow$ $T_2 = 310$ K

22. If a wire is stretched to make it 0.1% longer, its resistance will :

(1) increase by 0.2 % (2) decrease by 0.2 %

(3) decrease by 0.05 % (4) increase by 0.05 %

22. **(1)** $R = (\rho L / A)$, Volume $= LA =$ constant $\Rightarrow$ $\Delta L / L = (-\Delta A / A)$

$\Rightarrow$ $(\Delta R / R) = (\Delta L / L) - (\Delta A / A)] = 2(\Delta L / L) = 0.2\%$

23. **Direction :**

The question has paragraph followed by two statements, **Statement - 1** and **Statement - 2**. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement - 1 : When light reflects from the air – glass plate interface, the reflected wave suffers a phase change of π.

Statement - 2 : The centre of the interference pattern is dark.

(1) Statement -1 is true, Statement - 2 is true and Statement - 2 is the correct explanation of statement -1
(2) Statement -1 is true, Statement - 2 is true and Statement - 2 is not the correct explanation of Statement - 1
(3) Statement - 1 is false, Statement - 2 is true
(4) Statement - 1 is true, Statement-2 is false

23. **(1)**

24. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 ms^{-1}. The speed of the image of the second car as seen in the mirror of the first one is :

(1) $\frac{1}{15}$ ms^{-1} (2) 10 ms^{-1} (3) 15 ms^{-1} (4) $\frac{1}{10}$ ms^{-1}

24. **(1)** $[(1 / v) + (1 / u)] = (1 / f)$
$(-1 / v^2)\, v_{im} + (-1 / u^2) v_{0m} = 0$
$\Rightarrow \quad v_{im} = -(v^2 / u^2) v_{0m}$
$v_{im} = -(v / u)^2 v_{0m}$
$[(1 / v) + (1 / -280)] = (1 / 20)$
$-(1 / v) = [(1 / 20) + (1 / 280)] = (15 / 280)$
$v = (280 / 15)$
$\Rightarrow \quad v_{im} = -[280 / (15 \times 280)]^2\, v_{0m} = -(15 / 225) = -(1 / 15)\ ms^{-1}$

25. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is :

(1) 36.3 *eV* (2) 108.8 *eV* (3) 122.4 *eV* (4) 12.1 *eV*

25. **(2)** $E_n = (-13.6\, z^2 / n^2) eV$
$\Rightarrow \quad \Delta E = [+13.6 \times 9 \times (1 - (1 / 9))]$

$= + 13.6 \times 8 = 108.8\ eV$

26. The electrostatic potential inside a charged spherical ball is given by $\phi = a r^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is :

(1) $-6a\varepsilon_0 r$ (2) $-24\pi\, a\varepsilon_0$ (3) $-6\, a\varepsilon_0$ (4) $-24\pi\, a\varepsilon_0 r$

26. **(3)** $\phi = ar^2 + b \qquad E = -(d\phi / dr) = -(d(ar^2 + b) / dr) = -2ar$
flux $= (-2ar).\, 4\pi r^2 = (q / \varepsilon_0) \qquad q - 8\pi a r^3\, \varepsilon_0$
$\rho = -q / [(4 / 3)\pi r^3] = -6\, a\varepsilon_0$

27. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :

(1) $\pi \frac{v^4}{g^2}$ (2) $\frac{\pi}{2} \frac{v^4}{g^2}$ (3) $\pi \frac{v^2}{g^2}$ (4) $\pi \frac{v^2}{g}$

27. **(1)** area = πR^2_{max} when R_{max} = maximum horizontal range of water drops
$\Rightarrow$ area = $\pi [V^2 \sin 90° / g]^2 = [\pi V^4 / g^2]$

28. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J / kg / K) :
(1) 8.4 kJ (2) 84 kJ (3) 2.1 kJ (4) 4.2 kJ

28. **(1)** $\Delta Q = \Delta U + \Delta W$ $\Delta W = 0$ (expansion is ignored)
$\Rightarrow \Delta U = \Delta Q = m.S.\Delta\theta = (100 \times 10^{-3}$ Kg) (4184) (50 – 30)
= 418.4 × 20 = 8368 J = 8.4 KJ

29. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when (2 / 3) of it has decayed and time t_1 when (1 / 3) of it had decayed is :
(1) 14 min (2) 20 min (3) 28 min (4) 7 min

29. **(2)** $t_2 - t_1 = (2.303 / \lambda) \log_{10} [(2N / 3) / (N / 3)] = [(2.303 \times 20) / 0.693] \log_{10}(2)$
=20 minutes

30. This question has **Statement - 1** and **Statement - 2**. Of the four choices given after statements, choose the one that best describes the two statements.

Statement - 1 : A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement - 2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement - 1 is true, Statement -2 is true, Statement -2 is the correct explanation of Statement -1
(2) Satement - 1 is true, Statement - 2 is true, Statement - 2 is not the correct explanation of Statement - 1
(3) Statement - 1 is false, Statement - 2 is true
(4) Statement - 1 is true, Statement - 2 is false.

30. **(3)** $h\nu = h\nu_0 + K_{max}$ ...(1)
$h(2\nu) = h\nu_0 + K'_{max}$ ...(2)
where K_{max} is Kinetic energy after frequency is doubled
$\Rightarrow K'_{max} = h\nu_0 + 2K_{max}$ [using (1) & (2)]
$\Rightarrow K'_{max} > 2K_{max}$ $\Rightarrow ev'_0 > 2(ev_0)$
where v_0' is stopping potential after frequency is doubled
$\Rightarrow v'_0 > 2v_0$ $\Rightarrow$ Statement (1) is false
statement (2) is true because $K_{max} = h\nu - h\nu_0$ (linear equation)