B3.2-R3: BASIC MATHEMATICS

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$, find $C = A^T B^T$ and its determinant.

b) For what values of p are

$\vec{A} = p\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2p\hat{i} + p\hat{j} - 4\hat{k}$ perpendicular?

c) Determine the square root of $(3 + 4i)$, $i = \sqrt{-1}$.

d) Using 'Ratio Test' for the convergence of an infinite series, examine the convergence of the series

$$\frac{1}{2} + \frac{1}{2.2^2} + \frac{1}{3.2^3} + \frac{1}{4.2^4} + ...$$

e) Evaluate $\int_{-1}^{2} (x + x^2)dx.$

f) Test the differentiability of the function

$$f(x) = \begin{cases} x^2 & , & x \leq 1 \\ \sqrt{x} & , & x > 1 \end{cases}$$

at x=1.

g) Find the center and radius of the circle
$2x^2 + 2y^2 + 5x + 7y - 3 = 0$

(7x4)

2.

a) Reduce the matrix

$$\begin{pmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{pmatrix}$$

to normal form and hence find its rank.

b) Examine the consistency of the following system of equations:

$2x - 5y + 7z = 6$

$3x - 8y + 11z = 11$

$x - 3y + 4z = 3.$

c) Find the asymptotes of the curve

$y^3 + x^2y + 2xy^2 - y + 1 = 0$

(6+6+6)

3.

a) Find $\dfrac{dy}{dx}$, if $x^y \cdot y^x = k$, where k is constant.

b) Find the limit

$$\lim_{x \to 0} (e^x + x)^{1/x}$$

c) Find the maximum possible area of a right angled triangle whose hypotenuse is 5 cm long.

(6+6+6)

4.

a) Is the following series convergent?

$$\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8} + \dots$$

b) Evaluate the integral

$$\int_0^4 |x-2| \, dx$$

c) Put the equation
$$4x^2 + 9y^2 = 36$$
in the standard form and sketch it. Indicate its center, vertex and foci.

(6+6+6)

5.

a) If A = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, then verify that

$A^2 - 7A - 2I = 0$. Also find A^{-1}.

b) Find the value of the determinant

$$D = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}.$$

Compute $\dfrac{dD}{dx} \Big|_{x=1}$.

c) Evaluate

$$\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} \, dx$$

(6+6+6)

6.

a) Let α and β be the complex roots of unity, find the value of
$(1 - α)(1 − β)(1 − β^2)(1 − β^4)$

b) Find the projection of the vector

$2\hat{i} + \hat{j} + 6\hat{k}$ on the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

c) Find the equations of tangent and normal to the curve
$y(x − 2)(x - 3) − x + 7 = 0$
at the point where the curve meets the x-axis.

(6+6+6)

7.

a) Show that (log x) /x has a maximum value at x = e. Compute the maximum value.

b) If 2θ is the angle between two unit vectors $\vec{a}$ and $\vec{b}$, then show that $\left|\vec{a} − \vec{b}\right|$ = 2sinθ.

c) Representing the area by integral, find the area of the region bounded by the lines x = 4, y = 1 and x + 4y = 4.

(6+6+6)