B3.2-R3: BASIC MATHEMATICS

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Express

$$z = \frac{(\cos\theta + i\sin\theta)^3(\sin\theta + i\cos\theta)}{(\cos 2\theta - i\sin 2\theta)}$$

in the form a+ib, where a and b are real numbers

b) Find whether the vectors (1, 2, 3), (3, 4, 5) and (5, 6, 7) are linearly dependent or linearly independent.

c) If $y=x^x$, find the value of

$$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}$$

d) Discuss the continuity of the function f(x) at $x = 0$, where

$f(x) = \frac{|x|}{x}$, $x \neq 0$

$= 1$, $x = 0$.

e) Evaluate the integral $\int \frac{dx}{x(1 - x^{1/4})}$

f) For finding the shape of the hyperbola, trace the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

g) Find $\lim_{x \to 0} \frac{4^x - 5^x}{x}$

(7x4)

2.

a) Find the value of the determinant $\begin{vmatrix} 2 & \omega & \omega^2 \\ \omega & \omega^2 & 2 \\ \omega^2 & 2 & \omega \end{vmatrix}$

where ω is the cube root of unity.

b) If any three successive coefficients in the expansion of $(1 + x)^n$ are 36, 84 and 126, find n.

c) Find the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 2 & -1 \\ -1 & -2 & 6 \end{bmatrix}$.

(6+6+6)

3.

a) Find $\lim_{x \to 2} \left[\frac{x - 1}{x - 2} - \frac{1}{\ln(x - 1)} \right]$

b) Evaluate the definite integral $\int_0^{\pi/2} \left[\sqrt{\tan x} + \sqrt{\cot x}\right] dx$

c) Discuss the convergence of the infinite series $\sum_{n=1}^{\infty} \frac{3n}{4(n+1)}$

(6+6+6)

4.

a) Obtain the Taylor polynomial approximation of degree 4 to the function f(x) = x sinx, about the point x = 0.

b) If $f(x) = \begin{cases} x+2 & x<1 \\ 4x-1 & 1 \le x \le 3 \\ x^2+5 & x>3 \end{cases}$, examine whether $\lim_{x \to 1} f(x)$ and $\lim_{x \to 3} f(x)$ exist:

c) Let g(x) and h(x) be two differentiable functions such that h'(x) = -g(x) and g'(x) = h(x) for all x, $\alpha \le x \le \beta$. If $f(x) = [g(x)]^2 + [h(x)]^2$, then find f(α) given that f(β) = 2.

(6+6+6)

5.

a) Find all the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$.

b) Find the equation of the straight line passing through the point (-2, -3) and inclined at 60^0 to the line $x + \sqrt{3}\, y = 2$.

c) Find the equations of the tangents to the ellipse $x^2 + 9y^2 - 6x + 18y - 18 = 0$ which are parallel to the line x + y + 1 = 0.

(6+6+6)

6.

a) Find the equation of the normal at a point on the parabola $y^2 = 2x$, whose ordinate is 4.

b) Determine whether the following system of equations have a solution. If it has, then find the solution.

$$2x_1 + 3x_2 - 5x_3 = -1$$
$$6x_1 - x_2 + 3x_3 = 5$$
$$10x_1 - 5x_2 + 11x_3 = 11$$

c) Find the value of *a* so that the area bounded by the curves $y = x^2 - a^2$ and $y = a^2 - x^2$ is 72 units.

(6+6+6)

7.

a) Students in a college prepare a poster for display. The poster is to contain 200 square inches of printed matter, with margins of 4 inches at top and bottom, and of 2 inches on each side. Find the dimensions of the poster if the total area is to be minimum.

b) Let $\vec{a}$ = i + j – 2k, $\vec{b}$ = 2i + j + k and $\vec{c}$ = 4i + j – 2k. Find a vector $\vec{d}$ such that $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d}.\vec{a} = 0$.

c) How will you find out if the point (αβ) is outside, on or inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?

Hence, show that the triangle whose vertices are (1, 2), (3, -1) and (-2, 1) lies wholly inside the ellipse $x^2+2y^2=13$.

(6+6+6)