

MATHEMATICS

1. The sum to infinity of the progression $9 - 3 + 1 - \frac{1}{3} + \dots$ is
 - 1) 9
 - 2) $\frac{9}{2}$
 - 3) $\frac{27}{4}$
 - 4) $\frac{15}{2}$

2. If ${}^n C_{12} = {}^n C_6$ then ${}^n C_2 = \dots$
 - 1) 72
 - 2) 153
 - 3) 306
 - 4) 2556

3. The middle term in the expansion of $\left(x - \frac{1}{x}\right)^{18}$ is

 - 1) ${}^{18} C_9$
 - 2) $-{}^{18} C_9$
 - 3) ${}^{18} C_{10}$
 - 4) $-{}^{18} C_{10}$

4. If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

 - 1) $-\frac{15}{4}$
 - 2) $\frac{15}{4}$
 - 3) $\frac{9}{4}$
 - 4) 4

5. The digit in the units place in the number 7^{289} is

 - 1) 9
 - 2) 7
 - 3) 1
 - 4) 3

(Space for Rough Work)

6. When 2^{301} is divided by 5, the least positive remainder is

- 1) 4
- 2) 8
- 3) 2
- 4) 6

7. The contrapositive of "If two triangles are identical, then these are similar" is

- 1) If two triangles are not similar then these are not identical.
- 2) If two triangles are not identical then these are not similar.
- 3) If two triangles are not identical then these are similar.
- 4) If two triangles are not similar then these are identical.

8. The contrapositive of the inverse of $p \rightarrow \sim q$ is

- 1) $\sim q \rightarrow p$
- 2) $p \rightarrow q$
- 3) $\sim q \rightarrow \sim p$
- 4) $\sim p \rightarrow \sim q$

9. The converse of the contrapositive of $p \rightarrow q$ is

- 1) $\sim p \rightarrow q$
- 2) $p \rightarrow \sim q$
- 3) $\sim p \rightarrow \sim q$
- 4) $\sim q \rightarrow p$

10. If ω is a complex cube-root of unity then, $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is equal to

- 1) -1
- 2) 1
- 3) 0
- 4) ω

(Space for Rough Work)

11. The solutions of the equation $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ are

- 1) 3, -1
- 2) -3, 1
- 3) 3, 1
- 4) -3, -1

12. If $A = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$ then, $|AB|$ is equal to

- 1) 80
- 2) 100
- 3) -110
- 4) 92

13. The inverse of the matrix $\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$ is

- 1) $\frac{1}{11} \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$
- 2) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$
- 3) $\frac{1}{13} \begin{bmatrix} -2 & 5 \\ 1 & 3 \end{bmatrix}$
- 4) $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

14. The projection of the vector $2\hat{i} + \hat{j} - 3\hat{k}$ on the vector $\hat{i} - 2\hat{j} + \hat{k}$ is

- 1) $-\frac{3}{\sqrt{14}}$
- 2) $\frac{3}{\sqrt{14}}$
- 3) $-\sqrt{\frac{3}{2}}$
- 4) $\frac{3}{\sqrt{2}}$

15. A unit vector perpendicular to the plane containing the vectors $\hat{i} - \hat{j} + \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ is

- 1) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$
- 2) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$
- 3) $\frac{\hat{j} - \hat{k}}{\sqrt{2}}$
- 4) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

(Space for Rough Work)

16. If \hat{a} , \hat{b} and \hat{c} are mutually perpendicular unit vectors, then $|\hat{a} + \hat{b} + \hat{c}|$ is equal to

- 1) 3
- 2) $\sqrt{3}$
- 3) $\sqrt{a^2 + b^2 + c^2}/3$
- 4) 1

17. The identity element in the group $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} \mid x \in R, x \neq 0 \right\}$ with respect to matrix multiplication is

- 1) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
- 2) $\frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
- 3) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- 4) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

18. In the group $G = \{1, 3, 7, 9\}$ under multiplication modulo 10, the inverse of 3 is

- 1) 1
- 2) 3
- 3) 7
- 4) 9

19. In the group $\{0, 1, 2, 4, 5\}$ under addition modulo 6 a subgroup is

- 1) $\{0, 2, 5\}$
- 2) $\{1, 4, 5\}$
- 3) $\{0, 1, 3\}$
- 4) $\{0, 2, 4\}$

20. In the group $(Q^+, *)$ of positive rational numbers w.r.t. the binary operation $*$ defined

$a * b = \frac{ab}{3} \forall a, b \in Q^+$ the solution of the equation $5 * x = 4^{-1}$ in Q^+ is

- 1) $\frac{27}{20}$
- 2) $\frac{20}{27}$
- 3) $\frac{1}{20}$
- 4) 20

(Space for Rough Work)

21. $(0, -1)$ and $(0, 3)$ are two opposite vertices of a square. The other two vertices are
- 1) $(0, 1), (0, -3)$ 2) $(3, -1), (0, 0)$
3) $(2, 1), (-2, 1)$ 4) $(2, 2), (1, 1)$
22. The equation to the line bisecting the join of $(3, -4)$ and $(5, 2)$ and having its intercepts on the x -axis and the y -axis in the ratio $2 : 1$ is
- 1) $x + y - 3 = 0$ 2) $2x - y = 9$
3) $x + 2y = 2$ 4) $2x + y = 7$
23. The distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is
- 1) $2\sqrt{5}a$ 2) $\sqrt{10}a$
3) $10a$ 4) $5\sqrt{2}a$
24. The equation to the circle with centre $(2, 1)$ and touching the line $3x + 4y = 5$ is
- 1) $x^2 + y^2 - 4x - 2y + 5 = 0$ 2) $x^2 + y^2 - 4x - 2y - 5 = 0$
3) $x^2 + y^2 - 4x - 2y + 4 = 0$ 4) $x^2 + y^2 - 4x - 2y - 4 = 0$
25. The condition for a line $y = 2x + c$ to touch the circle $x^2 + y^2 = 16$ is
- 1) $c = 10$ 2) $c^2 = 80$
3) $c = 12$ 4) $c^2 = 64$
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(Space for Rough Work)

26. The two circles $x^2 + y^2 - 2x + 22y + 5 = 0$ and $x^2 + y^2 + 14x + 6y + k = 0$ intersect orthogonally provided k is equal to

- 1) 47
- 2) -47
- 3) 49
- 4) -49

27. The radius of the circle $x^2 + y^2 + 4x + 6y + 13 = 0$ is

- 1) $\sqrt{26}$
- 2) $\sqrt{13}$
- 3) $\sqrt{23}$
- 4) 0

28. The centre of the circle $x = 2 + 3 \cos \theta$, $y = 3 \sin \theta - 1$ is

- 1) (3, 3)
- 2) (2, -1)
- 3) (-2, 1)
- 4) (-1, 2)

29. The sum of the focal distances of any point on the conic $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

- 1) 10
- 2) 9
- 3) 41
- 4) 18

30. The eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$ is

- 1) $\frac{3}{4}$
- 2) $\frac{3}{5}$
- 3) $\frac{\sqrt{41}}{4}$
- 4) $\frac{\sqrt{41}}{5}$

(Space for Rough Work)

31. The ends of the latus-rectum of the conic $x^2 + 10x - 16y + 25 = 0$ are

- 1) (3, -4), (13, 4)
- 2) (-3, -4), (13, -4)
- 3) (3, 4), (-13, 4)
- 4) (5, -8), (-5, 8)

32. The equation to the hyperbola having its eccentricity 2 and the distance between its foci 8 is

- 1) $\frac{x^2}{12} - \frac{y^2}{4} = 1$
- 2) $\frac{x^2}{4} - \frac{y^2}{12} = 1$
- 3) $\frac{x^2}{8} - \frac{y^2}{2} = 1$
- 4) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

33. The solution of $\text{Sin}^{-1} x - \text{Sin}^{-1} 2x = \mp \frac{\pi}{3}$ is

- 1) $\pm \frac{1}{3}$
- 2) $\pm \frac{1}{4}$
- 3) $\pm \frac{\sqrt{3}}{2}$
- 4) $\pm \frac{1}{2}$

34. In a ΔABC if the sides are $a = 3, b = 5$ and $c = 4$, then $\text{Sin} \frac{B}{2} + \text{Cos} \frac{B}{2}$ is equal to

- 1) $\sqrt{2}$
- 2) $\frac{\sqrt{3} + 1}{2}$
- 3) $\frac{\sqrt{3} - 1}{2}$
- 4) 1

35. The value of $\text{Cos} (270^\circ + \theta) \text{Cos} (90^\circ - \theta) - \text{Sin} (270^\circ - \theta) \text{Cos} \theta$ is

- 1) 0
- 2) -1
- 3) $\frac{1}{2}$
- 4) 1

(Space for Rough Work)

36. If $12 \cot^2 \theta - 31 \operatorname{Cosec} \theta + 32 = 0$, then the value of $\sin \theta$ is

1) $\frac{3}{5}$ or 1

2) $\frac{2}{3}$ or $-\frac{2}{3}$

3) $\frac{4}{5}$ or $\frac{3}{4}$

4) $\pm \frac{1}{2}$

37. The circum-radius of the triangle whose sides are 13, 12 and 5 is

1) 15

2) $\frac{13}{2}$

3) $\frac{15}{2}$

4) 6

38. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ then

1) $x + y + xy = 1$

2) $x + y - xy = 1$

3) $x + y + xy + 1 = 0$

4) $x + y - xy + 1 = 0$

39. The general solution of $\sin x - \cos x = \sqrt{2}$, for any integer n is

1) $n\pi$

2) $2n\pi + \frac{3\pi}{4}$

3) $2n\pi$

4) $(2n + 1)\pi$

40. The amplitude of $\frac{1 + i\sqrt{3}}{\sqrt{3} + i}$ is

1) $\frac{\pi}{3}$

2) $\frac{\pi}{4}$

3) $\frac{2\pi}{3}$

4) $\frac{\pi}{6}$

(Space for Rough Work)

41. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are

- 1) $\sqrt{2}$ and $\frac{\pi}{6}$
- 2) 1 and 0
- 3) 1 and $\frac{\pi}{3}$
- 4) 1 and $\frac{\pi}{4}$

42. The real part of $\frac{1}{1+\cos\theta+i\sin\theta}$ is

- 1) $-\frac{1}{2}$
- 2) $\frac{1}{2}$
- 3) $\sqrt{2}$
- 4) $\frac{1}{\sqrt{2}}$

43. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ is equal to

- 1) $\frac{1}{2}$
- 2) $-\frac{1}{2}$
- 3) 0
- 4) 1

44. If $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ then $\frac{dy}{dx}$ is equal to

- 1) $\text{Sech}^2 x$
- 2) $\text{Cosech}^2 x$
- 3) $-\text{Sech}^2 x$
- 4) $-\text{Cosech}^2 x$

45. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2 + 2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$,

then the value of k is

- 1) 1
- 2) -2
- 3) 2
- 4) $\frac{1}{2}$

(Space for Rough Work)

46. If $y = \text{Tan}^{-1} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}$, then $\frac{dy}{dx}$ is equal to

1) $\frac{x^2}{\sqrt{1-x^4}}$

2) $\frac{x^2}{\sqrt{1+x^4}}$

3) $\frac{x}{\sqrt{1+x^4}}$

4) $\frac{x}{\sqrt{1-x^4}}$

47. If $x = \text{Sin } t, y = \text{Cos } pt$, then

1) $(1-x^2)y_2 + xy_1 + p^2y = 0$

2) $(1-x^2)y_2 + xy_1 - p^2y = 0$

3) $(1+x^2)y_2 - xy_1 + p^2y = 0$

4) $(1-x^2)y_2 - xy_1 + p^2y = 0$

48. If ST and SN are the lengths of the subtangent and the subnormal at the point $\theta = \frac{\pi}{2}$ on the curve $x = a(\theta + \text{Sin } \theta), y = a(1 - \text{Cos } \theta), a \neq 1$, then

1) $ST = SN$

2) $ST = 2SN$

3) $ST^2 = aSN^3$

4) $ST^3 = aSN$

49. If θ is the acute angle of intersection at a real point of intersection of the circle $x^2 + y^2 = 5$ and the parabola $y^2 = 4x$ then $\text{Tan } \theta$ is equal to

1) 1

2) $\sqrt{3}$

3) 3

4) $\frac{1}{\sqrt{3}}$

50. A spherical balloon is being inflated at the rate of 35 cc/min. The rate of increase of the surface area of the balloon when its diameter is 14 cm is

1) 7 Sq.cm/min

2) 10 Sq.cm/min

3) 17.5 Sq.cm/min

4) 28 Sq.cm/min

(Space for Rough Work)

51. $\int \frac{\sin(2x) dx}{1 + \cos^2 x} =$

- 1) $-\frac{1}{2} \text{Log}(1 + \cos^2 x) + C$
- 2) $2 \text{Log}(1 + \cos^2 x) + C$
- 3) $\frac{1}{2} \text{Log}(1 + \cos 2x) + C$
- 4) $C - \text{Log}(1 + \cos^2 x)$

52. $\int \frac{e^x (1 + \sin x)}{1 + \cos x} dx =$

- 1) $e^x \text{Tan}\left(\frac{x}{2}\right) + C$
- 2) $e^x \text{Tan } x + C$
- 3) $e^x \left(\frac{1 + \sin x}{1 - \cos x}\right) + C$
- 4) $C - e^x \text{Cot}\left(\frac{x}{2}\right)$

53. $\int \frac{1 + \tan x}{e^{-x} \cos x} dx = \dots\dots\dots$

- 1) $e^{-x} \text{Tan } x + C$
- 2) $e^{-x} \text{Sec } x + C$
- 3) $e^x \text{Sec } x + C$
- 4) $e^x \text{Tan } x + C$

54. $\int_{\pi/4}^{\pi/2} \text{Cosec}^2 x dx = \dots\dots\dots$

- 1) -1
- 2) 1
- 3) 0
- 4) $\frac{1}{2}$

55. $\int_0^{\pi/4} \text{Log}(1 + \tan x) dx =$

- 1) $\frac{\pi}{8} \log_e 2$
- 2) $\frac{\pi}{4} \log_2 e$
- 3) $\frac{\pi}{4} \log_e 2$
- 4) $\frac{\pi}{8} \log_e \left(\frac{1}{2}\right)$

(Space for Rough Work)

56. The area bounded by the parabola $y^2 = 4ax$ and the line $x = a$ and $x = 4a$ is

1) $\frac{35a^2}{3}$

2) $\frac{4a^2}{3}$

3) $\frac{7a^2}{3}$

4) $\frac{28a^2}{3}$

57. A population $p(t)$ of 1000 bacteria introduced into nutrient medium grows according to the

relation $p(t) = 1000 + \frac{1000t}{100 + t^2}$. The maximum size of this bacterial population is

1) 1100

2) 1250

3) 1050

4) 5250

58. The differential equation representing a family of circles touching the y -axis at the origin is

1) $x^2 + y^2 - 2xy \frac{dy}{dx} = 0$

2) $x^2 + y^2 + 2xy \frac{dy}{dx} = 0$

3) $x^2 - y^2 - 2xy \frac{dy}{dx} = 0$

4) $x^2 - y^2 + 2xy \frac{dy}{dx} = 0$

59. The area of the region bounded by the curve $9x^2 + 4y^2 - 36 = 0$ is

1) 9π

2) 4π

3) 36π

4) 6π

60. The general solution of the differential equation $(2x - y + 1)dx + (2y - x + 1)dy = 0$ is

1) $x^2 + y^2 + xy - x + y = C$

2) $x^2 + y^2 - xy + x + y = C$

3) $x^2 - y^2 + 2xy - x + y = C$

4) $x^2 - y^2 - 2xy + x - y = C$

(Space for Rough Work)